Cosmic Bubble Collisions

Outline

- Background
  - Expanding Universe: Einstein's Eqn with FRW metric
  - Inflationary Cosmology: model with scalar field
  - QFT→Bubble nucleation→Bubble collisions
- Bubble Collisions in Single Field Theory
  - Results: Classical Tunneling via "Free Passage"
- Concerns:
  - dissipiation via interactions with other fields
    - Include interaction with additional scalar field
    - Account for gravity
  - Revisit validity of FP itself for single field case
- Methods
  - Purely classical
    - Analytic
    - Numerical
  - Semiclassical

Expanding Universe

- Special relativity
  - correct definition of distance in flat spacetime is $ds^2 = dt^2 - dx^2 - dy^2 - dz^2$
  - index notation: $ds^2 = g_{\mu\nu} dx^\mu dx^\nu$ (repeated indices summed over)
  - here the metric, $g_{\mu\nu}$, is the 4x4 matrix: $diag(1, -1, -1, -1)$
- Universe is **homogeneous**, **isotropic**, and **expanding** on large scales → use FRW metric: $g_{\mu\nu} = diag(1, -a^2(t), -a^2(t), -a^2(t))$

Increasing a(t) means expansion. How does it evolve?

- GR: Einstein's equation relates curvature of spacetime to mass/energy content of spacetime

$$R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = 8\pi G T_{\mu\nu}$$

Combinations of components and derivs of components of metric

Cosmological constant

Stress energ tensor

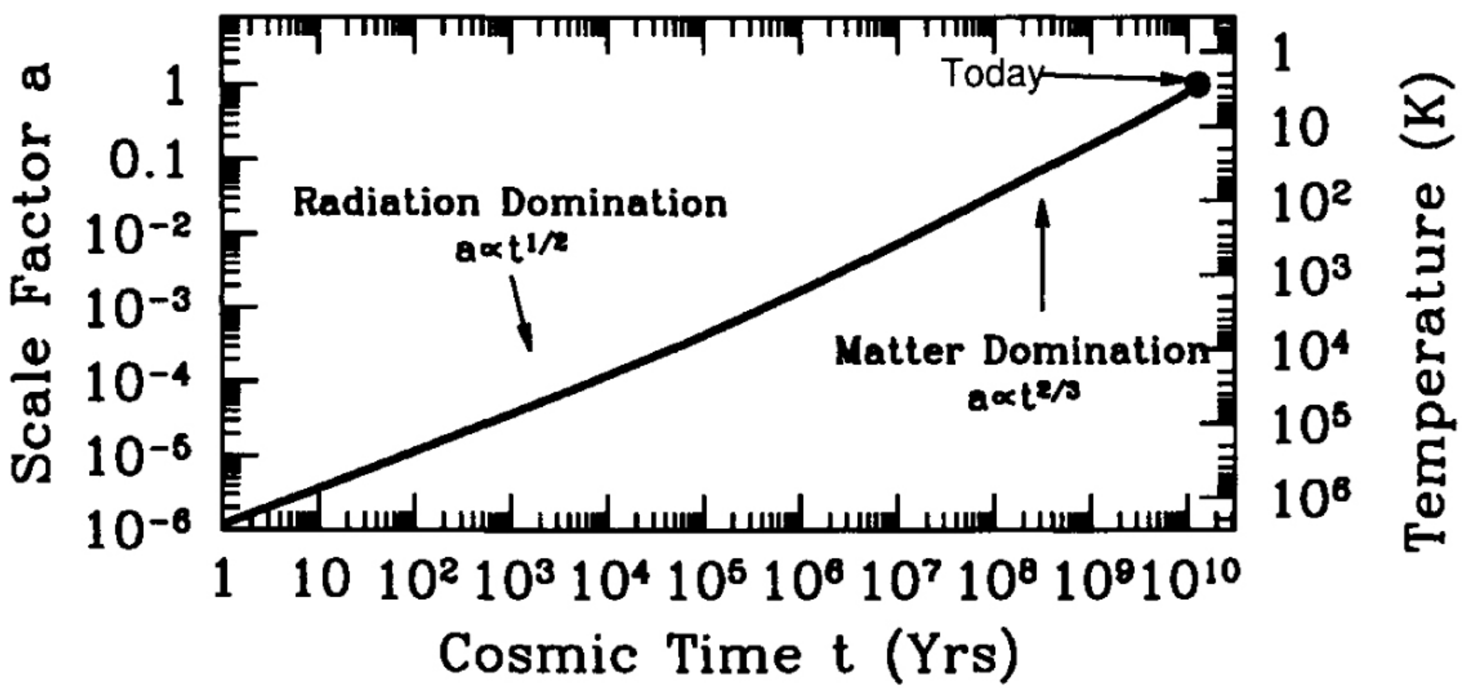
- For FRW: $$\boxed{\left(\frac{\dot{a}}{a}\right)^2 = \frac{8\pi G\rho}{3} + \frac{\Lambda}{3} - \frac{k}{a^2}}$$

Energy density

spatial curvature=0

$$H = \dot{a}/a$$

- Note: Λ equivalent to a constant energy density (does not dilute as universe expands)
- Universe with only Λ has **$a \propto e^{Ht}$** , other types?
  - ordinary (non-rel) matter: $\rho \propto a^{-3}$ → $a \propto t^{2/3}$
  - radiation: $\rho \propto a^{-4}$ → $a \propto t^{1/2}$
  - Only since recently has (today's) observed Λ been dominant form of energy density (~70%) in universe. Λ is nearly zero, but dominates since matter and radiation have become so diluted.



Horizon Problem/Inflation

- If we assume only radiation domination followed by matter domination we find that CMB photons originated from region not causally connected
- CMB photons arriving from every direction fit same BB curve, so have same temp to one part in 10^5

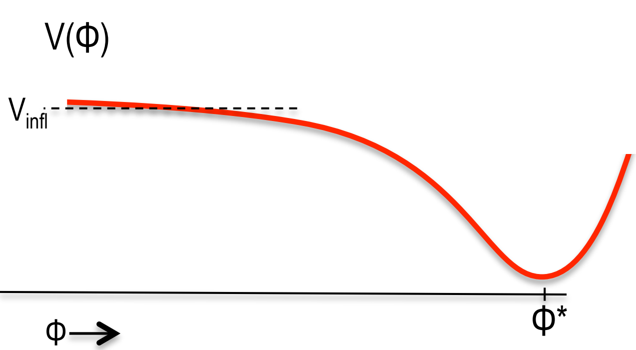
Solution: INFLATION

Short period of very **rapid exponential** expansion, i.e. large, positive before radiation domination. As inflation ends $\Lambda \rightarrow \Lambda_{today}$=small

- occurred at ~10^{-32}-10^{-33} s after big bang, lasted ~10^{-36} s
- a(t) increased by ~10^{27}, (vol increased by factor of 10^{78})

How should we model inflation?

- A scalar field, Φ(x), has energy density: $\rho = \frac{1}{2}\dot{\phi}^2 + \frac{1}{2}|\vec{\nabla}\phi|^2 + V(\phi)$
- Model inflation with homogeneous scalar field so is zero. We call this field the inflaton field. If dΦ/dt can be ignored rel to V(Φ), then inflaton field's ρ enters Friedmann eqn as V(Φ).

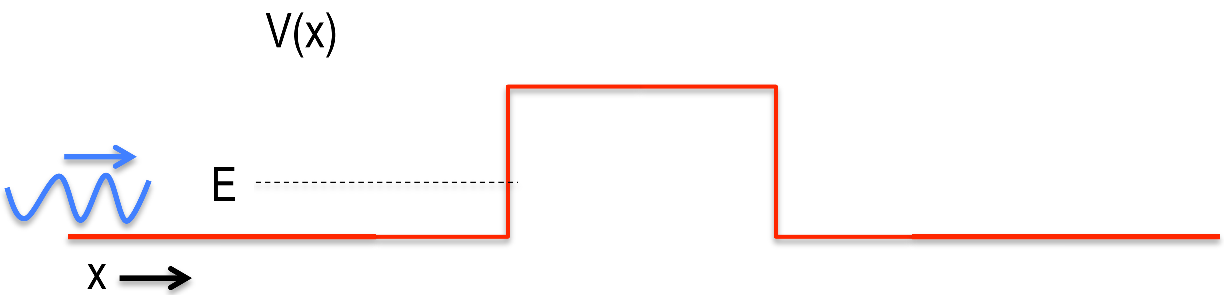


Slow-roll inflation:

1. dΦ/dt starts off small, V has small negative slope here, phi rolls down flat part of potential slowly (dΦ/dt stays small) until here so $\rho \approx V(\Phi) \approx V_{infl}$.
2. As Φ rolls down steeper part of potential dΦ/dt becomes significant, V(Φ) is decreasing from V_{infl} (inflation is ending). Interactions between Φ and other fields yield particle production in other fields as rolls into potential well and settles into Φ* (there is damping due to this particle production and hubble damping term in Φ field equation
3. V(Φ*)~0 corresponds to the small Λ we observe today

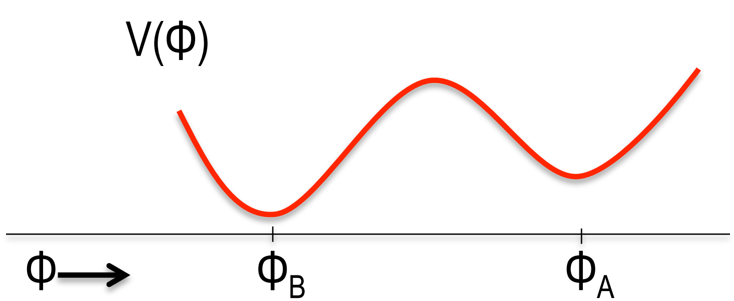
Quantum Effects

- The inflaton field is a quantum field
- Tunneling:
  - Regular Quantum Mechanics:



    - For IC corresponding to particle coming in from left with energy E, not all of the wavefunction will be reflected off the barrier. Nonzero transmission coeff (nonzero probability of finding particle to right of barrier).
  - Similarly a quantum field can develop regions that fluctuate into configurations outside of the basin of attraction field begins in

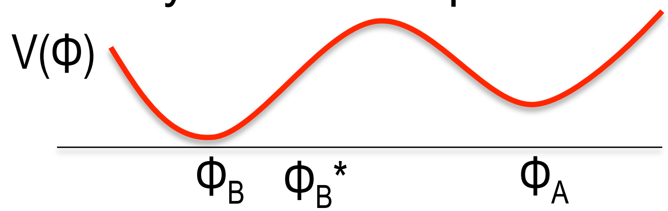
Consider a potential V(Φ) that looks like:



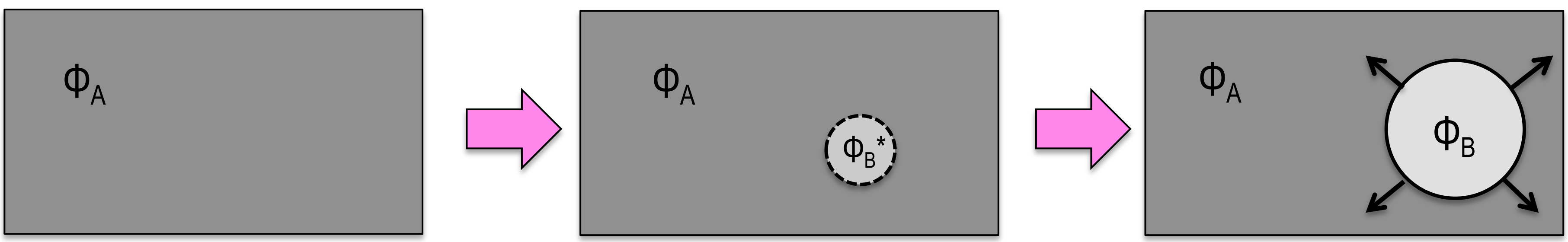
Bubble Nucleation

- Scalar field Lagrangian: $\mathcal{L}(\phi) = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi)$
- In Minkowski space variation of action yields this eqn of motion:

$$\boxed{g^{\mu\nu} \partial_\mu \partial_\nu \phi = -\frac{\partial V(\phi)}{\partial \phi}}$$



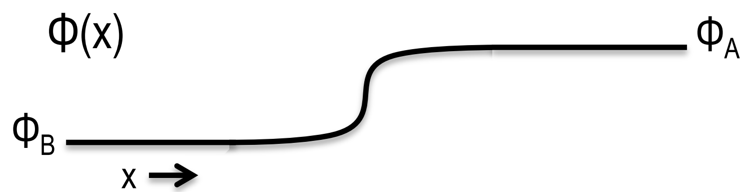
- Homogeneous classical field initially in Φ_A (with $d\Phi/dt=0$) stays in Φ_A
  - For FRW metric with $da/dt \neq 0$ the field eqn has a damping term nonzero, so Φ_A is stable attractor
- Quantum field initially homogeneous and equal to Φ_A develops small regions, or **bubbles**, in which field configuration fluctuates away from Φ_A
- Some fluctuations will be in basin of attraction of Φ_B, such as Φ_{B^*}



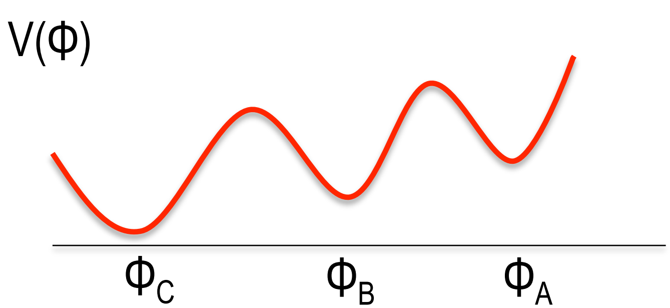
- $\Phi_B{}^*$ is far enough inside basin of Φ_B, the field inside bubble quickly evolves into Φ_B and this bubble "expands" (in sense that bubble walls move out, encompassing more and more space originally in Φ_A, now in Φ_B. This is because $V(\Phi_B)<V(\Phi_A)$, so outward pressure gradient
- Note: fluctuations into field configurations Φ' with $V(\Phi')>V(\Phi_A)$ collapse so we don't care about them

Bubble collisions

- Note: region separating inside of bubble (in Φ_B) and surrounding "sea" (in Φ_A) is finite. Bubble walls accelerate as move out, thin walls move near speed of light.
  - Field configuration in a thin wall can be solved for using "relaxation". schematically (in 1 spatial dim) looks like:



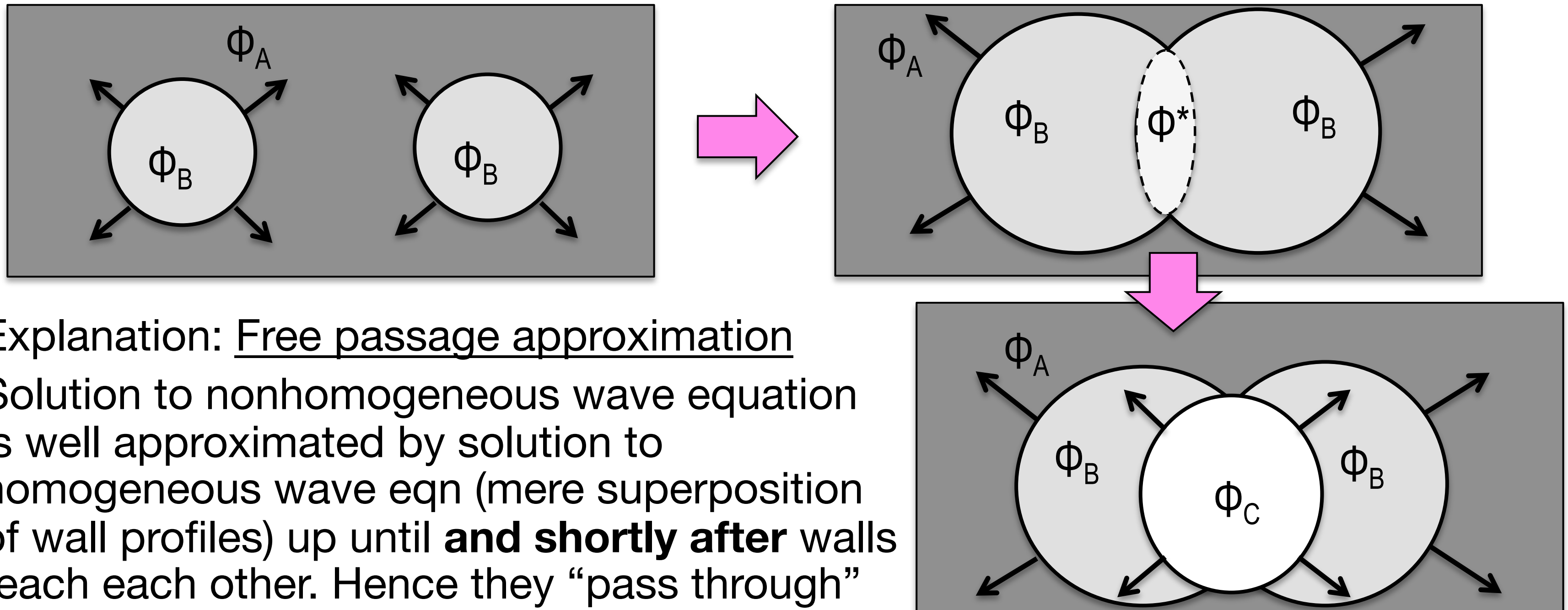
- Presumably these bubbles are nucleated all over.. So bubble collisions inevitably occur. What then?
  - Numerical results for single scalar field theory found **classical tunneling** could occur. Consider a potential with 3 local minima:



initial conditions corresponding to two expanding phiB bubbles in surrounding phiA are evolved numerically according to field eqn:

$$\boxed{g^{\mu\nu}\partial_\mu\partial_\nu\phi = -\frac{\partial V(\phi)}{\partial\phi}}$$

Bubble collisions



Explanation: <u>Free passage approximation</u>

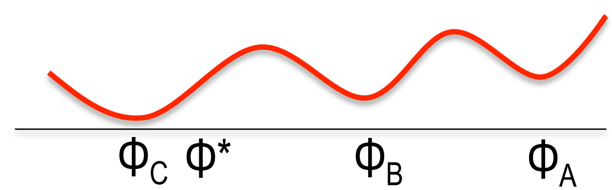
Solution to nonhomogeneous wave equation is well approximated by solution to homogeneous wave eqn (mere superposition of wall profiles) up until **and shortly after** walls reach each other. Hence they "pass through" each other.

After collision interior (overlap region) is in:

$\Phi^*=2\Phi_B-\Phi_A$

If **Φ^* is sufficiently far in basin of attraction of Φ_C** then field inside collision region evolves into Φ_C and expands.

And so we have a **classical mechanism for bubble nucleation.**



Concerns

- Most realistic avenues for energy dissipation are closed off to the model because it is a single field theory without gravity.
- Is free passage realistic? Or could it be an artifact of the simplicity of the model?
- Dissipation:

  1. Interactions

  – We know inflaton field interacts with other fields (fermion fields, gauge fields, etc)
  – Violent changes in a field coupled to other fields typically results in bursts of particle production (associated w/ add'l fields)
    - Ex: as inflation ends (violent change in inflaton field) energy is leaked into other fields in form of particle production

  Bubble collisions are violent events.. Expect particle production if we include interactions. Does opening this avenue for energy dissipation yield a different result?

  2. Gravitational Effects

  – The inflaton field (and any additional fields built into the model) source the stress energy tensor, hence affect the metric (which appears in field eqns themselves).
  – By not accounting correctly for this backreaction we ignore gravitational wave production which is another mechanism of energy dissipation AND is one that hypothetically could be observed

Interacting Field theory

- Simplest thing to do: introduce an additional scalar field, a stand in for quarks, etc
- New Lagrangian: $\mathcal{L}(\phi,\psi)=\frac{1}{2}g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi+\frac{1}{2}g^{\mu\nu}\partial_\mu\psi\partial_\nu\psi-V(\phi,\psi)$

→ Eqns of motion in Minkowski space:

$$\boxed{\begin{aligned} g^{\mu\nu}\partial_\mu\partial_\nu\phi &= -\frac{\partial V(\phi,\psi)}{\partial\phi} \\ g^{\mu\nu}\partial_\mu\partial_\nu\psi &= -\frac{\partial V(\phi,\psi)}{\partial\psi}\end{aligned}}$$

V ought to have 3 local min in Φ-Ψ plane, take $V(\Phi,\Psi)=V_1(\Phi)+V_{int}(\Phi,\Psi)+m^2\Psi^2/2$ where V_1 is the $\sim\Phi^6$ potential in single field case that has the 3 local min Φ_A, Φ_B, Φ_C

- Subtleties: not all interactions will provide a legitimate test of classical tunneling via free passage. Both fields would participate in FP since FP in multifield case involves superpositions of two component (scalar) vectors in field space.
  - Single field case: $\Phi^*=\Phi_A+2\Delta\Phi=2\Phi_B-\Phi_A$ had to be in basin of attraction of Φ_C to get classical tunneling
  - Analog in multifield: $2(\Phi,\Psi)_B-(\Phi,\Psi)_A$ has to be in basin of attraction of $(\Phi,\Psi)_C$

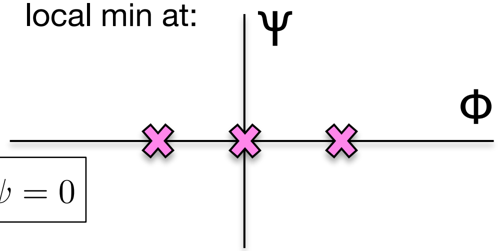
  → local minima in Φ-Ψ plane have to lie roughly on a line
- so looking at 2 interactions in particular: $V_{int}\sim\Phi^2\Psi^2$, $V_{int}\sim\Phi^3\Psi$

Summary of Methods

$V(\Phi,\Psi)=V_1(\Phi)+V_{int}(\Phi,\Psi)+m^2\Psi^2/2$:

$V_{int}=g\Phi^2\Psi^2/2$

local min at:



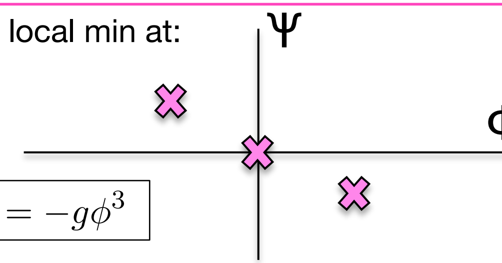
Ψ field eqn is KG with Φ dep mass:

$$\boxed{\Box\psi+(m^2+g\phi^2)\psi=0}$$

IC: Φ has right/left moving wall profiles, Ψ flat with small fluctuations otherwise trivial soln

$V_{int}=g\Phi^3\Psi$

local min at:



Ψ field eqn is KG with mass=m, and Φ dep forcing:

$$\boxed{\Box\psi+m^2\psi=-g\phi^3}$$

IC: BOTH Φ and Ψ have right/left moving wall profiles

- Purely Classical Treatment:
  - Analytic: for $g\Phi^3\Psi$

  Assume FP for Φ, solution to Ψ eqn given in terms of retarded Green's function. Consistent with E conservation?
  - Numerical:

  Don't assume Φ_{FP}, approx solution to the (2) coupled nonlinear wave equations for relevant ICs

  New: Revisit Free passage argument for single field case

- Semi-classical Treatment: for $\Phi^2\Psi^2$ interaction

Treat Φ field classically and as given by FP, Ψ field as quantized
  - Background Field Method: expand action in path integral about $\Psi=\Psi_{classical}$(which is Φ dep, so nonhomogeneous since Φ_{FP} is nonhomogeneous).Consistent does Φ_{FP} correspond to tremendous Ψ particle production that would violate E conservation?
  - Bogoliubov Transformations, assume Φ_{FP}, compute map between early time, and late time (Ψ) creation/annihilation operators→ compute late time particle content of early time vacuum. Difficulty here is that Φ_{FP} is nonhomogeneous. Consistent?

Gravitational Effects

- Field(s) backreact via affect on metric
- Vary action: $S = S_{Einstein-Hilbert} + \int \sqrt{-g}\, d^4x\, \mathcal{L}(\phi, \psi)$
- Eqns of motion:

$$\boxed{\begin{aligned} g^{\mu\nu}\partial_\mu\partial_\nu\phi + \frac{\partial^\mu\phi\partial_\mu\sqrt{-g}}{\sqrt{-g}} &= -\frac{\partial V(\phi,\psi)}{\partial\phi} \\ g^{\mu\nu}\partial_\mu\partial_\nu\psi + \frac{\partial^\mu\psi\partial_\mu\sqrt{-g}}{\sqrt{-g}} &= -\frac{\partial V(\phi,\psi)}{\partial\psi} \\ R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R &= 8\pi G T_{\mu\nu}(\phi,\psi) \end{aligned}}$$

- Highly nonlinear… → solve numerically
- Introduce new variables, spatial metric and extrinsic curvature, so that we can formulate Einstein's eqn as a Cauchy problem (ADM eqns)

- Does this result in significant gravitational wave production? Observable?