The effects of weakly 3-D equilibria on the MHD stability of tokamak pedestals

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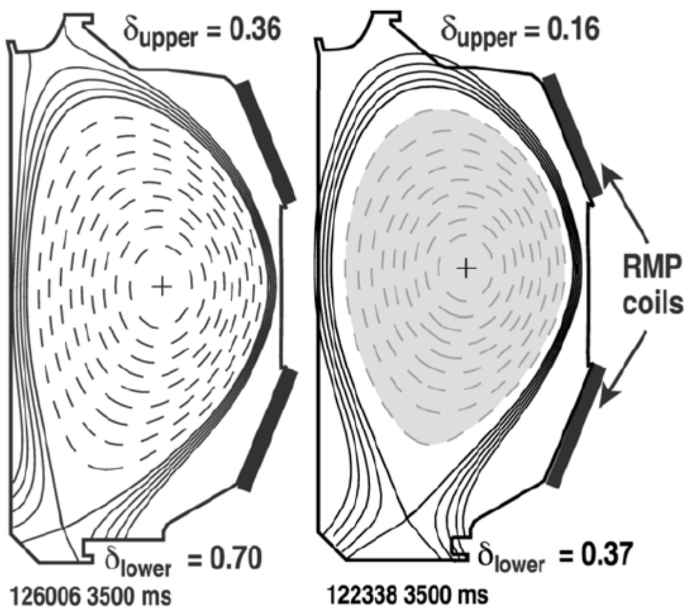
Theses

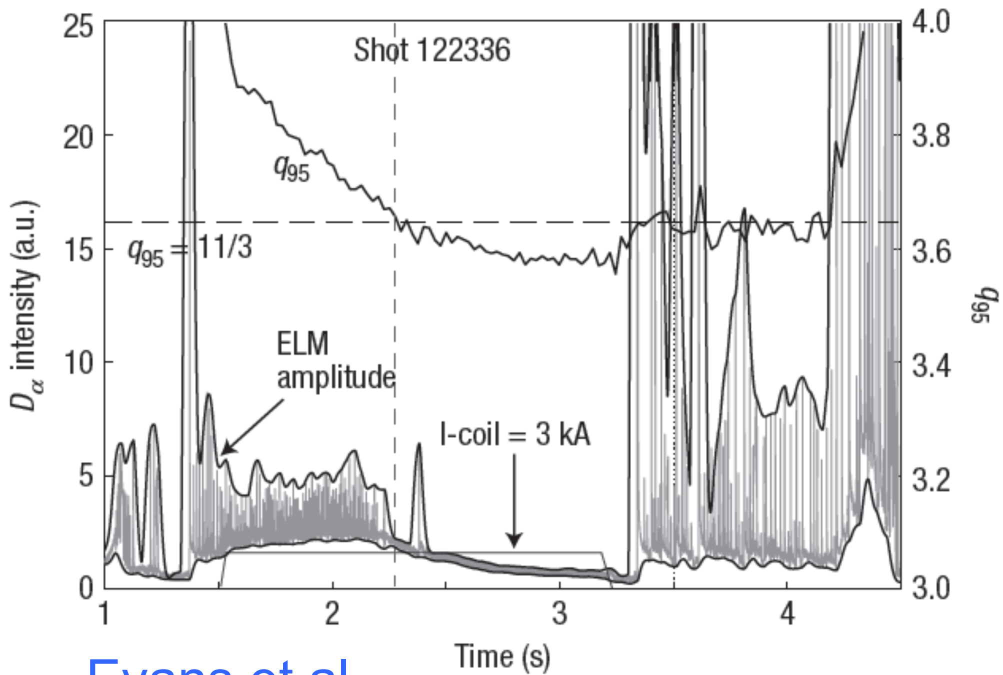
- The stability of MHD modes is evaluated in the presence of an equilibrium weakly perturbed by a topology-preserving 3-D distortion. Two classes of instabilities
  - Infinite-n ballooning stability
    - Strong modifications to marginal stability
    - Sensitivity to lower order rational surfaces
    - Indicative of any ‘local’ microinstability behavior
  - Finite-n ‘peeling-ballooning’ stability
    - Weak 3-D equilibrium is generally destabilizing
    - Modest changes to growth rates
    - Not as sensitive to variations in profiles
- Working towards a model for how 3-D applied fields can prevent ELMs in H-mode tokamaks.

Motivation: Understanding the physics of RMP suppression of ELMs

- Controlling edge localized modes (ELMs) is crucial for H-mode tokamak operation
  - Prominent control mechanism is the use of Resonant Magnetic Perturbations (RMPs)



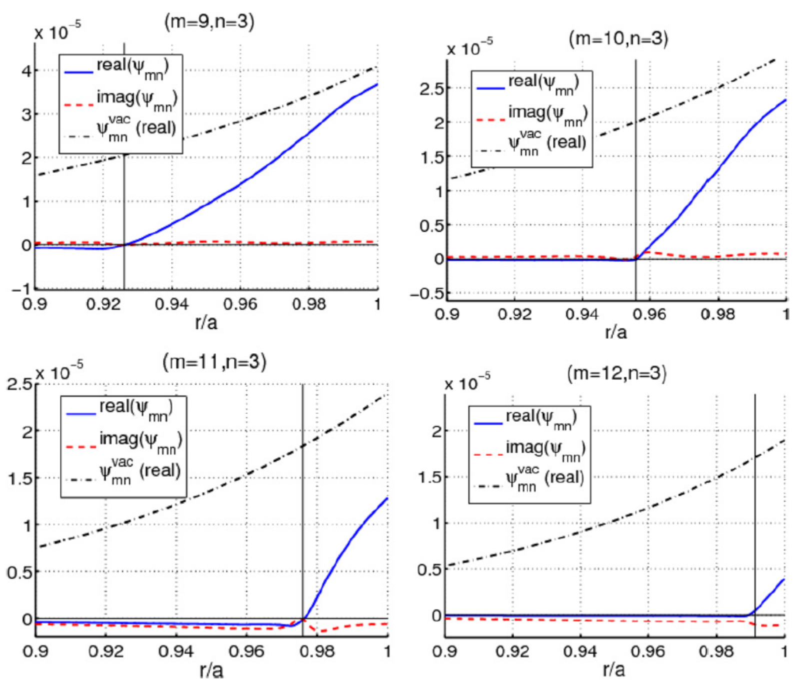
Evans et al, NF '08

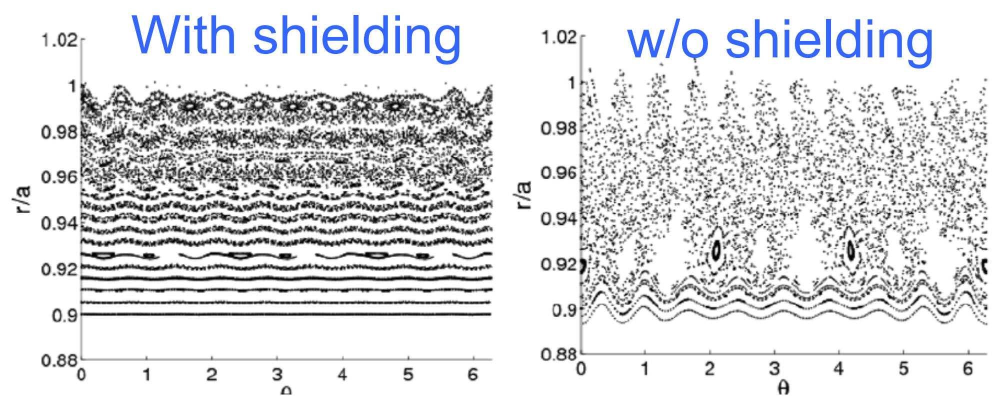


Evans et al, Nature Physics '06

Plasma rotation shields RMPs from producing stochastic magnetic fields

- Original intention of RMPs → stochastic magnetic fields → greatly enhance edge transport → eliminate drive for MHD stabilities
- However, plasma response is important → Plasma rotation provides shielding (Ferraro PoP '12, Liu et al NF '11, Becoulet et al NF '12 …)



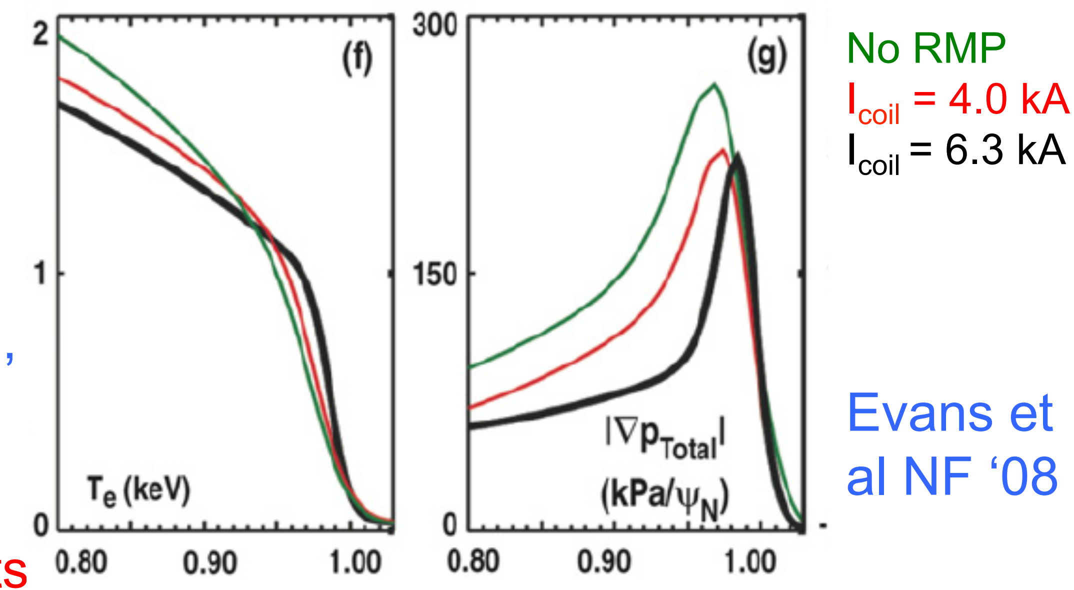


- With shielding, flux surface integrity restored

Nardon et al, NF '10

Profile data suggests stochastic mechanism is not present

- With RMP, electron temperature profile is not flattened in pedestal. Pedestal pressure gradient is unaffected RMP
  - Extent of pedestal altered with RMP
- What's happening?
  - Various ideas developing (Wade APS 2011, Callen et al PoP '12, …)
  - THIS TALK 3-D geometry affects pedestal stability → model for ELM suppression



Evans et al NF '08

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Outline

- Motivation and Background
- "Local" MHD stability with weakly 3-D equilibrium → proxy for micro-instability and anomalous transport
- "Global" MHD stability with weakly 3-D equilibrium
- Scenario for RMP stabilization

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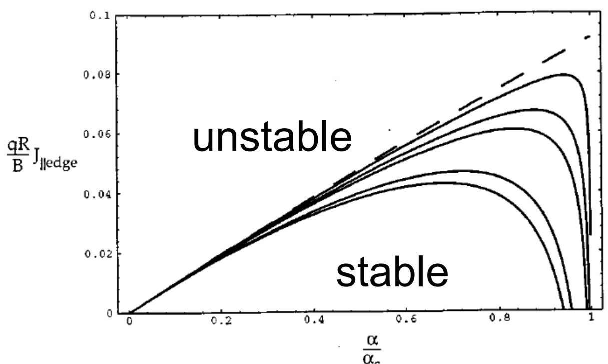
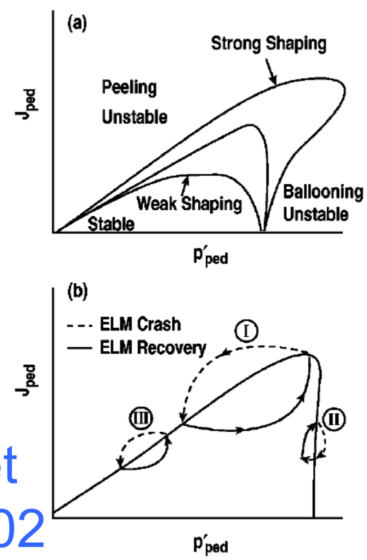
The intermediate-n stability of ideal MHD modes are thought responsible for ELM onset

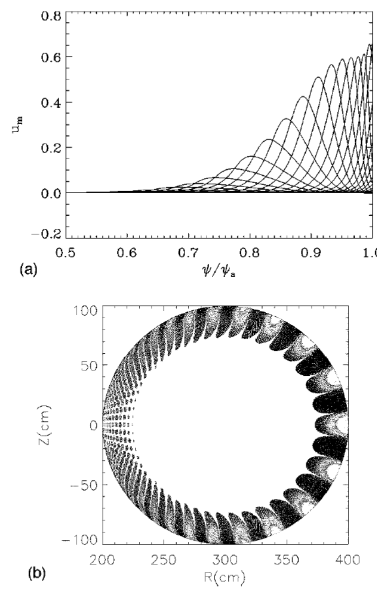
- Pedestal region → steep gradients, edge (bootstrap) current
  - Drives for ideal MHD instability
    - Edge currents (peeling modes)
    - Pressure gradient/bad curvature (ballooning modes)
  - At intermediate-n, both free energy sources available → "peeling-ballooning mode"
    - Prominent stability tool is ELITE



CCH et al, PoP '96



Snyder et al, PoP '02



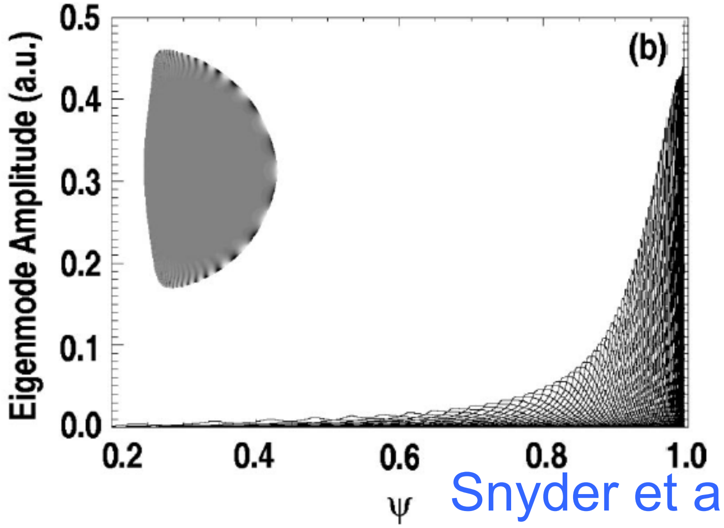
Wilson et al, PoP '02

Both 'local' and 'global' MHD instabilities are thought to play a role in pedestal physics

- MHD elements thought to be important in pedestal (Snyder et al '09)

Intermediate-n (~10) 'peeling-ballooning' modes --- global mode structure

'Local' micro-instabilities --- "∞-n" ideal ballooning used as a proxy for KBM



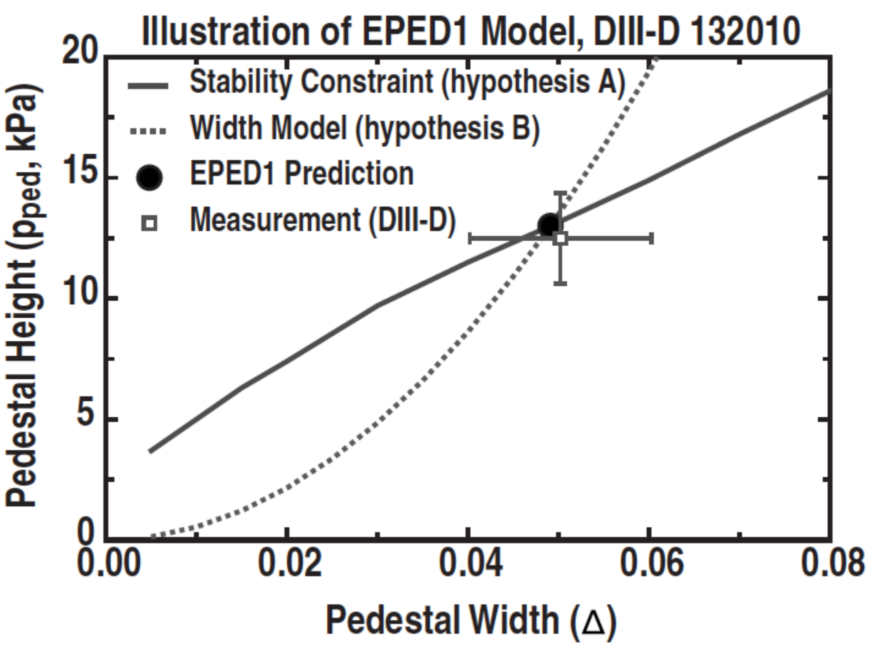
Snyder et al, PoP '02

Ballooning drive common to many microinstabilities (KBM, ITG, RBM,…)
→Curvature drive for a mode that twists with sheared B $\kappa_n + \Lambda \kappa_g$

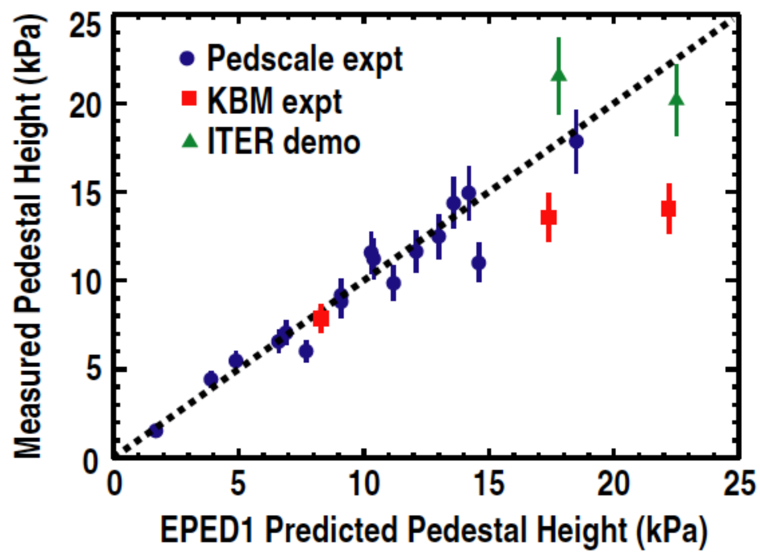
$$\Lambda = \frac{g^{\psi\psi}}{B} \int \frac{dl}{B} \frac{B^2}{g^{\psi\psi}} s$$

EPED1 model utilized to predict self-consistent pedestal height & width

- EPED1 model incorporates two constraints to self-consistently determine pedestal height and width
  - Two constraints: Peeling-Ballooning, KBM



Snyder et al, PoP '09



Groebner et al, PoP '10

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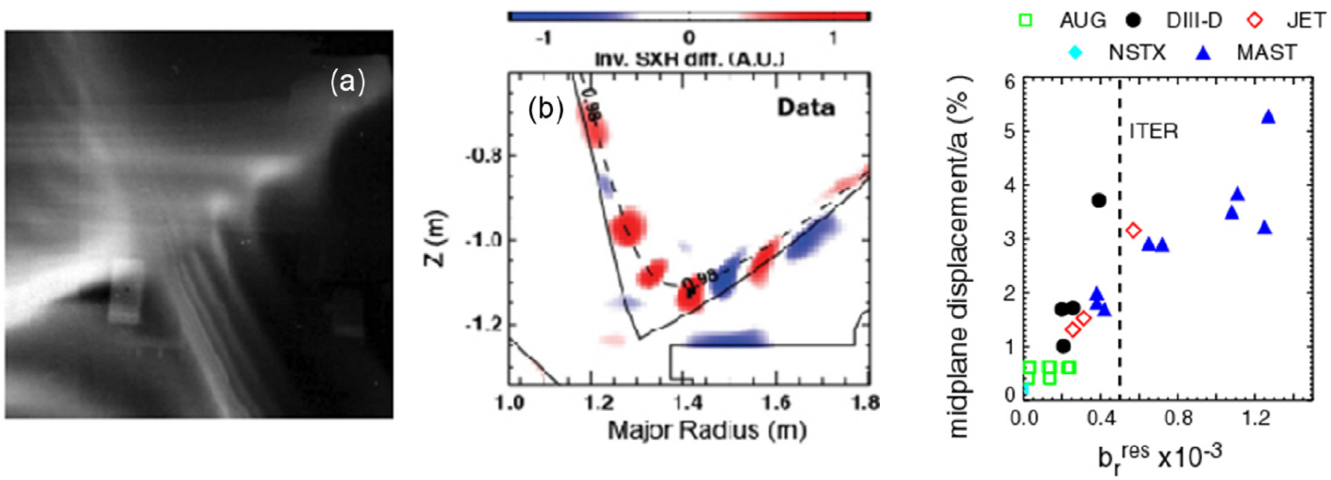
3D flux surface deformations have been measured when non-axisymmetric fields are applied

- From MAST, DIII-D, substantial 3-D deformations have been observed when non-axisymmetric fields are applied



Kirk et al, PPCF ‘13

- Deformations produced by RMP induced stable “kink-like” response
  - This effect can be modeled for MHD stability calculations of 3-D equilibrium

Understanding the role of 3-D fields on pedestal properties crucial for interpreting RMP physics

- Applied RMPs
  - Rotation provides shielding (small islands)
  - Substantial 3-D deformation of MHD equilibrium

- Pedestal properties determined by:
  - Micro-instabilities (e. g., KBMS)
  - Peeling-ballooning stability (n ~ 5-30)

- This work → Effect of shielded non-axisymmetric magnetic fields
  - Local MHD stability?
  - Global modes?

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For local instabilities, local 3-D MHD equilibrium theory is employed

- Using local 3-D equilibrium theory, shaped tokamak equilibria with small 3-D distortions are constructed
- Local 3-D equilibrium theory determined by two profiles and flux surface parameterization in straight-field line coordinates (CCH, '00)

$$R = R(\Theta) + \sum_{M} \gamma_i \cos(M_i \Theta - N_i \zeta)$$

$$Z = Z(\Theta) + \sum_{i} \gamma_i \sin(M_i \Theta - N_i \zeta)$$

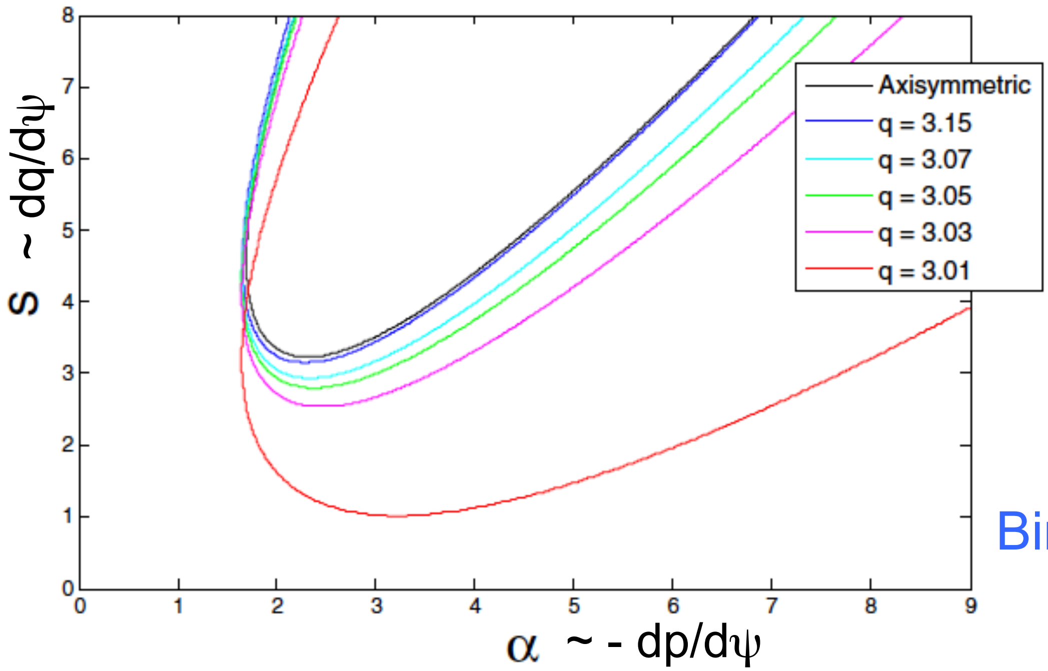
  - Topologically toroidal flux surfaces
  - Axisymmetric + small 3-D distortions
    - $R(\Theta)$, $Z(\Theta)$ ~ shaped Miller equilibria
    - $\delta B_\rho / B_0 \sim \Sigma_i \gamma_i (q M_i - N_i)/R_0$

3-D MHD equilibrium can be destabilizing to local (n→ ∞) ideal MHD modes

- Shielded 3-D magnetic perturbations can destabilize local microinstability properties and potentially enhance transport
  - Infinite-n ideal MHD ballooning stability boundary is strongly perturbed by 3-D fields
  - Most sensitive as rational surface approached

$A = 3.17$,
$\kappa = 1.66$
$\delta = 0.416$
$S_\kappa = 0.7$,
$s_\delta = 1.37$,
$d_r R_0 = -0.354$

$\gamma_i = 10^{-3}$
$M_i = 4,5,6,\ldots 14$
$N_i = 3$



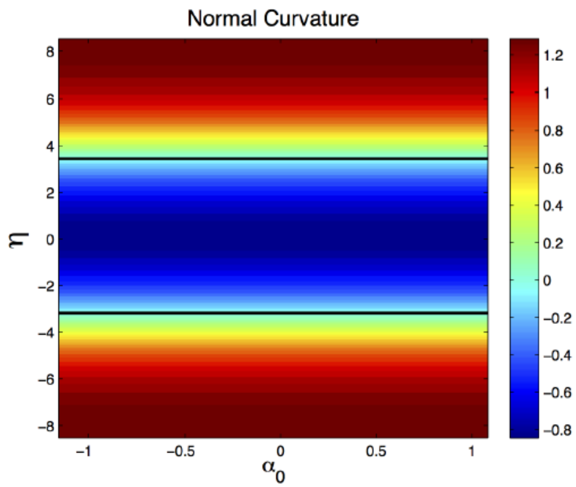
Bird and CCH, NF '13

3-D components modify important geometric coefficients

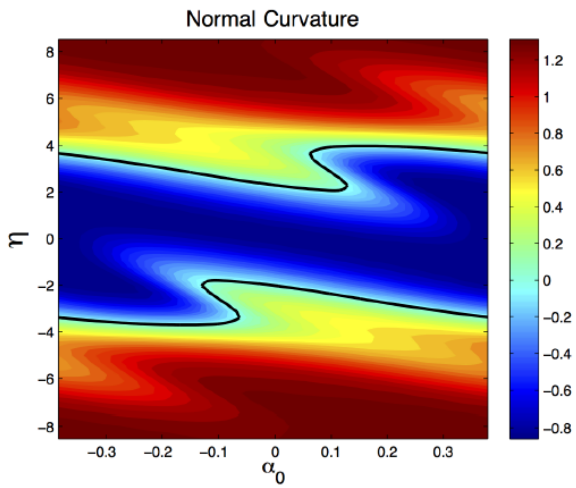
- Ideal ballooning stability --- balance of pressure/curvature drive with field line bending
  - Normal and geodesic curvature $\vec{\kappa} = \kappa_n \hat{n} + \hat{\kappa}_g$
  - Local shear $s = \hat{b} \times \hat{n} \cdot \nabla \times (\hat{b} \times \hat{n})$

- With 3-D fields, we have modest corrections to the curvature vector

- Axisymmetric:



3D deformation:



3-D fields have a significant effect on the local shear

- Local shear is related to parallel currents and normal torsion

$$s = \frac{\mu_o \vec{J}\cdot\vec{B}}{B^2} - 2\tau_n$$

  - 3-D fields produce modest corrections to τ_n
  - 3-D fields produce large changes to Pfirsch-Schlüter currents

$$\frac{\mu_0 \vec{J}\cdot\vec{B}}{B^2} = \sigma + \frac{dp}{d\psi}\lambda$$

$$\vec{B}\cdot\nabla\lambda = 2\mu_0\kappa_g \frac{|\nabla\psi|}{B}$$

    - With 3-D fields

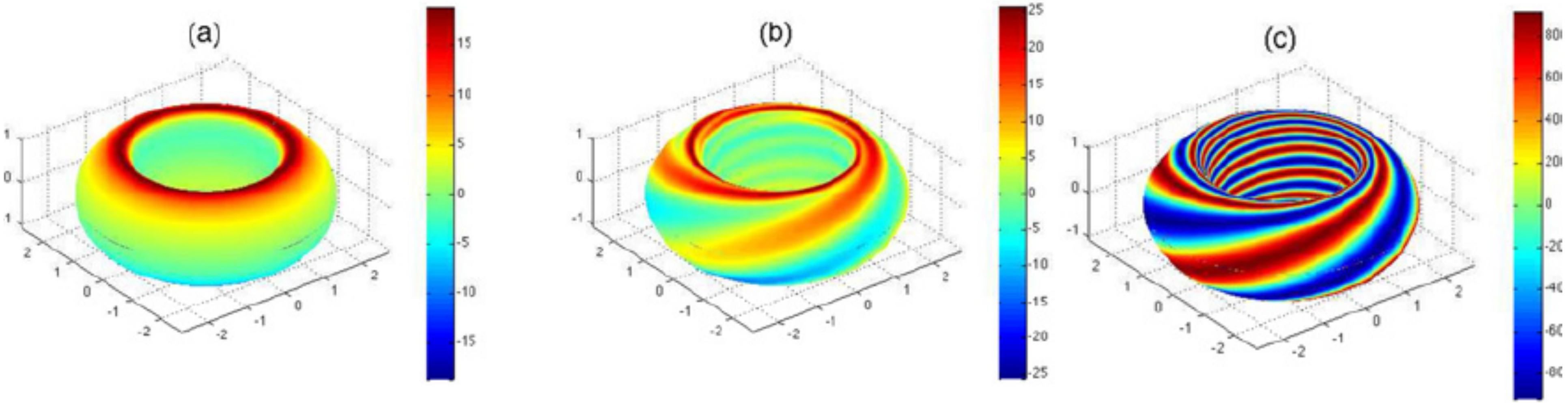
$$\kappa_g = \sum_{mn} \kappa_{gmn} e^{im\theta - in\zeta}$$

$$\lambda_{mn} \sim \frac{dp}{d\psi}\frac{\kappa_{gmn}}{q - m/n}$$

Local shear is sensitive to resonances

- Contours of integrated local shear: $\Lambda = \frac{g^{\psi\psi}}{B}\int \frac{dl}{B}\frac{B^2}{g^{\psi\psi}}s$

axisymmetric | Add 3-D, q = 3.15 | Add 3-D, q = 3.01



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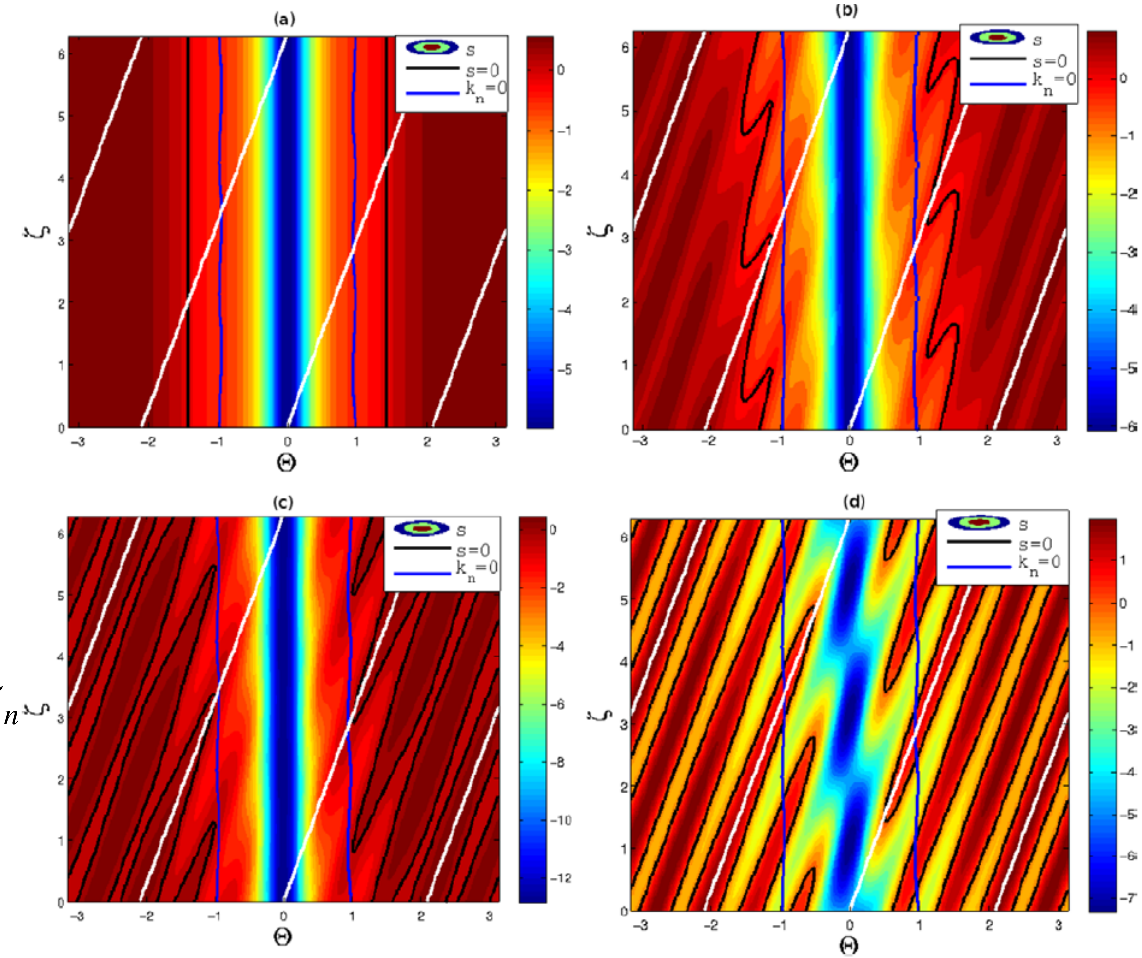
Stability affected by 3-D modulations to local shear

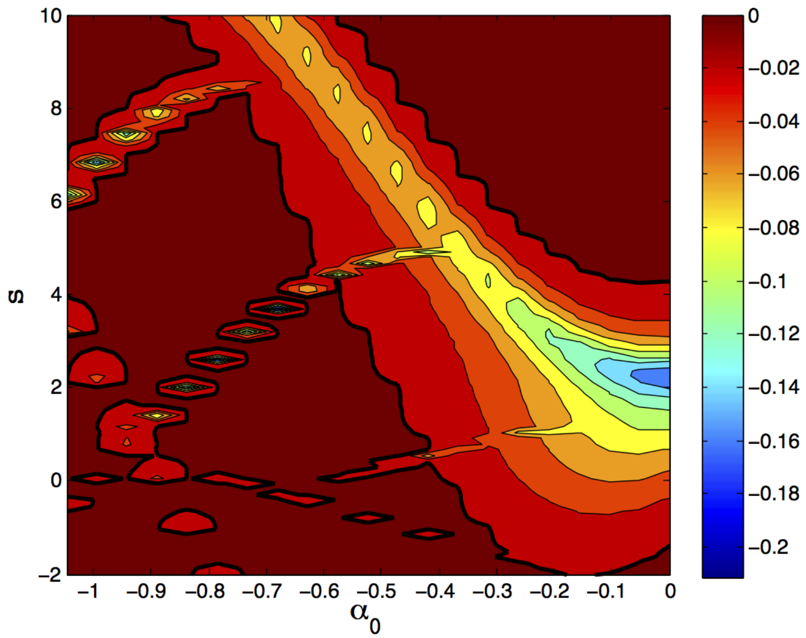
- Local shear contours: a) axisymmetric b) + 3-D, q = 3.07 c) + 3-D, q = 3.03 d) + 3-D, q = 3.01
  - Eigenmodes resides in regions with small local shear

$$s = \hat{b}\times\hat{n}\cdot\nabla\times(\hat{b}\times\hat{n}) = \frac{J_{\parallel}}{B} - 2\tau_n$$



In 3-D equilibrium, local eigenvalues are field line dependent

- Contours of growth rate as a function of field line label and s



q = 11/3 + 0.1

- – Flux tube analysis at different field line labels
  - Some flux tubes are more unstable than others
- – Turbulence sees the entire surface
  - Extension under development to model entire surface

  → Full surface GENE code

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To calculate global mode stability, local 3-D equilibrium is not sufficient

- Consider the presence of a 3-D equilibrium defined over a finite **volume** of the plasma
- Proper construction of 3-D equilibrium with RMP is an open topic
  - Variety of tools employed (Turnbull, PoP '13)
- **B** = $\mathbf{B}_o$ + δ**B**, |δ**B**|/|**B**| << 1
  - $\mathbf{B}_o$ = axisymmetric equilibrium
  - δ**B** ~ $e^{-iN\zeta}$ = small 3-D distortion
- MHD equilibrium to O(δ)

$$\vec{J} \times \vec{B} = \nabla p$$

$$\vec{J}_0 \times \vec{B}_0 = \nabla p_0$$

$$\delta\vec{J} \times \vec{B}_0 + \vec{J}_0 \times \delta B = \nabla \delta p$$

  - 3-D distortion governed by marginal linear ideal MHD response

Small 3-D distortion allows for a perturbation approach

- The ideal MHD force operator can be separated to O(δ)

$$\rho \frac{\partial^2 \vec{\xi}}{\partial t^2} = \vec{F}_0(\vec{\xi}) + \delta \vec{F}(\vec{\xi})$$

  - **F_0** = force operator of axisymmetric equilibrium
  - **δF** ~ $e^{-iN\zeta}$ = correction to force operator due to 3-D distortion
- Eigenvector and eigenvalue for axisymmetric equilibrium

$$\omega_{k0}^2 \rho \vec{\xi}_{k0} = -\vec{F}_0(\vec{\xi}_{k0}) \qquad \vec{\xi}_{k0} \sim e^{-in_k \zeta}$$

- To construct the eigenfunction for the full system, the eigenfunctions for the axisymmetric equilibrium can be used as a basis set

$$\vec{\xi}_j = \sum_k a_{jk} \vec{\xi}_{k0}$$

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A perturbation approach can used to estimate the effect of the 3-D field on ideal MHD spectrum

- Inserting eigenvalue expansion into ideal MHD force operator

$$\omega_j^2 \rho \sum_k a_{jk} \vec{\xi}_{k0} = \rho \sum_k \omega_{k0}^2 a_{jk} \vec{\xi}_{k0} - \sum_k a_{jk} \delta \vec{F}(\vec{\xi}_{k0})$$

  - Orthogonality properties→ matrix equation for coupling coefficients

$$\omega_j^2 a_{jm} = \omega_{m0}^2 a_{jm} + \sum_k a_{jk} V_{mk}$$

  - 3-D field effects enter through matrix element V_{mk}

$$V_{mk} = -\frac{\int d^3x \, \vec{\xi}_{m0}^* \cdot \delta \vec{F}(\xi_{k0})}{\left(\int d^3x \rho |\vec{\xi}_{m0}|^2\right)^{1/2} \left(\int d^3x \rho |\vec{\xi}_{k0}|^2\right)^{1/2}}$$

$$V_{km}^* = V_{mk}$$

Generally, 3-D equilibrium distortions are destabilizing

- For weak coupling, $a_{jk} = \delta_{jk} + O(\delta)$, off-diagonal coupling coefficients

$$a_{jj} \cong 1 \qquad a_{jk} \cong \frac{V_{kj}}{\omega_{j0}^2 - \omega_{k0}^2}$$

  – Eigenfunction

$$\vec{\xi}_j \cong \vec{\xi}_{j0} + \sum_k \frac{V_{kj}}{\omega_{j0}^2 - \omega_{k0}^2} \vec{\xi}_{k0}$$

  – Eigenvalues

$$\omega_j^2 \cong \omega_{j0}^2 + \sum_k \frac{|V_{jk}|^2}{\omega_{j0}^2 - \omega_{k0}^2}$$

  - Notice, for most unstable (or least stable) eigenvalue

  → $\omega_{j0}^2 < \omega_{k0}^2$ → 3-D correction term is destabilizing

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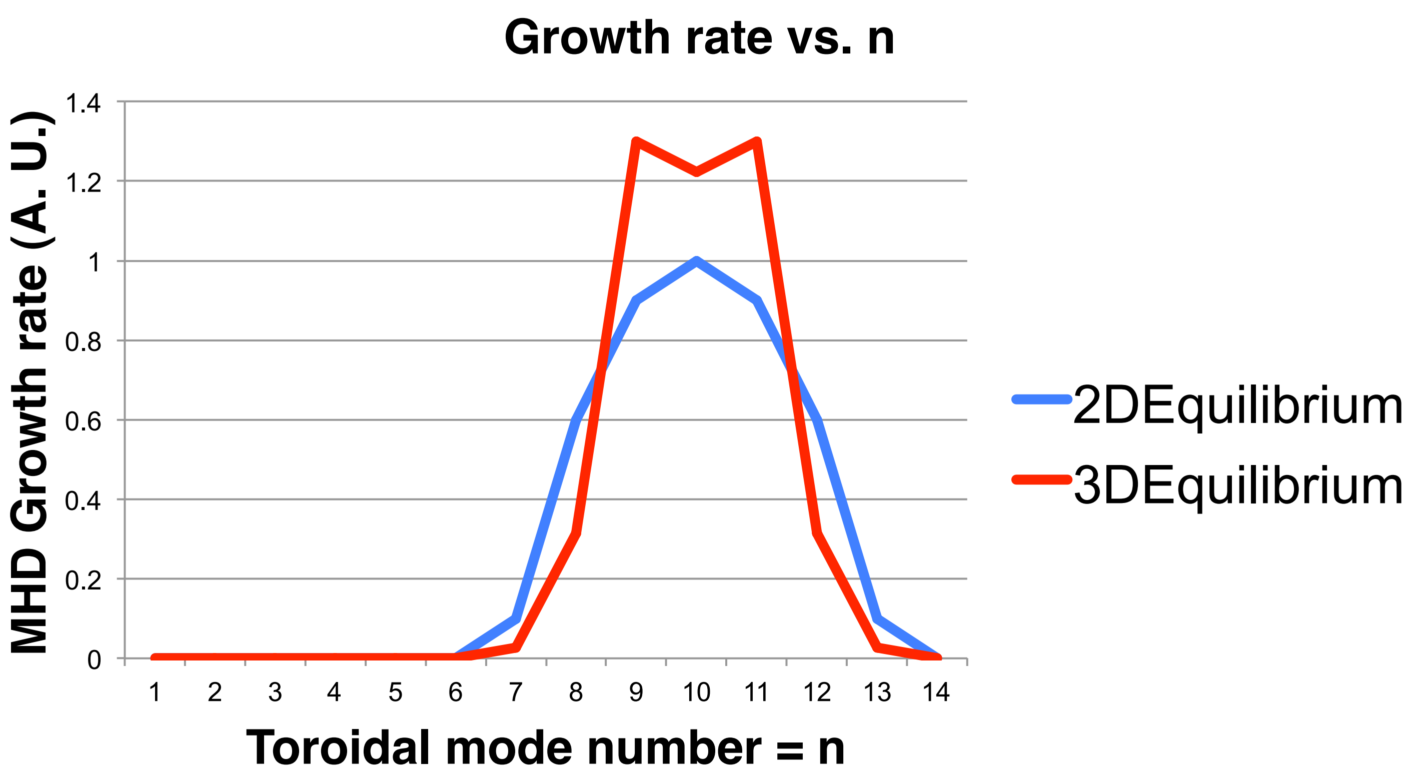
An example spectrum demonstrates the destabilizing property of the 3-D distortion

- Academic example using prescribed 2-D spectrum and approximate solution for V_{jk} with $\delta\mathbf{B} \sim e^{-i3\zeta}$ (N = 3)



The linear eigenfunctions have 3-D structure

- Linear eigenfunctions have coupled 3-D harmonics

$$\vec{\xi}_j = \vec{\xi}_{j0} + \sum_k \frac{V_{kj}}{\omega_{j0}^2 - \omega_{k0}^2} \vec{\xi}_{k0}$$

$\vec{\xi}_{j0} \sim e^{-in_j\zeta}$, $\quad \vec{\xi}_{k0} \sim e^{-in_k\zeta}$

  – For example with an N = 3 equilibrium distortion

$$\vec{\xi}_j = \vec{\xi}_{j0} + \frac{V_{j+3\,j}}{\omega_{j0}^2 - \omega_{j+3,0}^2} \vec{\xi}_{j+30} + \frac{V_{j-3\,j}}{\omega_{j0}^2 - \omega_{j-3,0}^2} \vec{\xi}_{j-30} + \ldots$$

$\vec{\xi}_{j0} \sim e^{-in_j\zeta}$, $\quad \vec{\xi}_{j+30} \sim e^{-i(n_j+3)\zeta}$, $\quad \vec{\xi}_{j-30} \sim e^{-i(n_j-3)\zeta}$

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Matrix elements can be calculated using perturbed 3-D equilibrium

- $\delta \mathbf{F}(\xi)$ due to 3-D equilibrium

$$\delta\vec{F}(\vec{\xi}) = \delta\vec{J}\times[\nabla\times(\vec{\xi}\times\vec{B}_0)] + \vec{J}_0\times[\nabla\times(\vec{\xi}\times\delta\vec{B})]$$

$$+\frac{1}{\mu_0}\{\nabla\times[\nabla\times(\vec{\xi}\times\vec{B}_0)]\}\times\delta\vec{B} + \frac{1}{\mu_0}\{\nabla\times[\nabla\times(\vec{\xi}\times\delta\vec{B})]\}\times\vec{B}_0 + \nabla\cdot(\vec{\xi}\cdot\nabla\delta p + \gamma\delta p\nabla\cdot\vec{\xi})$$

- Matrix elements can be calculated

$$V_{jk} = \int d^3x(\delta\vec{J}\cdot\vec{C}_1 + \delta\vec{B}\cdot\vec{D}_1 + \nabla\cdot\vec{R}_1 + E_1)$$

$$\vec{C}_1 = \vec{\xi}^*_{j0}\times\tilde{\vec{B}}_{k0} \qquad \tilde{\vec{B}}_{k0} = \nabla\times(\vec{\xi}_{k0}\times\vec{B}_0)$$

$$\vec{D}_1 = -\frac{1}{\mu_0}\vec{\xi}^*_{j0}\times(\nabla\times\tilde{\vec{B}}_{k0}) - \frac{1}{\mu_0}\vec{\xi}_{k0}\times(\nabla\times\tilde{\vec{B}}^*_{j0}) + \vec{\xi}_{k0}\times[\nabla\times(\vec{\xi}^*_{j0}\times\vec{J}_0)]$$

$$\vec{R}_1 = -\frac{1}{\mu_0}(\vec{\xi}^*_{j0}\times\vec{B}_0)\times[\nabla\times(\vec{\xi}_{k0}\times\delta\vec{B})] - \frac{1}{\mu_0}(\delta\vec{B}\times\vec{\xi}_{k0})\times\tilde{\vec{B}}^*_{j0} + (\delta\vec{B}\times\vec{\xi}_{k0})\times(\vec{\xi}^*_{j0}\times\vec{J}_0)$$

$$-\vec{\xi}^*_{j0}\vec{\xi}_{k0}\cdot\nabla\delta p - \gamma\delta p\vec{\xi}^*_{j0}\nabla\cdot\vec{\xi}_{k0}$$

$$E_1 = \nabla\cdot\vec{\xi}^*_{j0}(\vec{\xi}_{k0}\cdot\nabla\delta p + \gamma\delta p\nabla\cdot\vec{\xi}_{k0})$$

For 3-D equilibrium with shielded RMP fields, an approximate matrix element can be calculated

- For applications to RMP, consider shielded resonant component response → persistence of eddy currents at rational surfaces

$$\delta\vec{J} = \vec{B}_0 \frac{1}{\mu_0} \sum_{MN} \lambda_M e^{iM\Theta - iN\xi} \delta(q - \frac{M}{N}) + c.c.$$

  – In this limit, dominant contribution to the matrix element

$$V_{jk} = \int d^3x\, \delta\vec{J} \cdot \vec{\xi}^*_{j0} \times \tilde{\vec{B}}_{k0} = \sum_M (\lambda_{M,n_{j0}-n_{k0}} C^M_{n_{j0}n_{k0}} + \lambda^*_{-M,n_{k0}-n_{j0}} C^{-M}_{n_{k0}n_{j0}})$$

$$C^M_{n_{j0}n_{k0}} = -\frac{4\pi^2}{\mu_0 N_{j0,k0}} \int_0^{2\pi} \frac{d\Theta}{2\pi} e^{iM\Theta} [\frac{\vec{\xi}^*_{j0} \cdot \vec{B}_0 \times \tilde{\vec{B}}_{k0}}{q'\vec{B}_0 \cdot \nabla\Theta}]_{q=M/(n_{j0}-n_{ko})}$$

$$= -\frac{4\pi^2}{\mu_0 N_{j0,k0}} \int_0^{2\pi} \frac{d\Theta}{2\pi} e^{iM\Theta} [X^*_j X_k (1 + \frac{\partial}{\partial\Theta} \frac{g^{\psi\Theta}}{q' g^{\psi\psi}}) - X^*_j \frac{\sqrt{g}}{in_k q'} \vec{B}_0 \cdot \nabla \frac{\partial X_k}{\partial\psi} - \frac{\partial X^*_j}{\partial\psi} \frac{\sqrt{g}}{in_j q'} \vec{B}_0 \cdot \nabla X_k + ..]_{q=M/(n_{j0}-n_{ko})}$$

  – Characteristic amplitudes $V/\omega_0^2 \sim 0.3$
  – Proper calculation requires knowledge of 3-D distortion including plasma response (M3D, NIMROD) and ideal MHD eigenfunction from associated axisymmetric equilibrium (ELITE)

3-D equilibria affects local MHD stability and global MHD stability in different ways

- Effects of a weakly 3-D equilibrium on MHD stability properties:
  - 'Local' mode stability
    - Order unity changes to marginal profiles
    - Generally destabilizing
    - Sensitivity to low-order rational surfaces
  - 'Global' mode stability
    - Destabilizing
    - Modest changes to growth rates
    - Not as sensitive to variations in profiles

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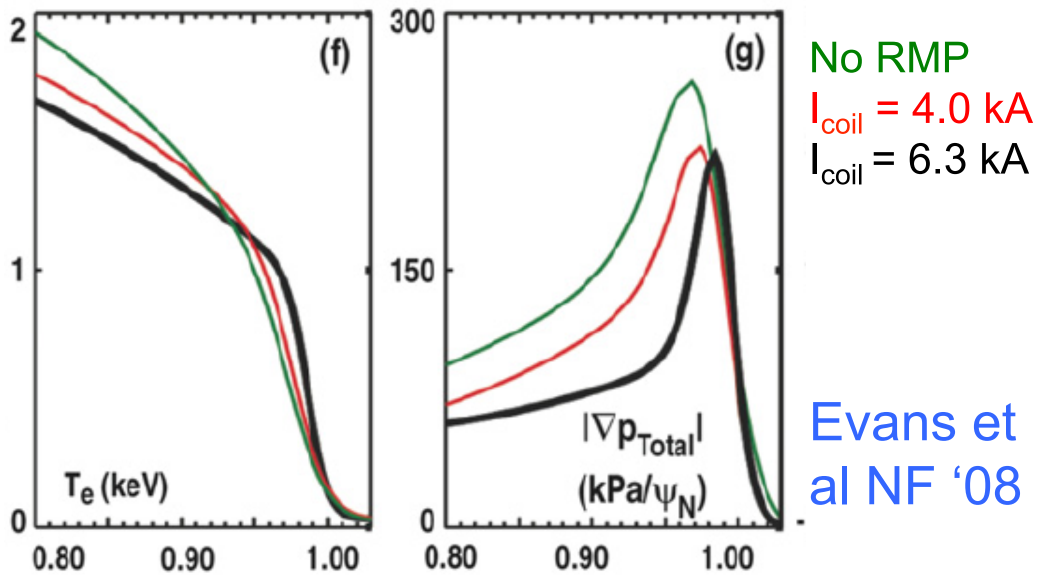
Stability calculations suggest a possible scenario for ELM suppression from RMP

- Recall, RMP suppresses radial propagation of steep pedestal gradient
  - Sensitive to q
- Hypothesis:

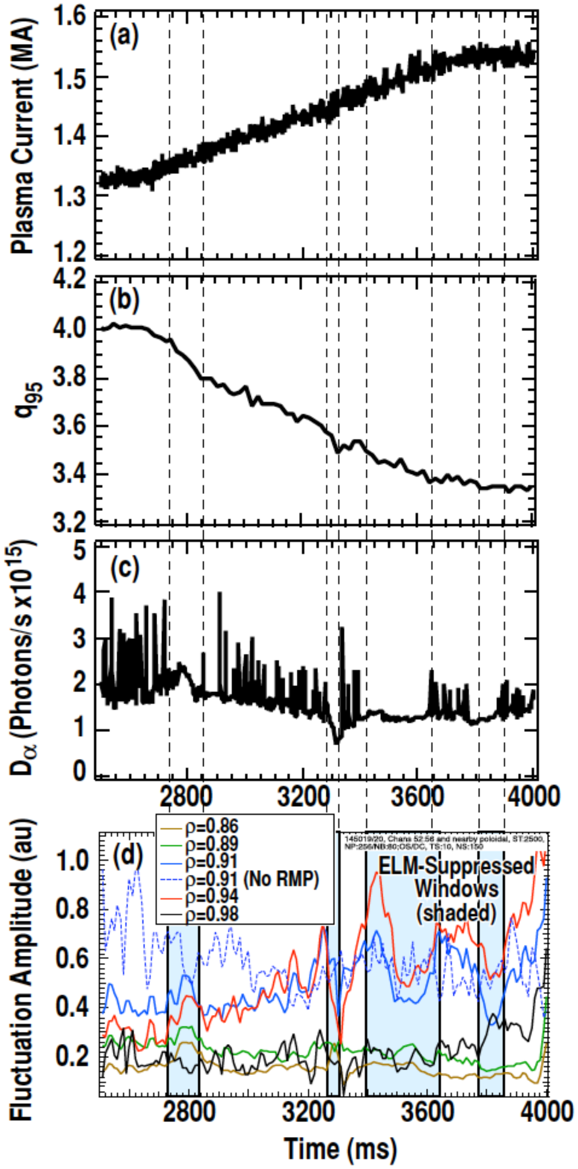
Necessary condition for ELM suppression is a near resonant $J_{||PS}$ at pedestal top → O(1) corrections to local shear



Evans et al NF '08

  - Enhanced transport at pedestal top → flattens pressure gradient, halts the pedestal, keeps profile from exciting intermediate-n MHD modes
  - 3-D effects alter peeling-ballooning stability, but not significantly enough to appreciably alter stability threshold

Suppression of turbulence is observed with ELM suppression



- From BES measurements, density fluctuation reductions correspond to ELM suppression windows in q

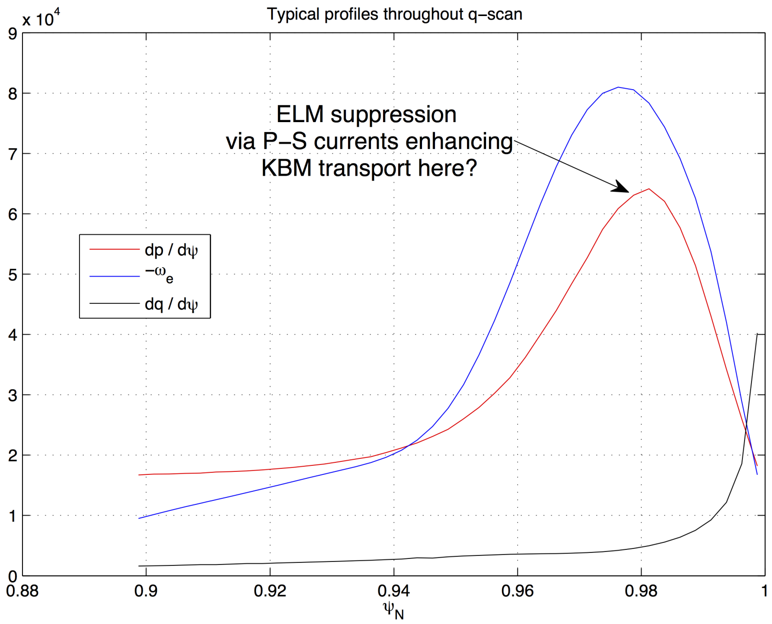
  With ELM suppression:

  - For $0.86 < \rho < 0.96$, large fluctuation at ELM suppression initiation, subsequent drop in amplitude
  - Modest changes to fluctuations in pedestal $\rho \sim 0.98$

McKee et al, NF '13

Initial analysis of near resonant Pfirsch-Schluter currents rely on M3D-C1 analysis

- Sequence of equilibria similar to shot 126006
  - Courtesy N. Ferraro, 3-D response constructed using M3D-C1
  - Track locations of low order rational surfaces near pedestal
  - Use local 3-D equilibrium theory to construct local shear properties
  - Track where destabilizing 3D effects are strongest and how they move with q_{95}.

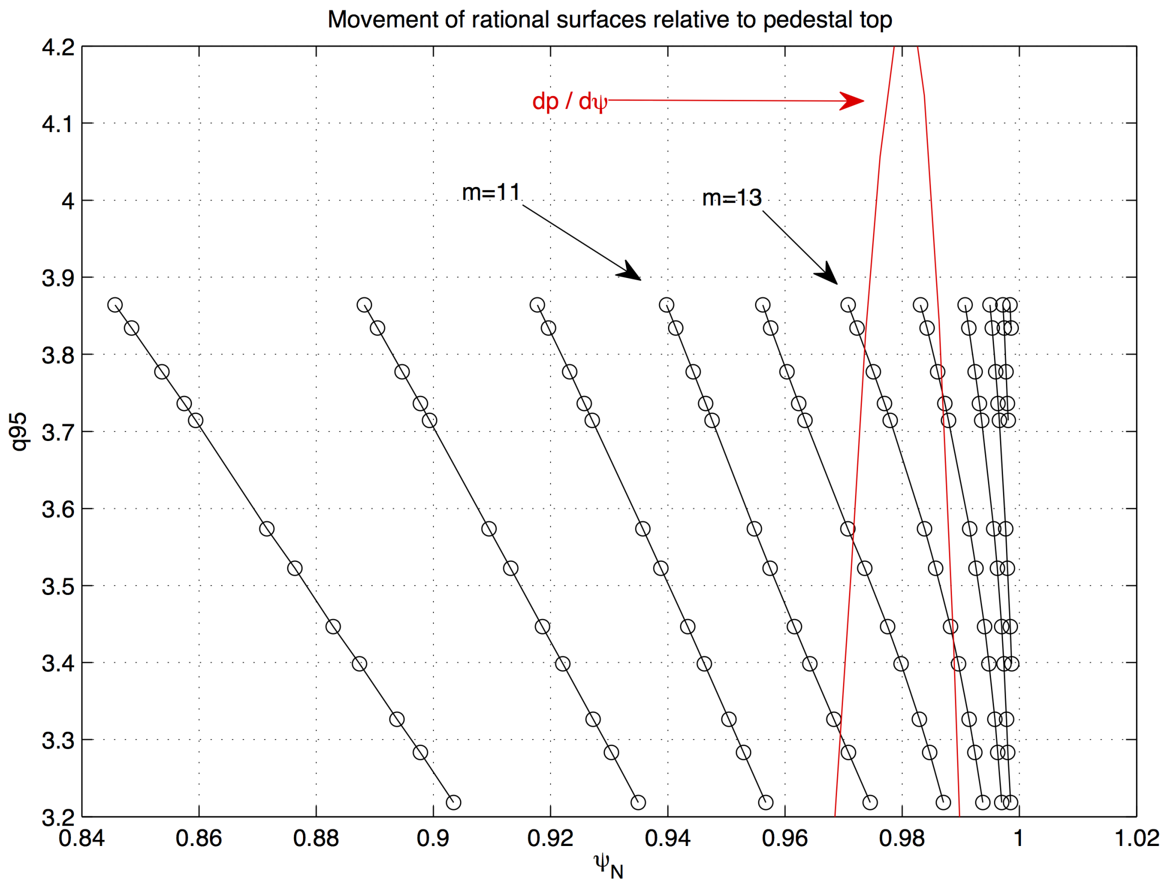


» $dp/d\psi$, $dq/d\psi$ roughly stay the same during q_{95} scan

» Rotation present → shielding physics operative

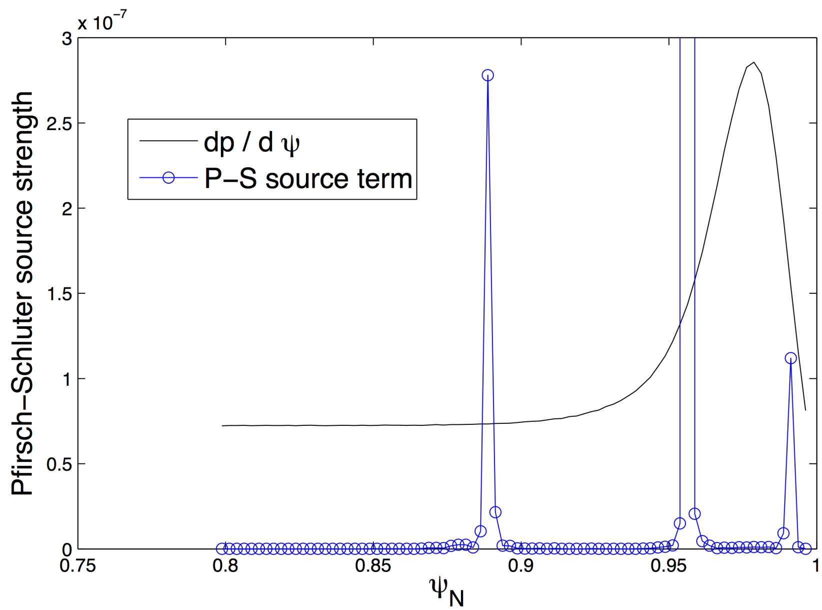
Rational surface movement during scan suggests possible suppression windows

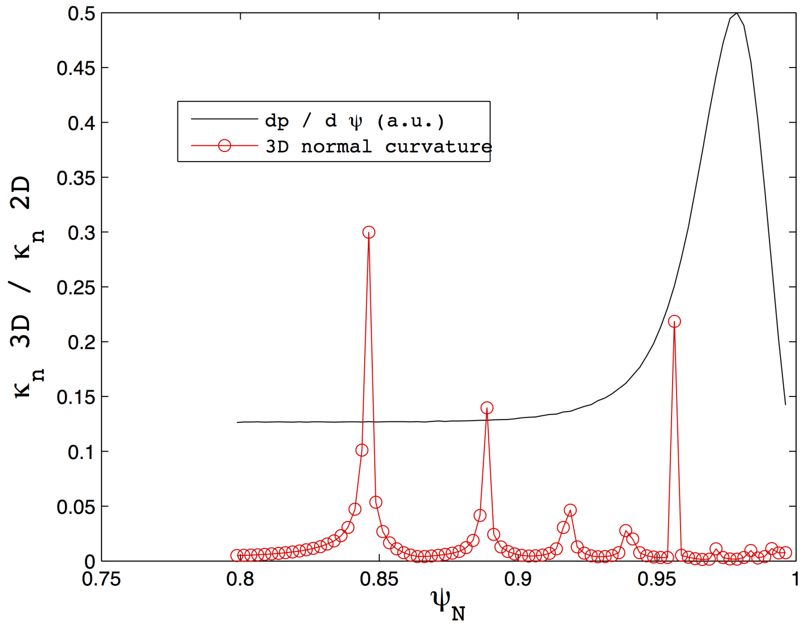
- Applied N = 3 **B**-field with broad range of m
- 3 ranges of q_{95} where lower order Surface goes through $\psi_n \sim 0.97$
  - $q_{95} \sim 3.8\text{-}3.9$
  - $q_{95} \sim 3.5\text{-}3.6$
  - $q_{95} \sim 3.2\text{-}3.3$



Geometric coefficients monitered during scan

- Example for case with large 3-D modulations of local shear near pedestal top





  - Initial conclusion --- applied 3-D fields on DIII-D are big enough to produce interesting changes to geometry
    - Note: not every resonant surface has a large variation in local shear

Future work: stability properties of 3-D equilibria

- Initial work has identified cases with interesting 3-D deformations of flux surfaces
- Infinite-n ballooning stability boundaries → quantify the 3-D effect
- Gyrokinetic simulations with GENE
  - Flux tubes
  - Full surface

Summary

- 3-D deformations of the magnetic flux surface shape can have important effect on MHD stability and transport
  - Effects are purely geometric → not reliant on any particular physics model
- 3-D deformations can produce O(1) changes to local MHD stability boundaries, sensitive to q resonanses
  - Indicates applied **B**-fields can directly affect microinstability/ anomalous transport
- 3-D deformations produce modest changes to global MHD modes' growth rate spectrum
- Suggests a possible scenario for RMP induced ELM suppression
  - Consistent with sensitivity to q_{95} and collisionality
- Beginning analyses for DIII-D cases indicate 3-D effects are big enough to matter