The origins of tokamak density limit scalings

Presented at the Columbia University
In the City of New York
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Outline

- Review of the Greenwald limit
  - A highly studied phenomenon
- Radiation island physics
  - Rebut radiation driven island theory
  - Local island threshold
  - Profile model - the relationship to the Greenwald limit
- Island growth analysis
- Radiation physics
- Experimental evidence for the model
- Summary and future work

Where does the Greenwald limit come from?

- The empirical tokamak operational limit (also known as the Greenwald limit) relates the maximum achievable line average plasma density to the circular-equivalent current density

$$\bar{n}_e(10^{20}\,m^{-3}) < \frac{I_p}{\pi a^2}(MA/m^2)$$

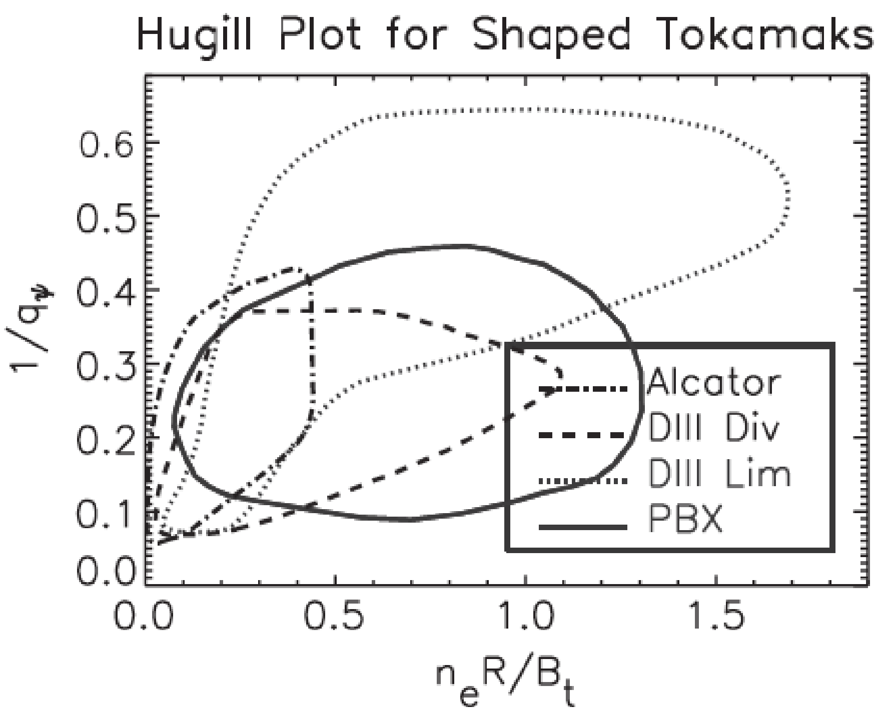
  – A radiative limit should scale as $P^{1/2}$
- The Greenwald limit is a fairly robust result

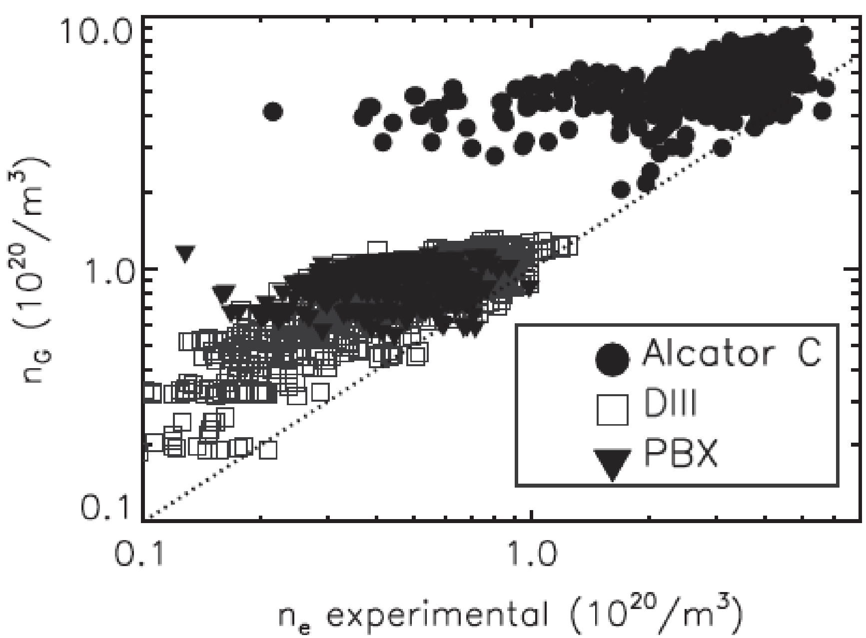
Puzzles associated with the Greenwald limit

1) The scaling is universal, but the phenomenon appears to be associated with radiative collapse and tearing modes, which have complicated dependencies on plasma parameters
2) If the physics is associated with radiative collapse, why is the density limit so weakly dependent on heating power?
3) Why is the limit only weakly dependent on Z_{eff}?
4) The collapse is associated with the onset of magnetic islands, so why does the limit not depend on plasma shaping or q (both which are known to affect MHD stability)?
5) Why is the density limit power scaling different in stellarators?
6) Why are tearing modes associated with a radiative collapse?

Evolution of the density limit

- The form of the density limit changed as databases from multiple tokamaks were amassed
- Hugill plot used q out of deference to MHD – works fine for circular cross-section machines
- Greenwald showed MHD shaping factor doesn't matter

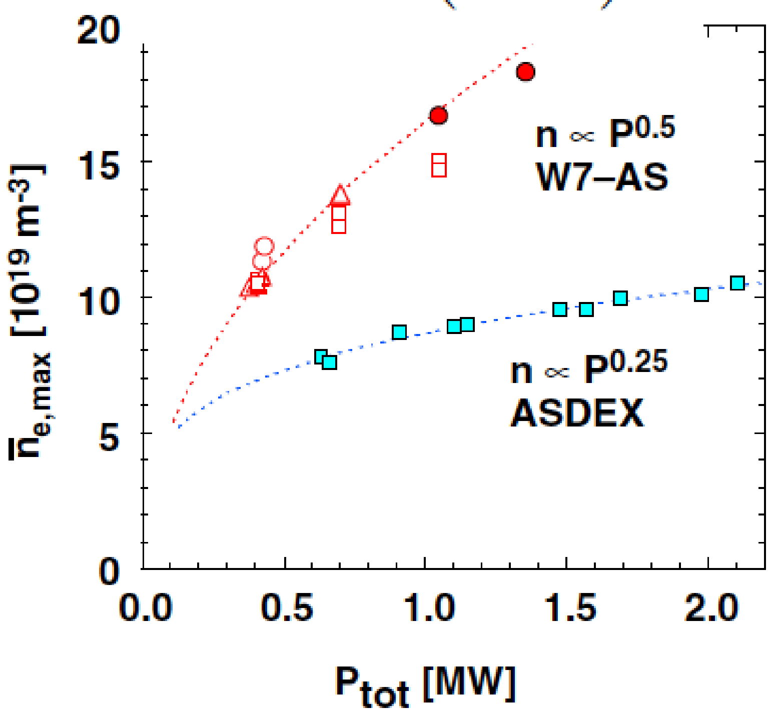




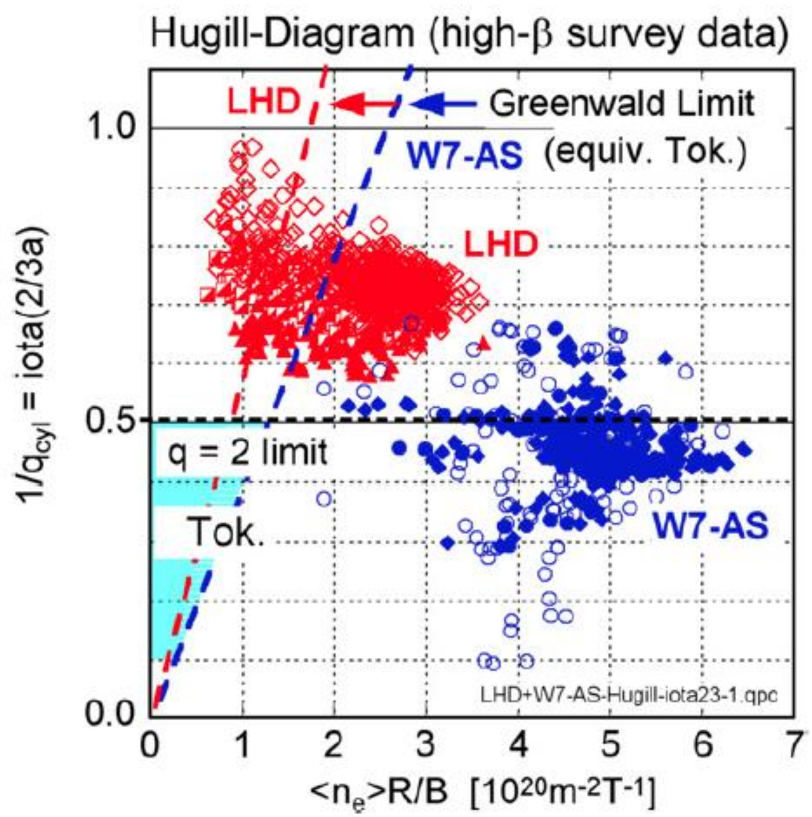
Stellarators are different than tokamaks

- Density limit clearly does not obey tokamak scalings
- Stellarator density limit is given by the Sudo limit

$$n_{\rm lim} = 0.25 \left(\frac{P_{\rm in} B_{\rm T}}{a^2 R} \right)^{0.5}$$



M. Greenwald, et al., Plasma Phys. Control. Fusion **44** (2002) R27–R80



A. Weller, et al., Nucl. Fusion **49** (2009) 065016

Radiation increases at the Greenwald limit

- Radiation physics matters!
  - Why doesn't the Greenwald limit depend on heating power?
- Collapse is not associated with fixed P_{rad}/P_{tot}

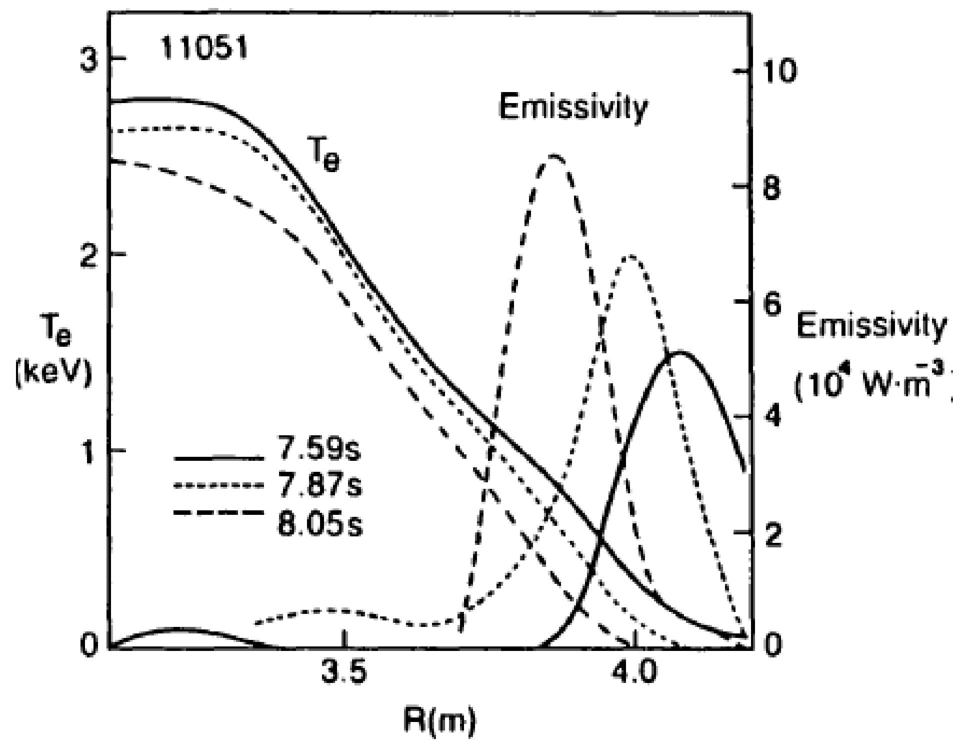
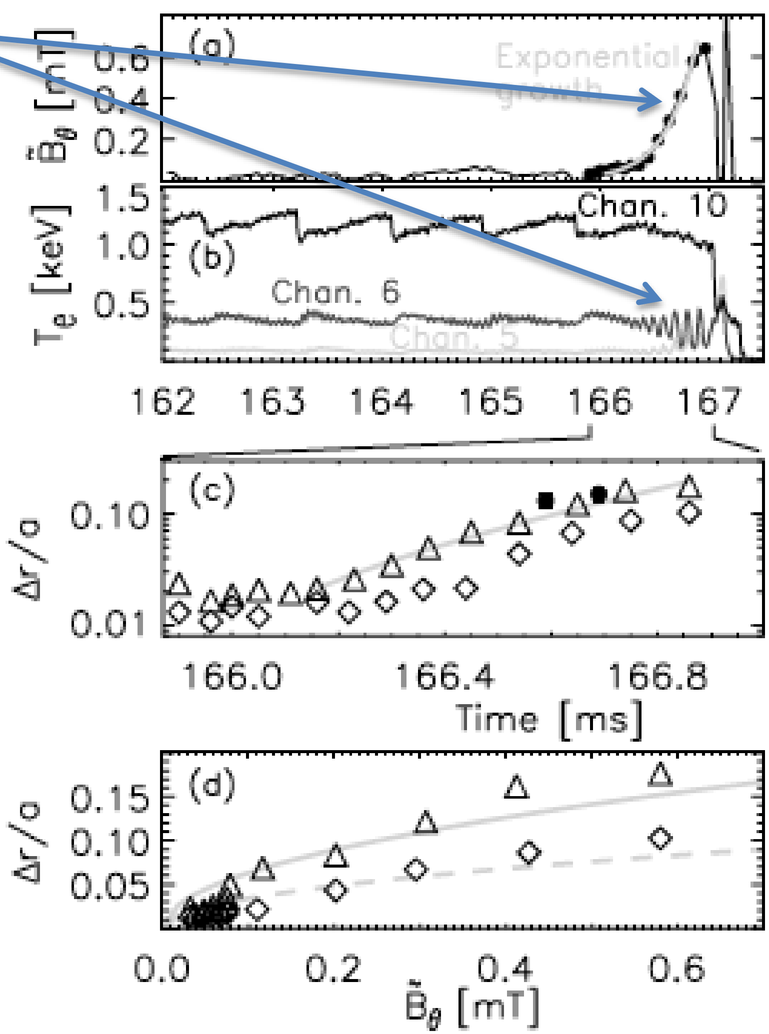


FIG. 12. Temperature and radiated power profiles during the plasma contraction.

*J. A. Wesson, R. D. Gill, M. Hugon, F. C. Schuller, J. A. Snipes, et al., Nucl. Fusion **29** (1989) 641

Tearing modes precede the density limit collapse

- MHD mode preceding collapse is ubiquitous
- Explained by Wesson as a classical Δ' change caused by the l_i increase
  - Classical Δ' is a sensitive parameter -> not a robust effect
  - Classical tearing modes grow like $t^{1/2}$ and saturate (R. B. White, et al. 1971)
  - Wesson model has not been successfully modeled from a stability point of view



F. Salzedas, et al., PRL **88** (2002) 075002

Summary of issues

- Associated with radiation - but not heating power
  - Relatively insensitive to Z_{eff}
- Current matters (like MHD) but shape doesn't (not like MHD)
- MHD tearing modes occur
- An apparently complex phenomenon is universal

The islands at the density limit have been identified as possibly radiation driven

- Suttrop et al. did extensive study on ASDEX-U (1997)
- Did not draw a causal connection between islands and the density limit
- Did say "A number of experimental observations suggest that growth of the (3,1) island can be assisted or driven by a radiation instability from the island"

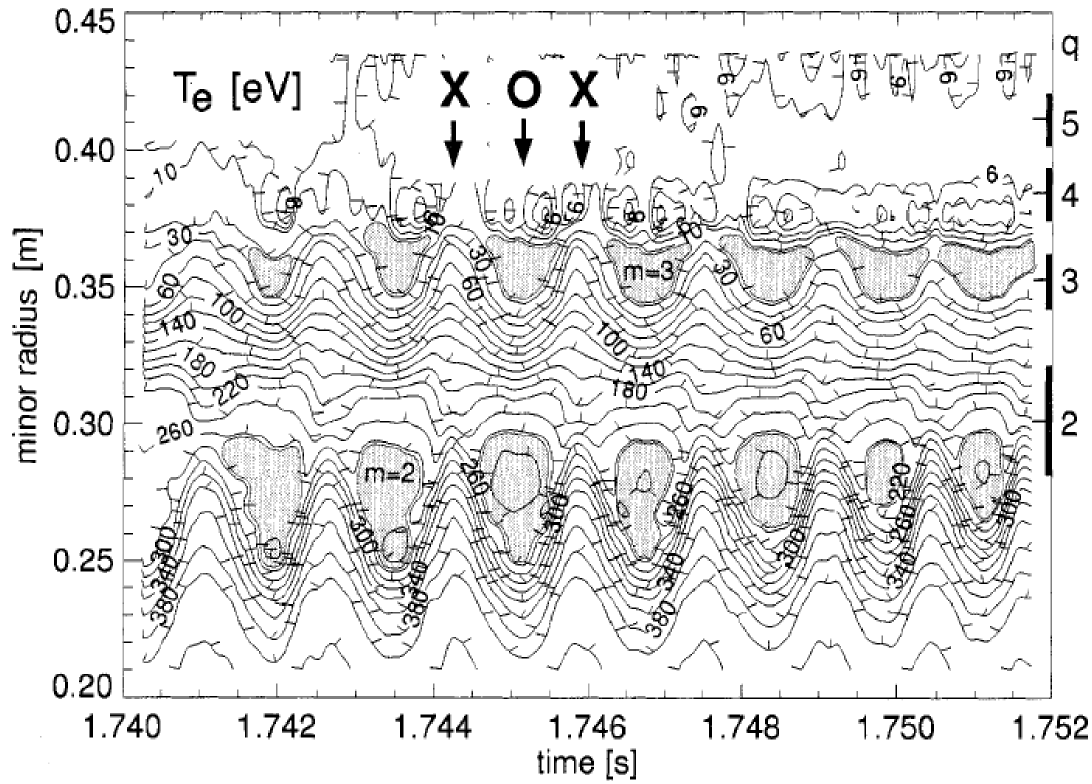
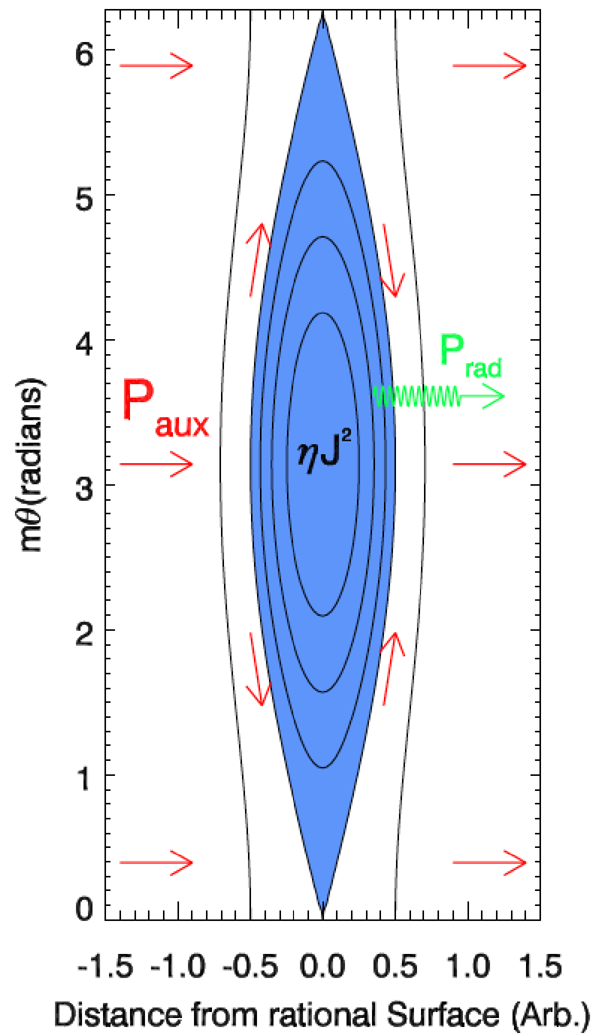


FIG. 2. Reconstruction of coupled (2,1) and (3,1) islands from T_e measurements in a time interval during current profile contraction between two minor disruptions. Islands recognized by regions of flat T_e are marked by shaded areas. While the (3,1) island grows, the (2,1) island shrinks. $q(r)$ is derived from equilibrium reconstruction at $t = 1.75$ s with radial uncertainties indicated.

Radiation driven islands

- The island is magnetically insulated from it's surroundings
- So radiation can cool the island,
- Lower temperature leads to increased resistivity
- The lowered current enhances the helical current perturbation
- The island then grows causing the process to continue.



P. H. Rebut and M. Hugon, Plasma Physics and Controlled Nuclear Fusion Research 1984 (Proc. 10th Int. Conf. London, 1984), Vol. 2, IAEA, Vienna, 197, (1985).

Radiation drive in the MRE

- Power balance in the island

$$n_e \chi_\perp \nabla T_e A_{island} = \delta P * V_{island}$$

  - where A_{island} is the surface area of the integrated over the inside and outside of the island and V_{island} is the volume of the island.

- Relate the current to the temperature using resistivity and use Rutherford *Δ*□ formula

$$\frac{dJ}{J} = - \frac{3dT}{2T} \qquad \Delta' = 16 k_1 \frac{\delta J}{swJ}$$

- Find the radiation drive term

$$\Delta' = 3 \frac{r_s s_I}{s} \frac{\delta P}{n_e \chi_\perp T_e} w$$

P. H. Rebut and M. Hugon, Plasma Physics and Controlled Nuclear Fusion Research 1984 (Proc. 10th Int. Conf. London, 1984), Vol. 2, IAEA, Vienna, 197, (1985).

Modified Rutherford equation with radiation

Radiation term

$$\frac{k_0}{\eta}\frac{dw}{dt} = \Delta' r_s - C_1\left(\frac{w}{w^2 + w_\chi^2}\right) + \frac{C_2}{w^3} + C_3 w$$

Rutherford term Pressure driven current Polarization current

- For now, ignore the bootstrap and polarization terms (consider low to moderate β_p)
- The MRE then becomes:

$$\frac{k_0}{\eta}\frac{dw}{dt} = \Delta' r_s + C_3 w$$

Where: $C_3 = 3\left(r_s s_I / s\right)\left(dP / \left[n_e c_{\wedge} T_e\right]\right)$

Exponential growth

- The radiation term changes sign when $\delta P = 0$ or

$$P_{rad} = P_{island}$$

Radiation drive term changes sign when island cools

$$P_{rad} < hJ^2 \quad or \quad n_e E_{ave} u_{eZ} < \frac{m_e u_{ei}}{e^2 n_e} J^2$$

$$n_e < \sqrt{\frac{m_e}{e^2 E_{eff}} \frac{u_{ei}}{u_{eZ}}} J$$

- Assume ohmic heating dominates inside of the island
- Auxiliary power is shunted around the island by parallel conduction, consistent with density limit being independent of heating power
  - Constant temperature island boundary
- Quantity in square root is nearly independent of temperature*
- Reminiscent of the Greenwald limit

*F. W. Perkins and R. A. Hulse, Phys. Fluids **28** (1985) 1837.

The onset of the density limit is determined by collisional processes

- The quantity under the square root is the radiative drift velocity loss per electron
  - Depends on the species mix of the plasma
  - Is nearly independent of plasma temperature

$$n_e < \sqrt{\frac{m_e}{e^2 E_{eff}} \frac{u_{ei}}{\left(u_{eZ}\right)_{eff}} J}$$

or

$$n_e < \frac{J}{e \Delta \mathrm{v}_{rad}} = f(Z) J$$

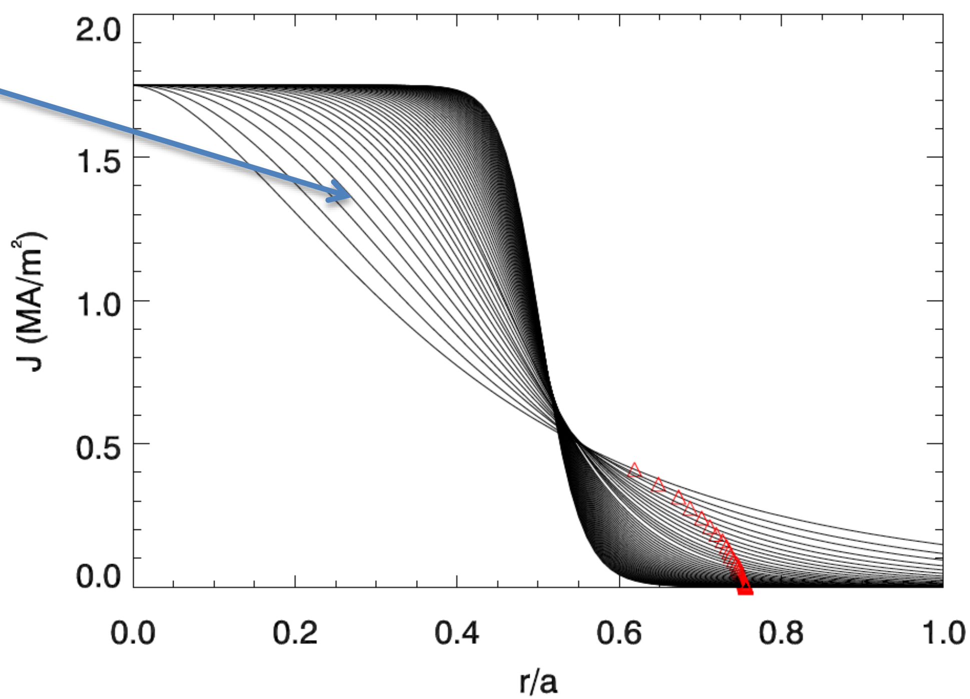
Simple cylindrical model relates local density and current to global values

- Use a simple profile model

$$J = \frac{J_0}{\left(1 + \left(\frac{r}{r_0}\right)^{2\nu}\right)^{1+\frac{1}{\nu}}}$$

- Assume parabolic density profile
- Still to many variables
  - Need additional information to determine J(r) at the density limit

Current profiles used in simple density limit model at constant-q



D. A. Gates and L. Delgado-Aparicio, Phys. Rev. Letters **108** (2012) 165004

Current profile peaking at the density limit

- Corresponding to the density limit there is a corresponding (simultaneous) l_i-limit
- Fit this curve with a line

$$l_i = 0.12 q_{edge} + 0.6$$

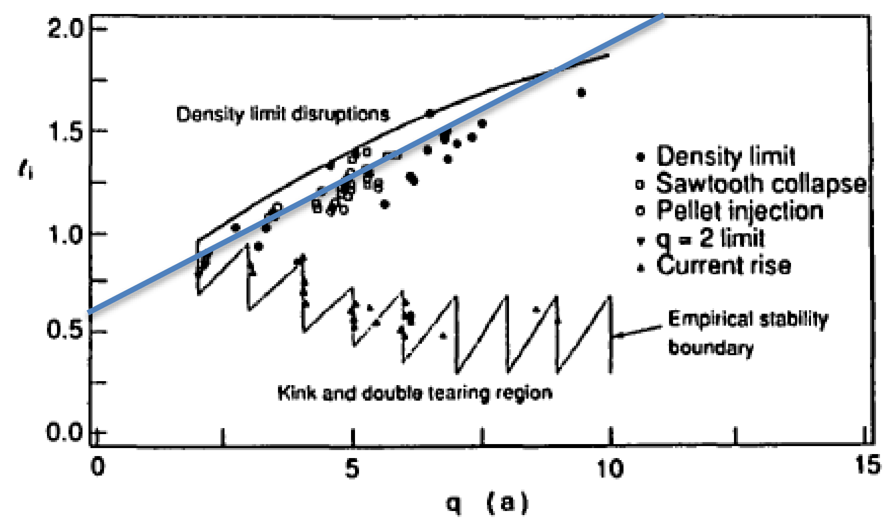


FIG. 6. Empirical stability diagram for JET, showing the ℓ_i–q plane. The lower boundary (dotted) indicates the stability boundary for rotating MHD modes during the current rise. The upper boundary (solid) indicates the region where major disruptions occur. The symbols indicate the onset of quasi-stationary modes in various situations.

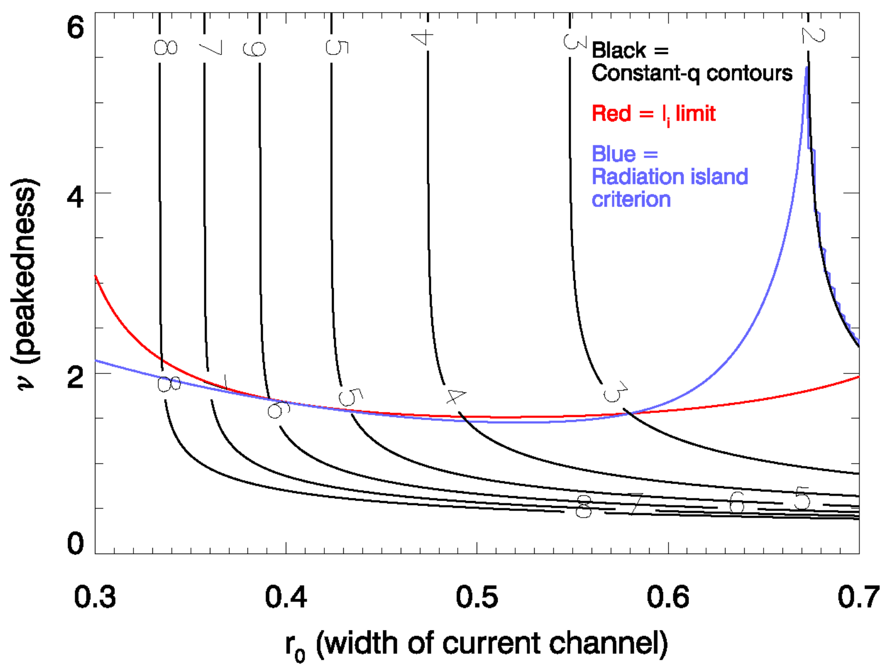
*J. A. Wesson, R. D. Gill, M. Hugon, F. C. Schuller, J. A. Snipes, et al., Nucl. Fusion **29** (1989) 641

A contour of constant local power balance correspond with the contour of maximum l_i

- Indicates that the local and the global scaling laws are co-linear if the current profile corresponds with the l_i observed at the density limit
- Can use the fit to evaluate the numerical agreement between the two limits
- Assuming carbon as the dominant impurity and Z_{eff} = 2, one finds:

$$\frac{f(Z)\pi a^2 J(r_{m/n})}{I_{\rm tot}\left(1-\frac{r^2_{m/n}}{a^2}\right)}\frac{\bar{n}_e}{n_e(0)}\sim 1.7$$

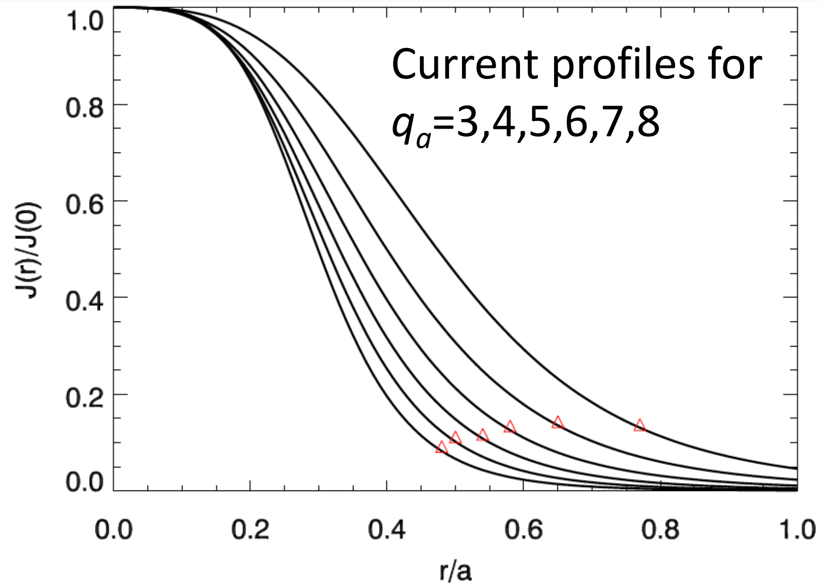
Compared to expectation of ~1



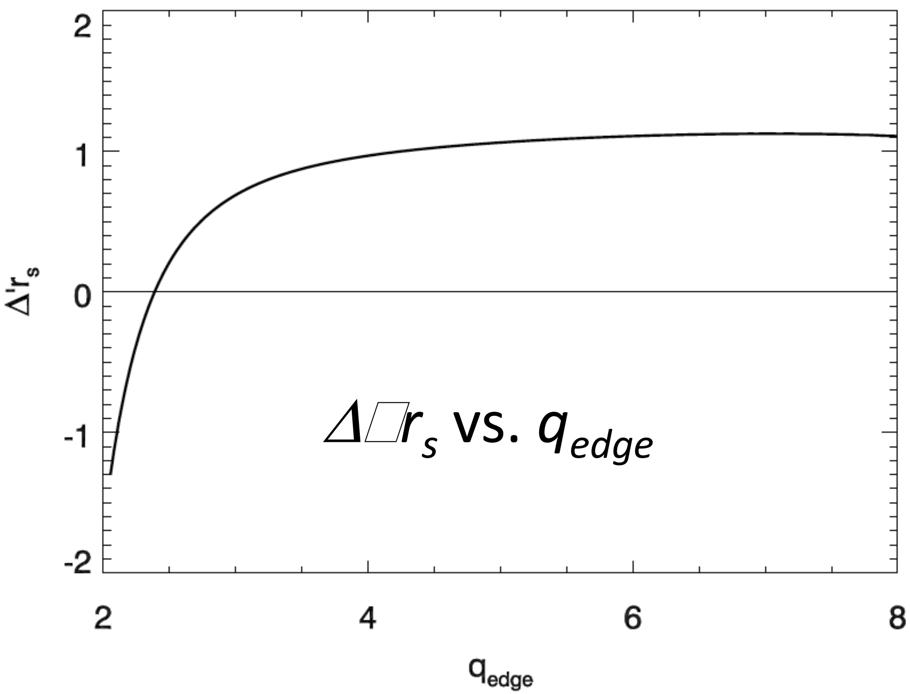
Contour plot of the total plasma current (black) as a function of the profile parameters ν and r_0. Also shown in the plot are the contour of the current profile peaking at the density limit and the best fit contour of the radiation driven island criterion

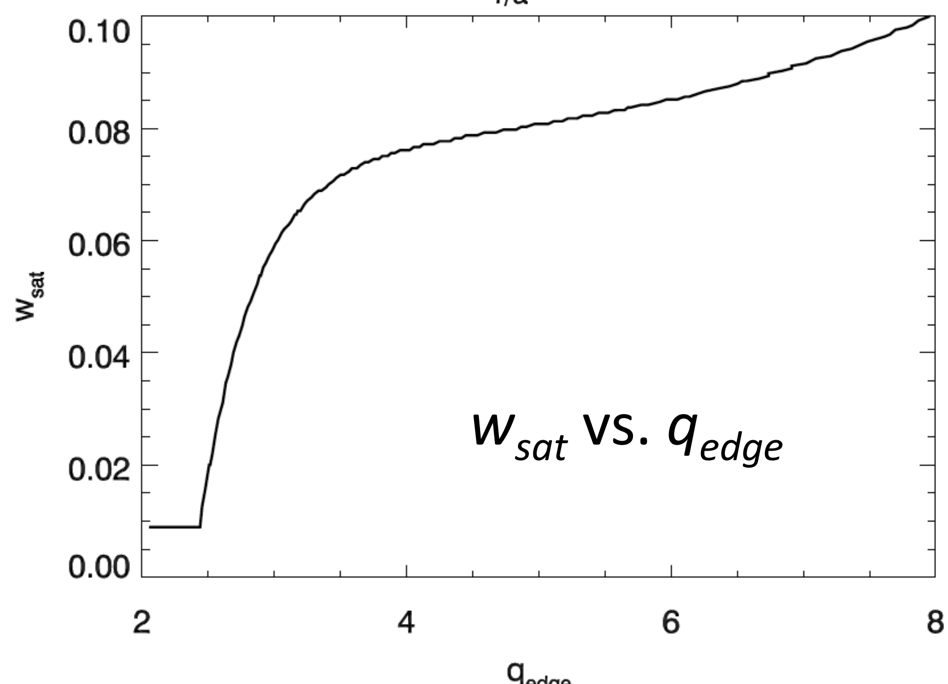
Evaluate Δ' for current profiles at the density limit

- Δ' is small and positive, resultant island saturates at small size
- Conclude Δ' provides a small saturated seed island



D. A. Gates, L. Delgado-Aparicio, R. B. White, accepted for publication, Nucl. Fusion (2013)





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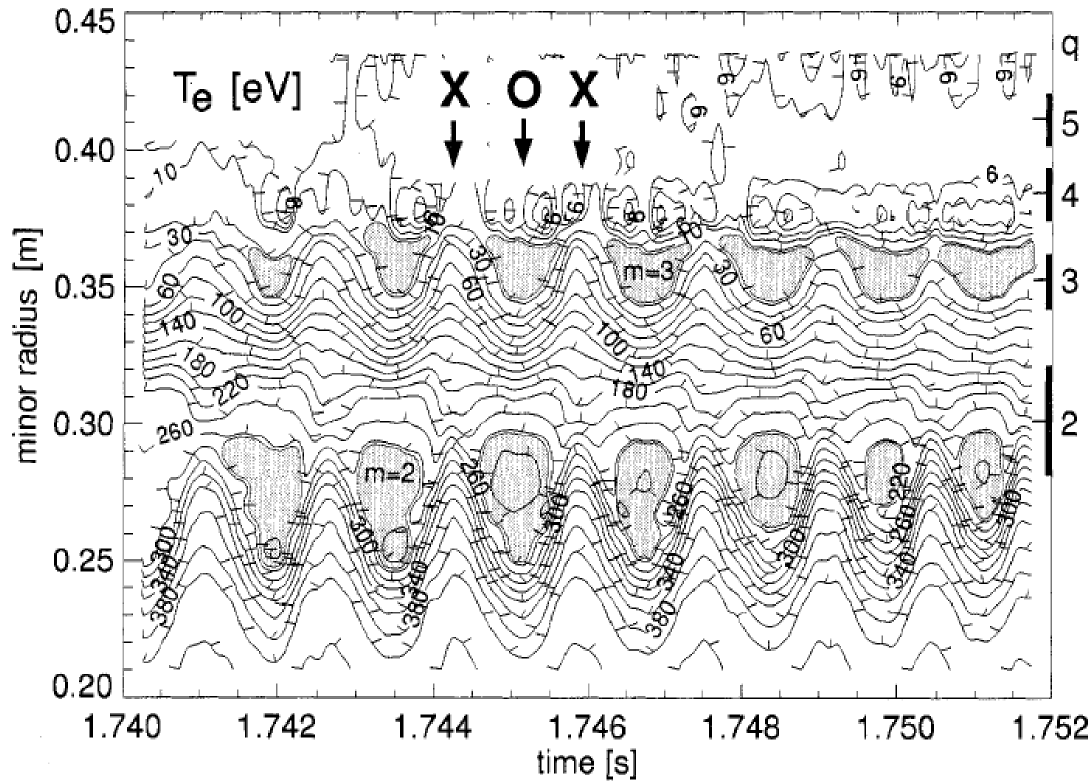

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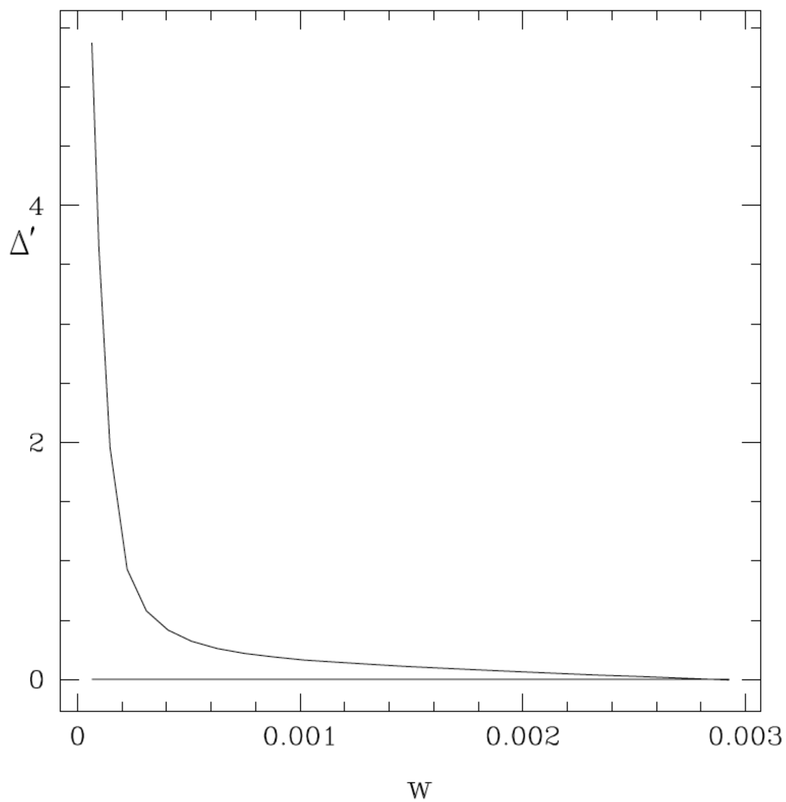
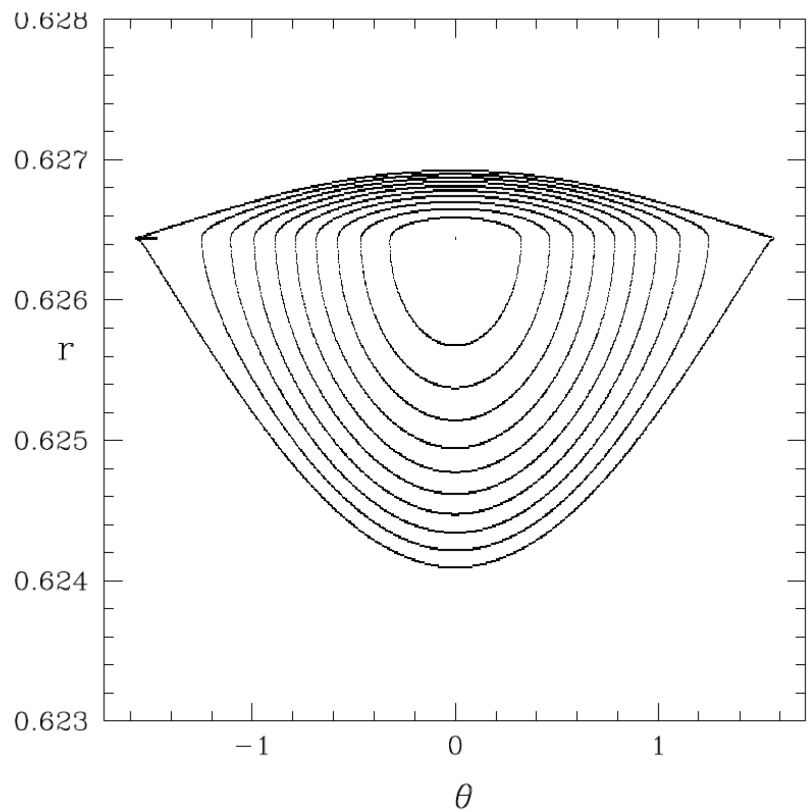
Island asymmetry is due to the peaked current profile

- Asymmetric islands with flattened temperature profiles saturate at larger island width

Δ′(w) (White et al, 73)



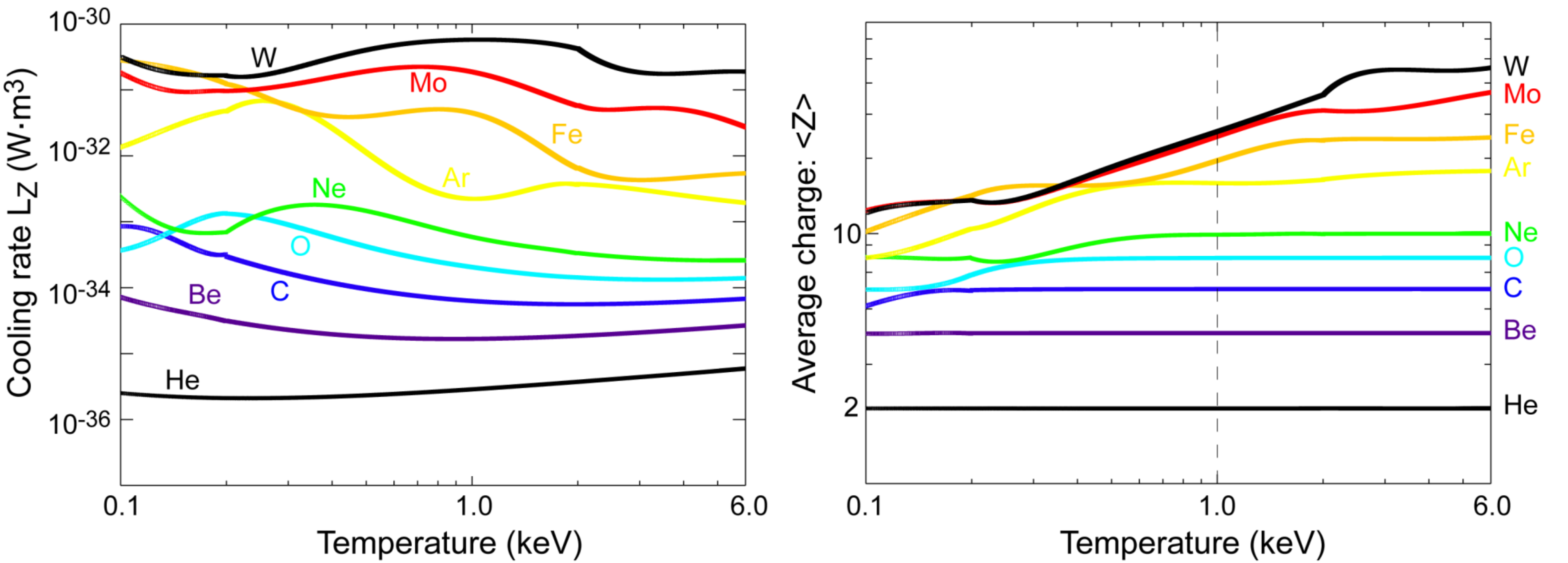
Asymmetric linear eigenfunction



Cooling rates L_Z and average charge <Z> can be obtained using the average ion model (AIM)

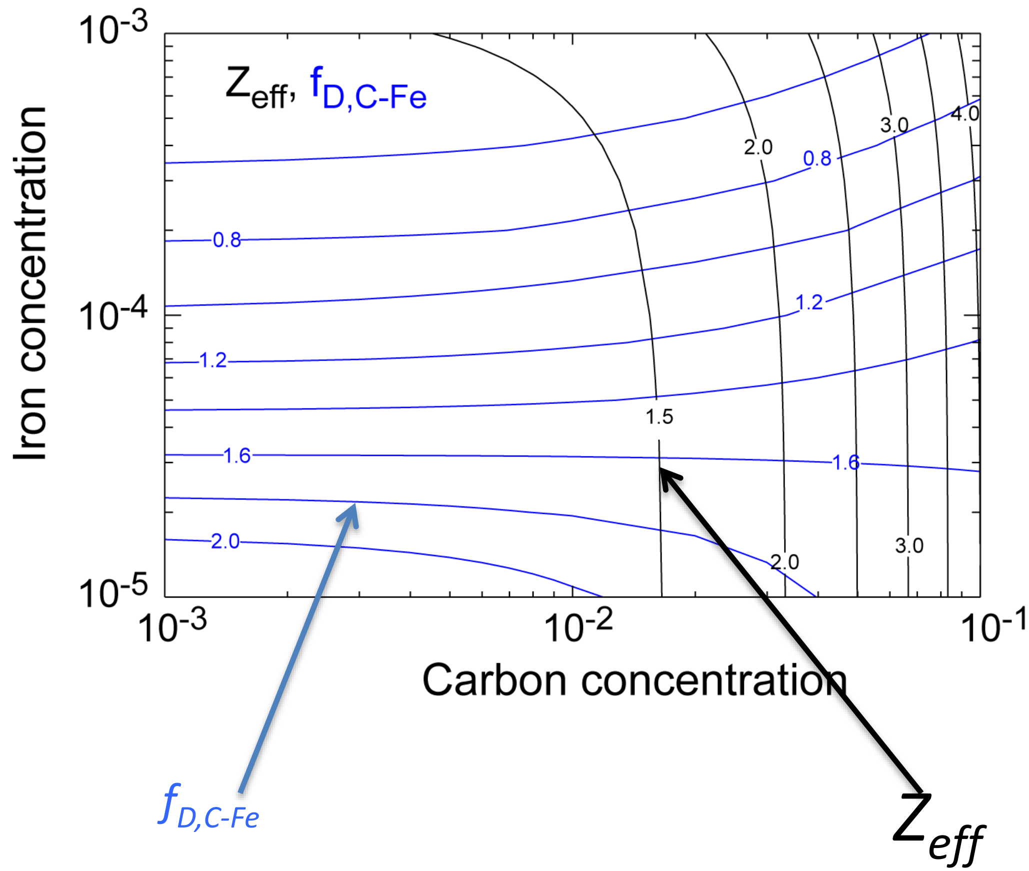
D. E. Post, *et al.*,
Steady-state radiative cooling rates for low-density, high temperature plasmas, Atomic Data and Nuclear Data Tables, **20**, 397-439, (1977).

$$P_{rad_i} = n_e n_{Z_i} L_{Z_i}$$



How does impurity mix affect the the density limit?

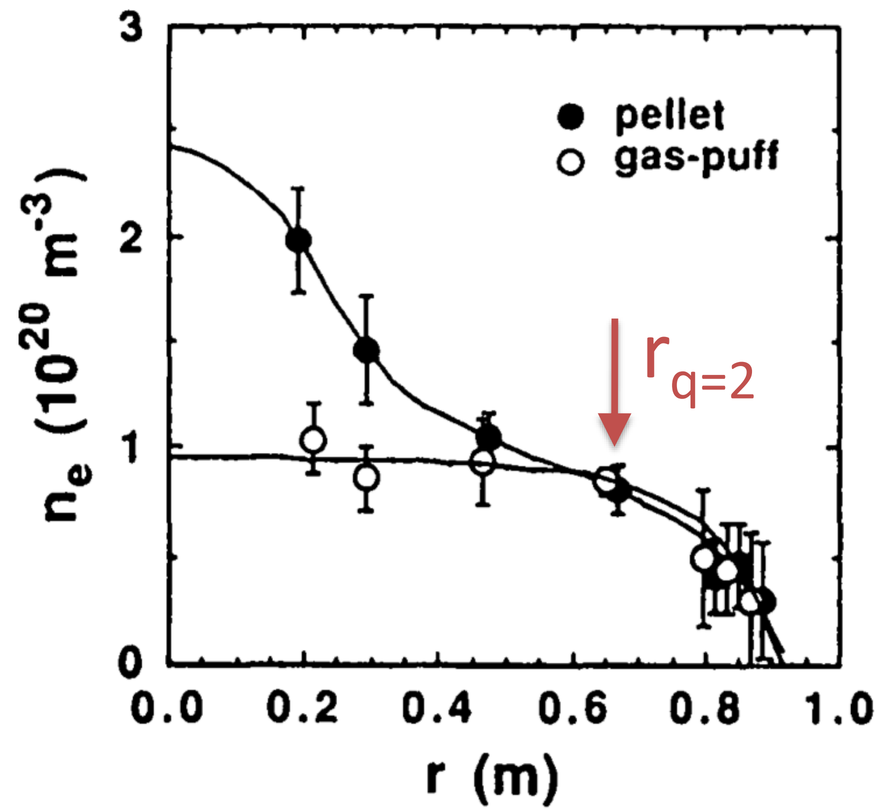
- The value of the density limit coefficient is also nearly independent with Z_{eff}
- Quantitative comparison to data required to verify impurity mix dependence



Density limit can be exceeded (1)

- Central fueling doesn't induce density limiting phenomena
- The density limit can be extended by
  - central fueling
  - edge pumping
  - edge transport mods
- The density at the q=2 surface is preserved

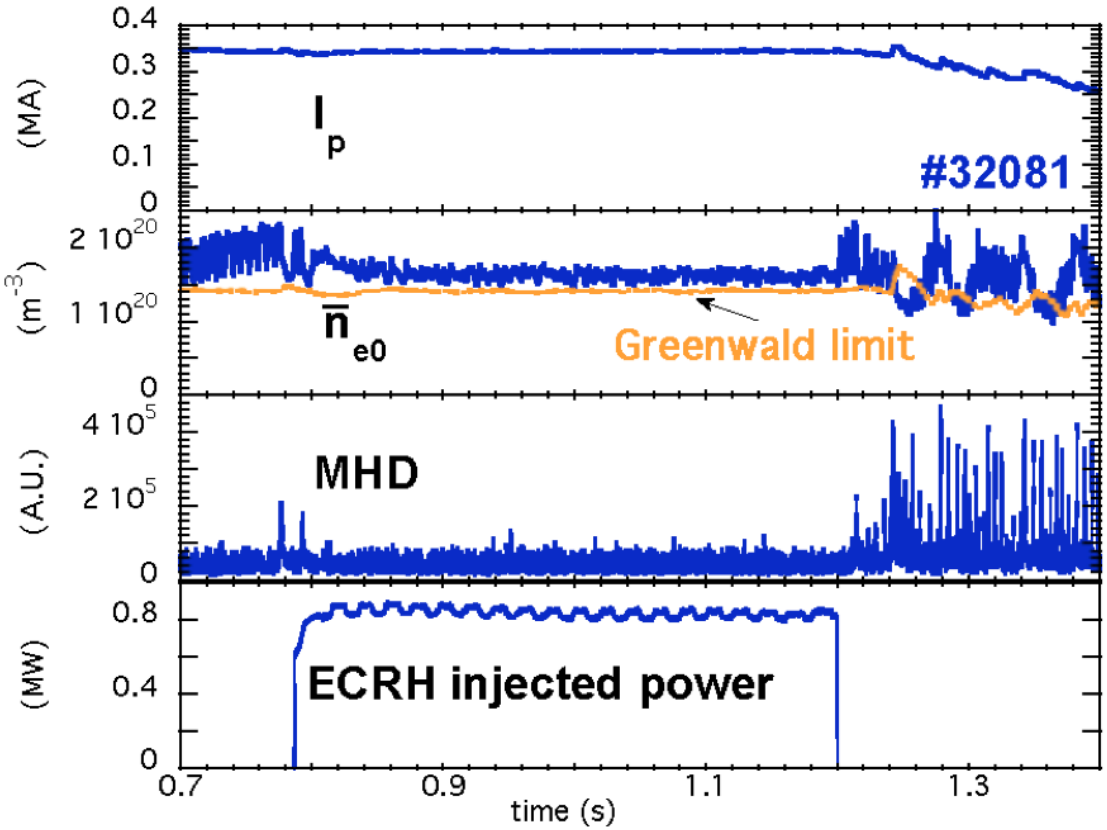
Results from JT-60U showing $n_e \sim 2^*n_G$ with pellet fuelling

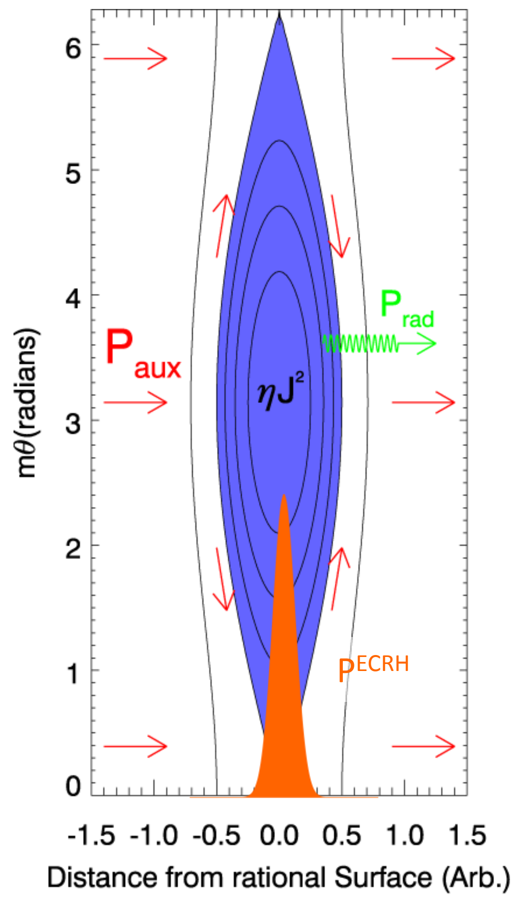


Y. Kamada, *et al.*, Nucl. Fusion, **31**, (1991) 1827

Local ECRH stabilizes 2/1 modes in Ohmic plasmas avoiding disruptions and achieving higher density limit

Tested in FTU, ASDEX-U, RTP (to be re-visited)





F. Salzedas, NF’02, B. Esposito, *et al., PRL*’08, NF’09,
G. Granucci, *et al.*, AIP’09.

$$\eta J^2 + P^{ECRH} \gtrsim P_{rad}$$

Ongoing and future work

- Ongoing work
  - Verify effect with full non-linear (cylindrical) simulation - Brennan
  - Complete analytic theory - White
  - Paper describing the impurity dependence of radiation – Delgado-Aparicio
- Next steps
  - Full simulation in toroidal geometry including radiation model
  - reproduce Greenwald limit using experimental parameters

Implications and future plans

- This theory provides a testable quantitative prediction of the density limit based on local measurements and points to methods for exceeding the limit and controlling disruptions
  - Important for ITER
- Theory predicts exponentially growing islands with a sudden robust current dependent onset condition
  - Consistent with a robust density limit and observed rapidly growing 2/1 tearing mode that is absent in stellarators
  - Dependent on the existence of inductive current drive
  - May also help explain other disruptions
- Need to directly verify local power balance
  - Data analysis proceeding on NSTX
  - Experiments proposed on DIII-D, EAST, KSTAR