MST and the Reversed Field Pinch

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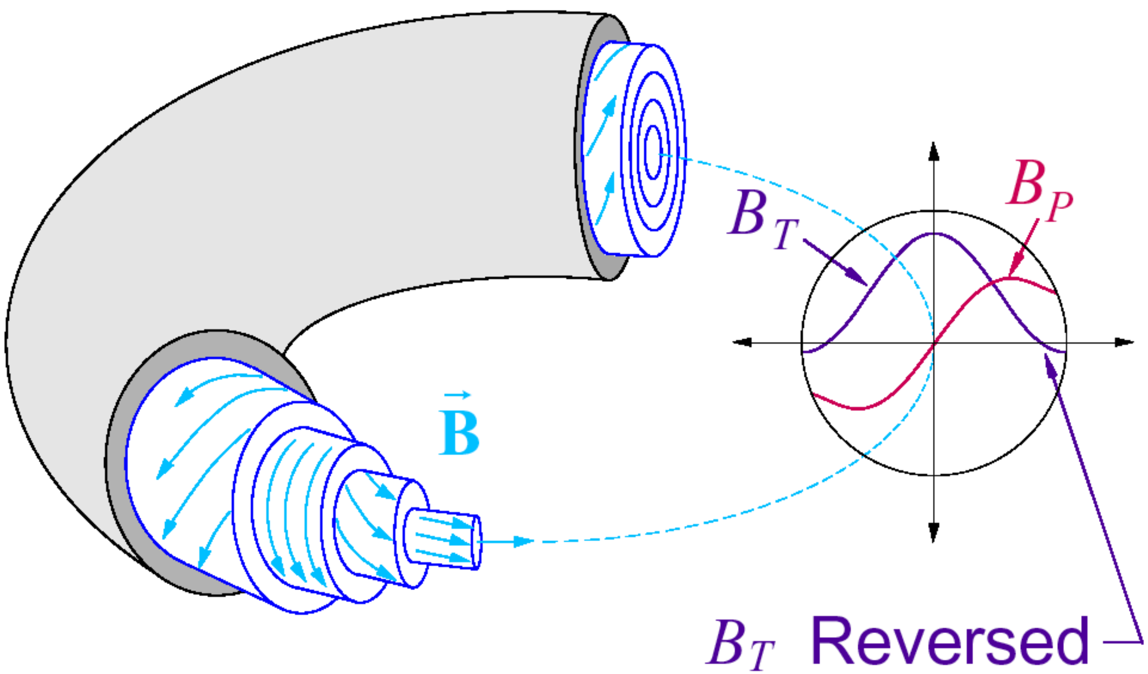
Outline

- Tutorial-level review of tearing stability, magnetic relaxation, and transport in the RFP
- Ion-related physics topics
- A few details on RFP fusion perspectives (as time allows)

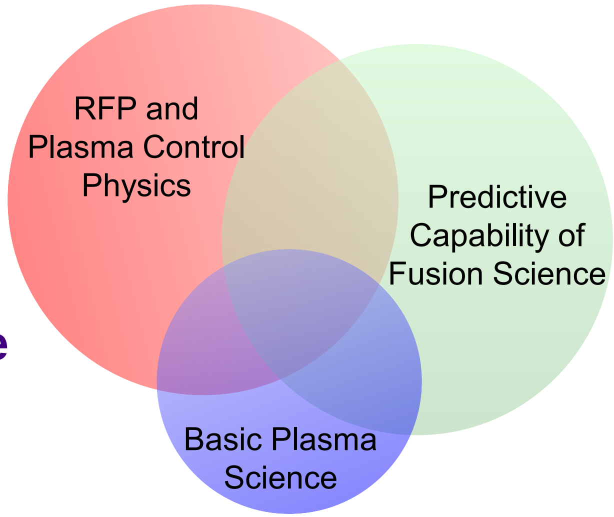
The Reversed Field Pinch magnetic configuration

- Magnetic field is generated primarily by the plasma current
- Small externally applied field:
  - Advantages for fusion
  - Large magnetic shear and weaker toroidal (neoclassical) effects, explores complementary/overlapping parameter space w/t the tokamak and stellarator
  - Basic science: magnetic self-organization and nonlinear plasma physics



Three synergistic research mission goals capture MST and the RFP's opportunities in fusion and plasma science

- **Advance the physics and control of the RFP plasma configuration**
  - MST is one of two large RFP experiments
  - Centerpiece of U.S.'s proof-of-principle program



- **Advance the predictive capability of fusion science**
  - Physics closely related to tokamak and stellarator, leveraged by RFP geometry
  - Validation of key physics models and codes
  - Development and application of advanced diagnostics

- **Discover basic plasma physics and its links to astrophysics**
  - Magnetic self-organization
  - Processes: magnetic reconnection, particle energization, momentum transport, magnetic turbulence

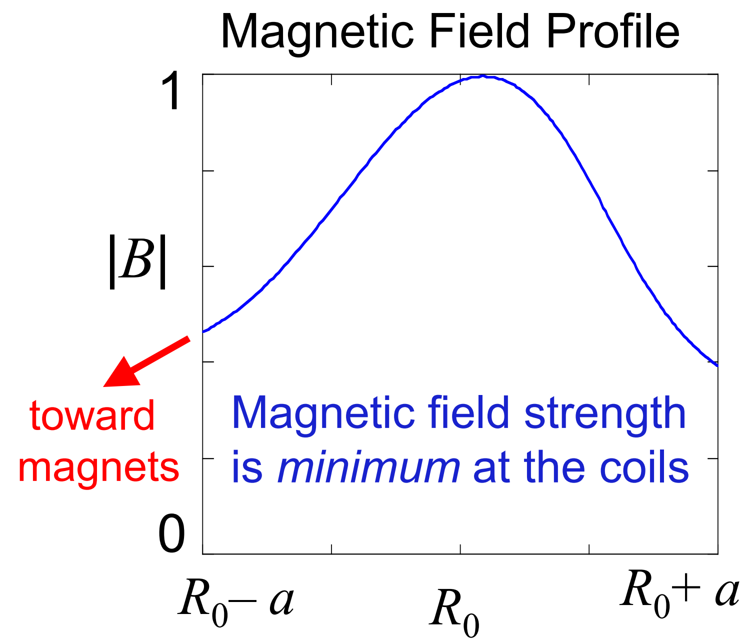
RFP's fusion advantages derive from the concentration of magnetic field within the plasma and small applied toroidal field

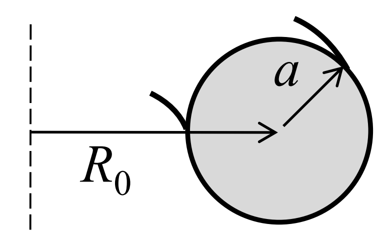
- Small field at the magnets ➡□ copper possible
  - 1/10th the magnetic pressure at the magnets compared to high safety factor (e.g., tokamak)
  - $\frac{B_{\text{coil}}}{\langle B \rangle} \sim \frac{2}{3}$ (*and* $B_{\text{T,coil}} \longrightarrow 0$)
  - Large fusion beta demonstrated:

    $$b_{\text{fusion}} \sim \langle p \rangle / B_{\text{coil}}^2 = 28\% \; ; \; \langle b \rangle = 10\%$$

- Large plasma current density
  - Ohmic ignition is possible

    ➡ No plasma-facing auxiliary heating components
  - High particle density limit, $n_{\text{GW}} \sim I_{\text{p}} / pa^2$





RFP's fusion advantages derive from the concentration of magnetic field within the plasma and small applied toroidal field

Why might these features be important?

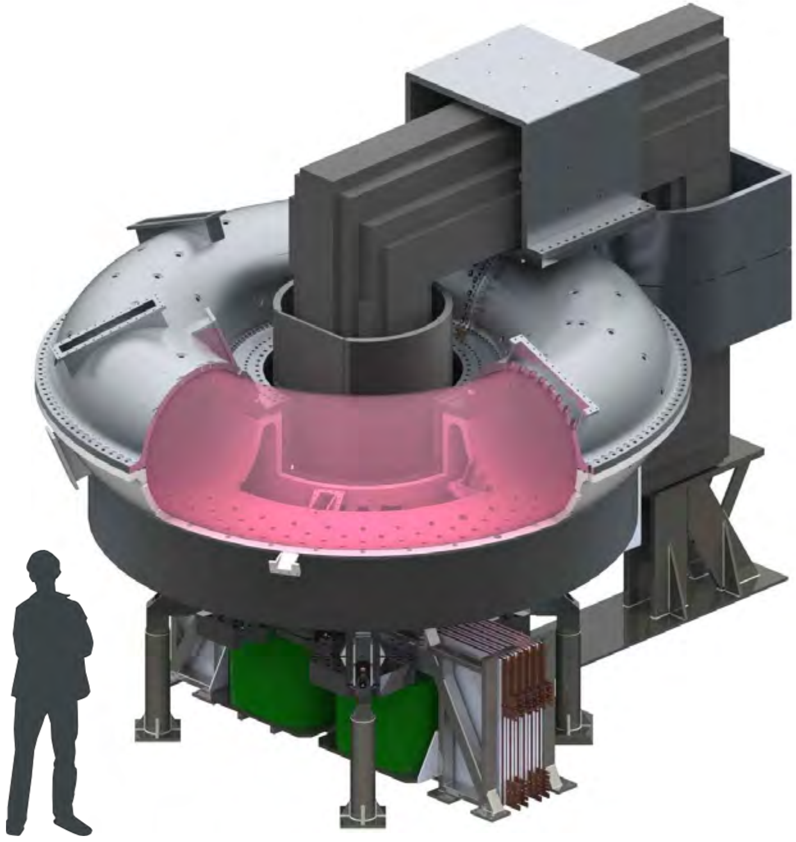
Example: Auxiliary heating is required for tokamak and stellarator plasmas, and it will likely be difficult to make RF antennas that maintain steady operation adjacent to 2.5 GW fusion plasma (a development "gap" for the tokamak & stellarator)

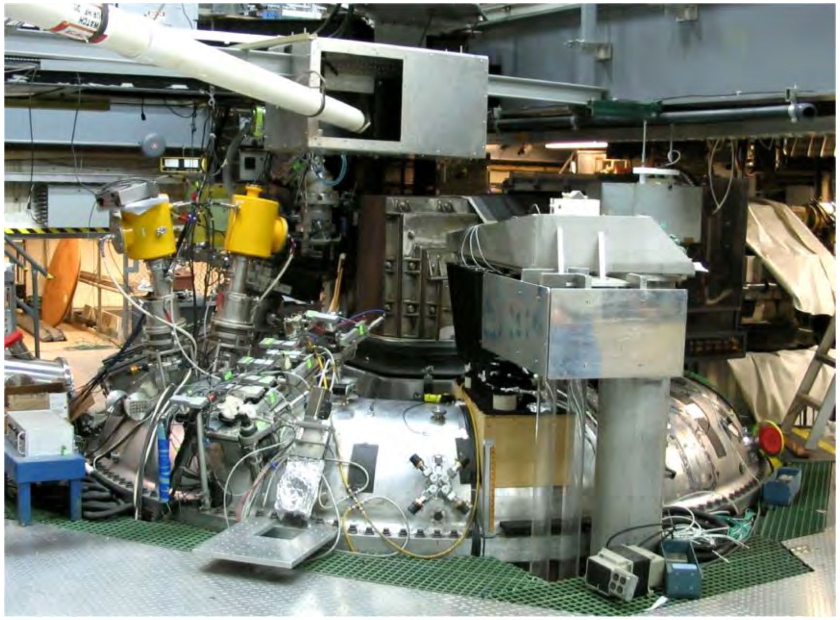
The RFP uses Ohmic heating, which is efficient and nearly invisibly crosses smooth material boundaries. It's available in large quantity only at low safety factor, $q \sim B_t / I_p$ (which the RFP explores)

Reliability and maintainability of a fusion power system are essential, but difficult to assess until high-performance plasmas of very long duration become routine

The Madison Symmetric Torus (MST)

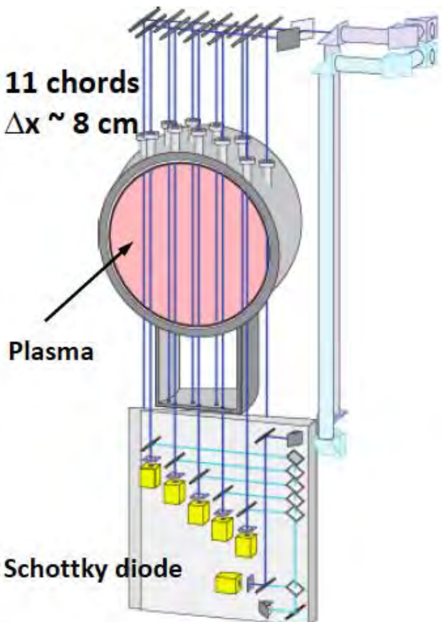
- Major and minor radii: R=1.5 , a=0.5 m
- Plasma current and magnetic field: I_p < 0.6 MA , $B(a)$ < 0.23 T
- Plasma density and temperature: $n \sim 10^{19}$ m^{-3} , $T_e \sim T_i \sim$ 2-3 keV



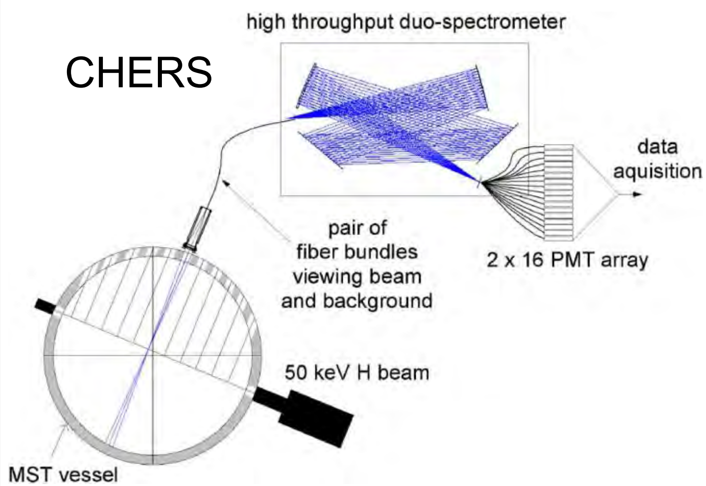


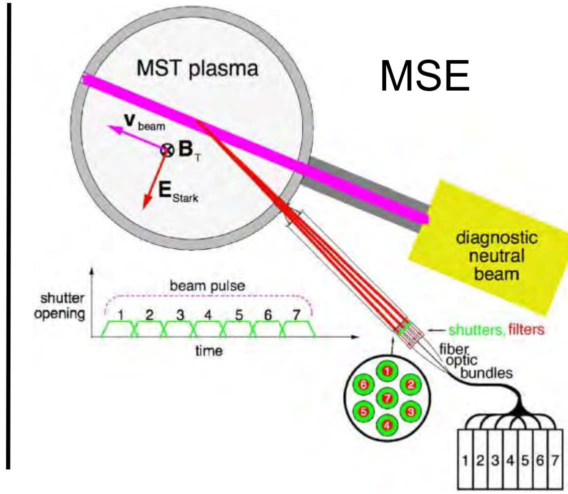
MST is equipped with advanced diagnostics, many state-of-the-art that impact designs elsewhere

FIR Interferometer-Polarimeter-Scattering (UCLA)

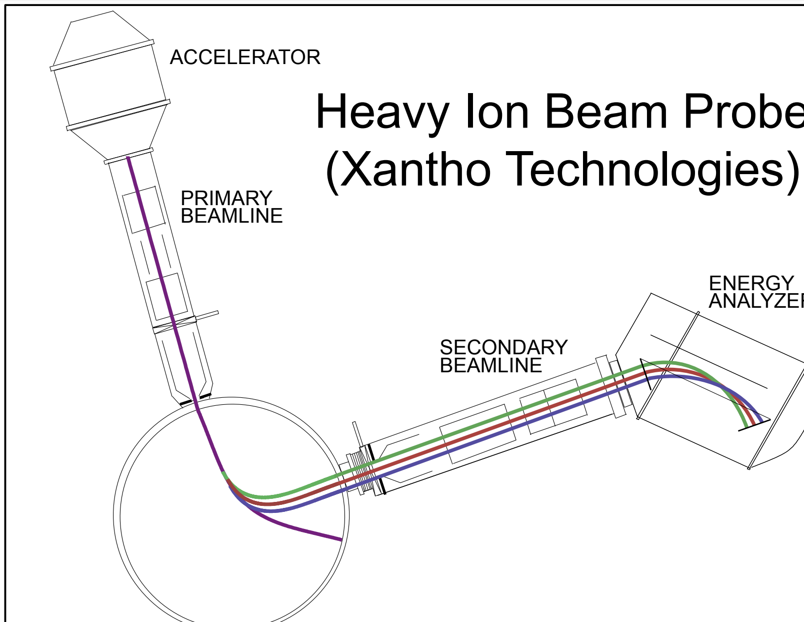


Diagnostic Neutral Beam (Active Spectroscopy)

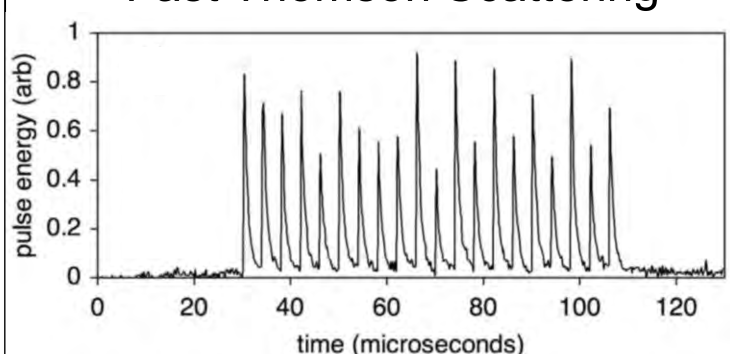




Heavy Ion Beam Probe (Xantho Technologies)

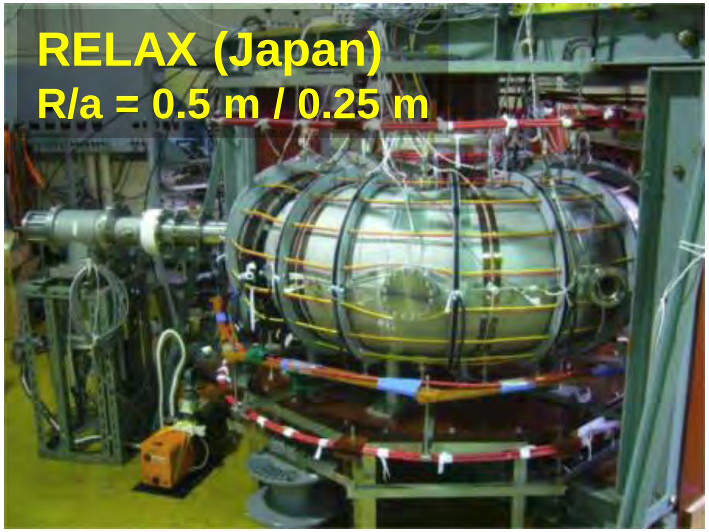


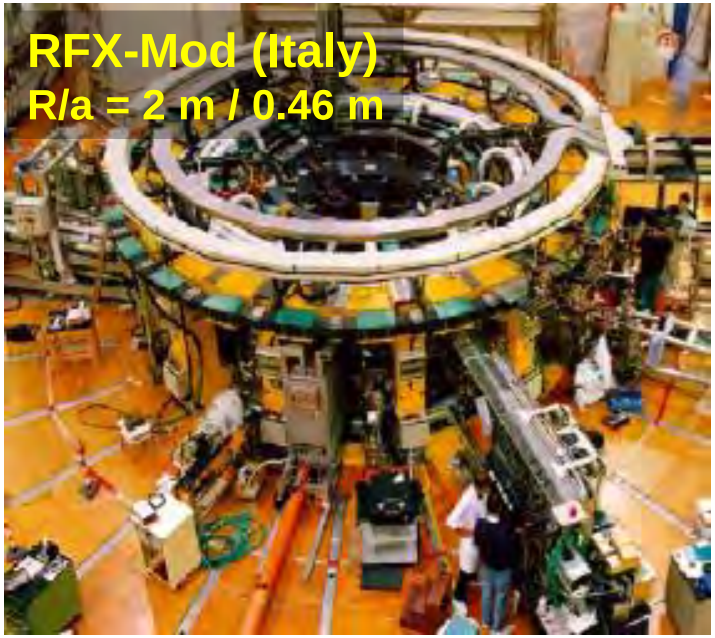
Fast Thomson Scattering

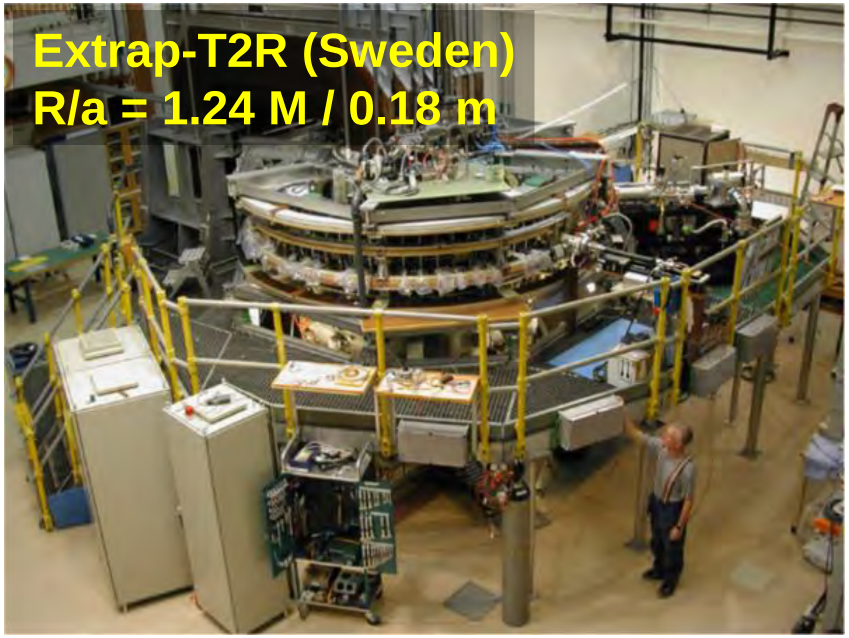


Other RFP experiments in the world program

- RFX-Mod
  - High current, 2 MA
  - Self-organized helical configuration
  - Feedback control of RWMs
- Extrap-T2R
  - Feedback control of RWMs
  - High power density, all-metal wall
- RELAX
  - Low aspect ratio, A=2







A new RFP program at the University of Science and Technology of China, Hefei

- First plasma in 2015; parts now being assembled in new experimental hall
- Shell system suitable for plasma-boundary studies, e.g., high-Z, lithium
- Advanced active mode control (Phase 2)
- Advanced inductive current drive and 3D effects in helical state
- Collaboration with EAST team on the design and construction

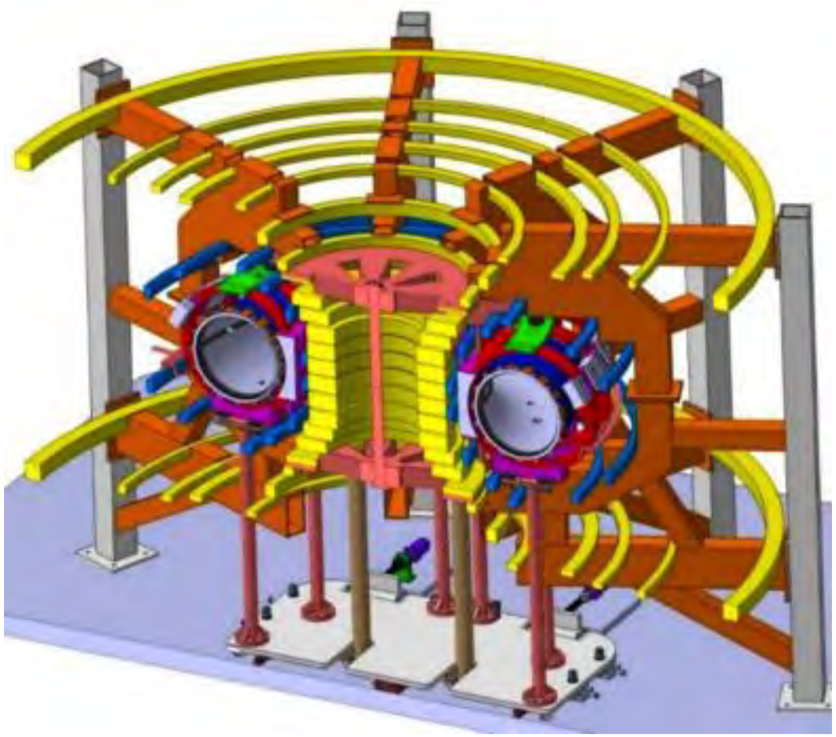
Keda Toroidal Experiment (KTX)

R = 1.4 m

a = 0.4 m

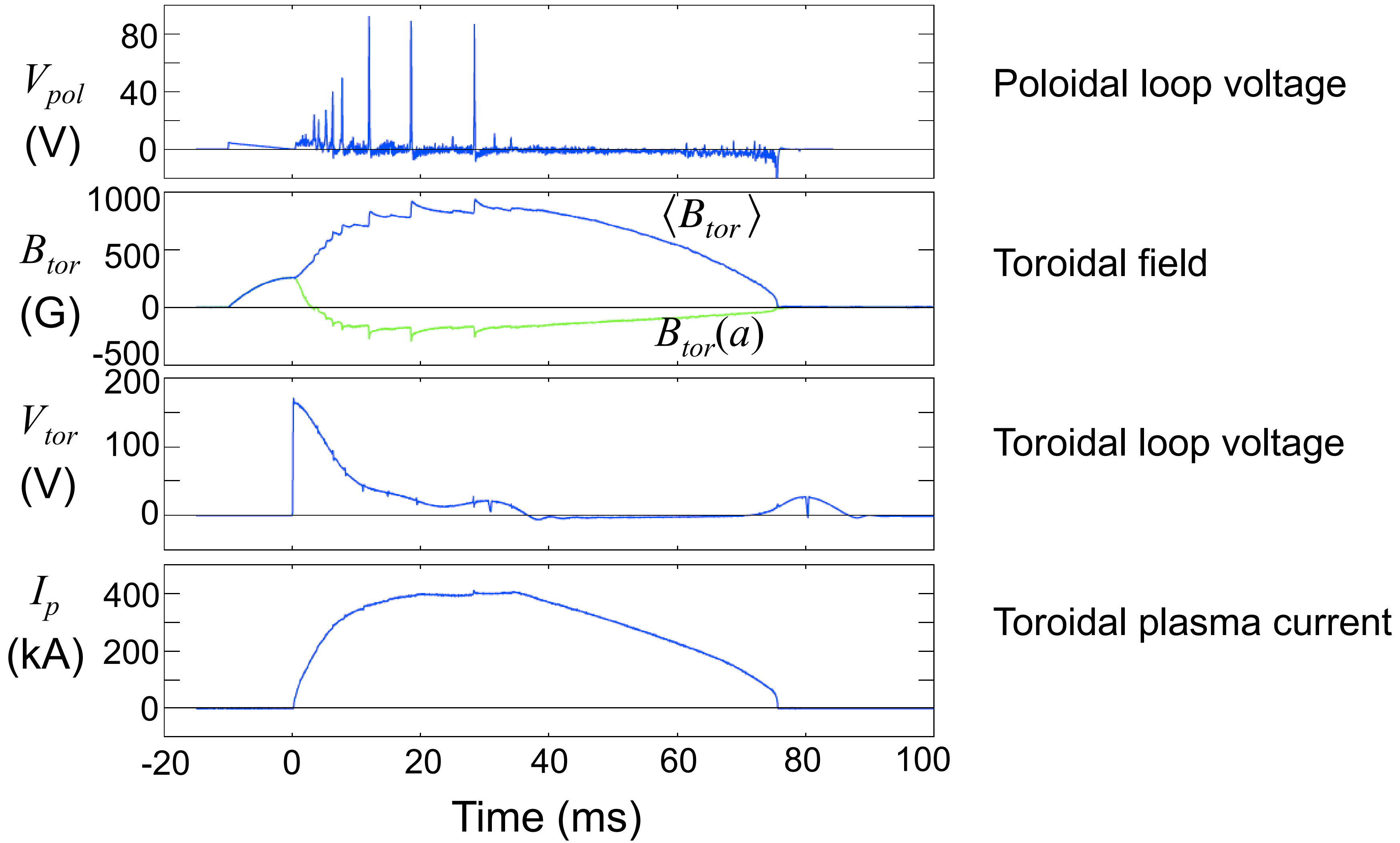
I_p = 0.5 ~ 1.0 MA

Φ_{mag} = 5 V-s

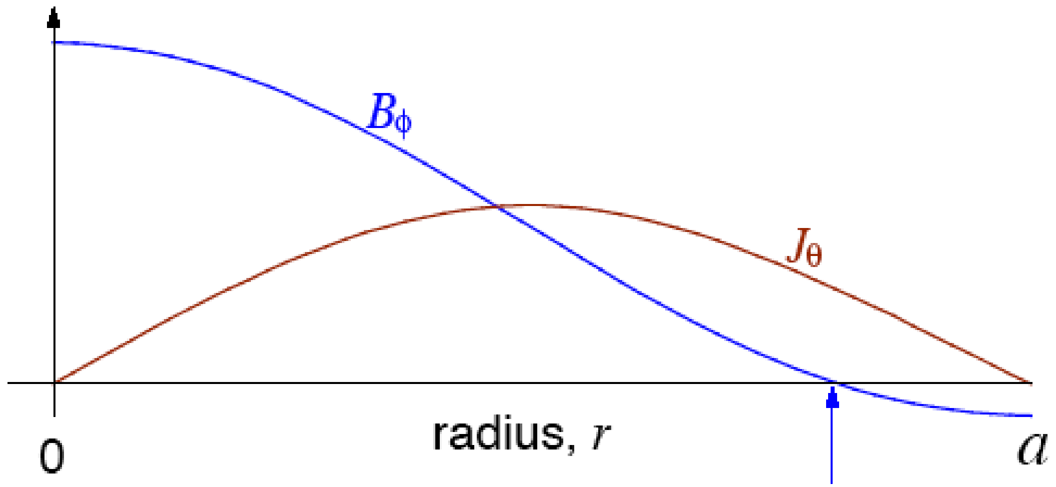


Tearing Modes
and
Relaxation Physics

Reversed BT forms with sufficiently large plasma current, and persists as long as induction is maintained



However, a reversed-BT should not be in equilibrium



reversal implies: $(\nabla \times B)_\theta = \frac{1}{R}\frac{\partial B_r}{\partial \phi} - \frac{\partial B_\phi}{\partial r} \neq 0$

$\Rightarrow J_\theta \neq 0$, which for finite η

$$\Rightarrow E_\theta = -\frac{\partial}{\partial t}\int B_\phi \, dS \neq 0$$

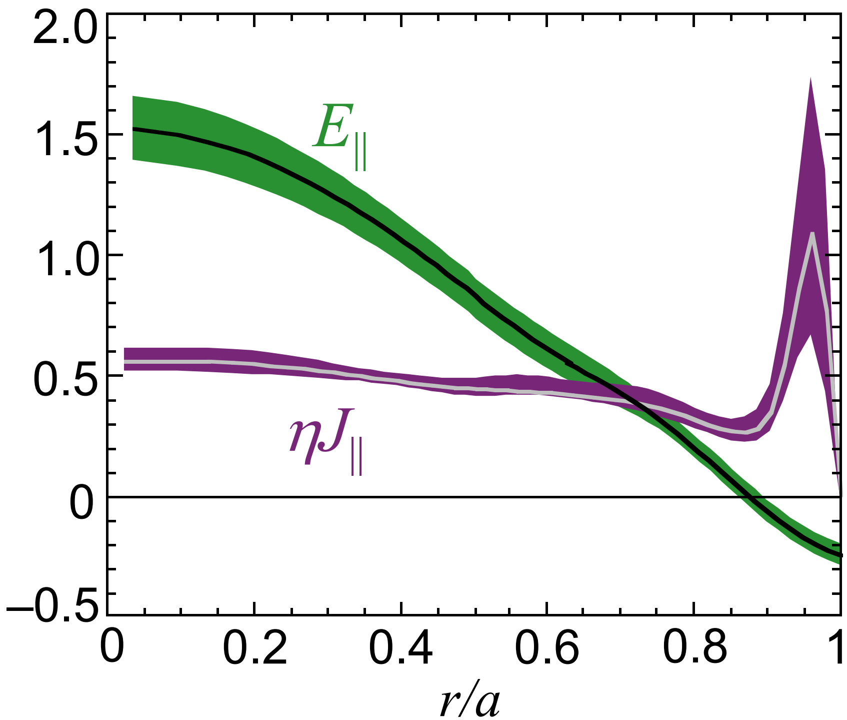
(non steady-state)

An apparent imbalance in Ohm's law is related to the surprising BT reversal

- Basic MHD Ohm's law: $\mathbf{E} + \mathbf{V} \times \mathbf{B} = \eta \mathbf{J} \Rightarrow E_{||} = \eta J_{||}$ and $\mathbf{V}_{\perp} = \mathbf{E} \times \mathbf{B} / B^2$

- There is less current in the core than could be driven by $E_{||}$, and more current in the edge than should be driven by $E_{||}$

  ⇒ current profile is relatively flatter than the inductive drive



Measured via MSTFIT equilibrium reconstructions

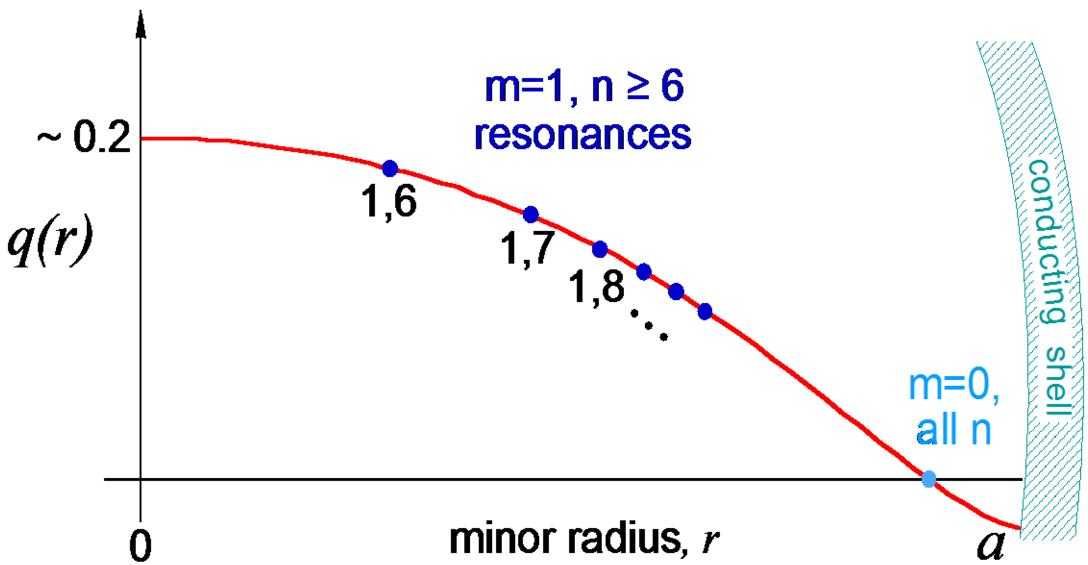
Tearing instability underlies the RFP's nonlinear dynamics

Tearing resonance:

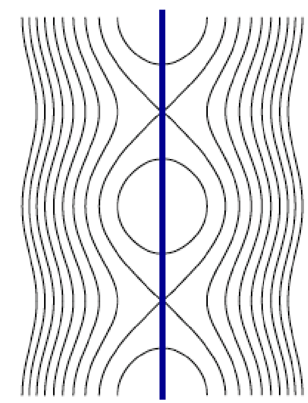
$$0 = \mathbf{k} \times \mathbf{B} = \frac{m}{r} B_q + \frac{n}{R} B_f \quad \Rightarrow \quad q(r) = \frac{rB_f}{RB_q} = \frac{m}{n}$$

m = poloidal mode number

n = toroidal mode number

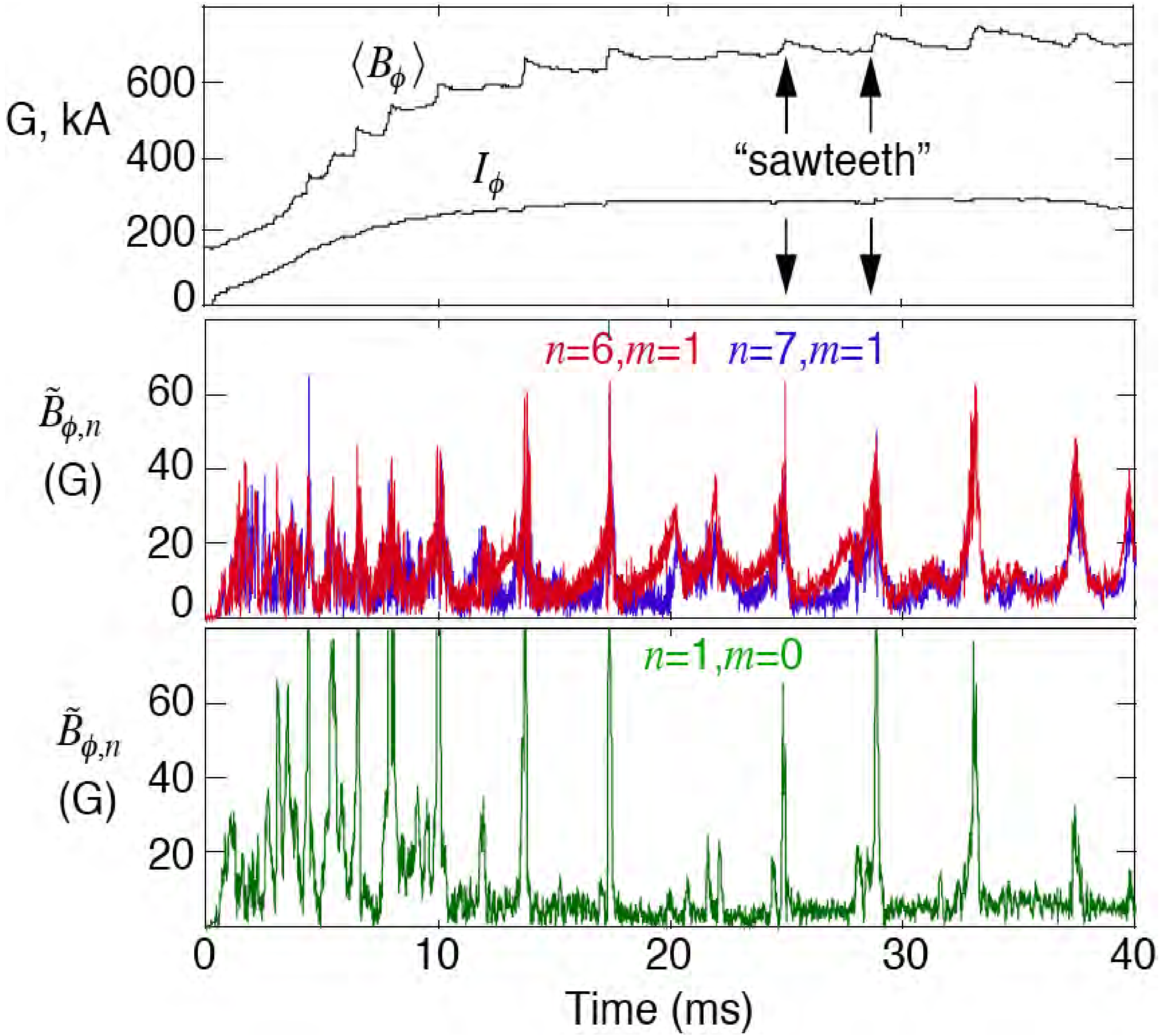


Magnetic Island



$q=m/n$

Tearing process is quasi-periodic, appearing as a sawtooth relaxation cycle



Only a couple of unstable modes

Linearly stable modes energized by nonlinear 3-wave coupling

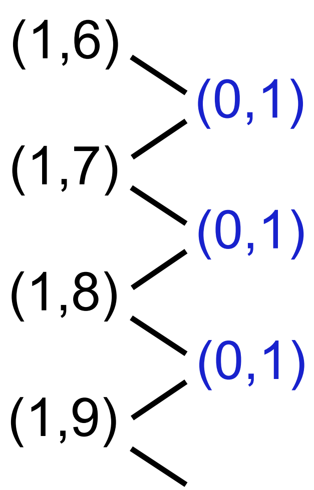
$$k_3 = k_1 \pm k_2$$

m = 0 mode facilitates nonlinear cascade and global core-to-edge communication

- Nonlinear coupling: $Q(\mathbf{k}_3) \sim Q(\mathbf{k}_1) \times Q(\mathbf{k}_2)$

$$\Rightarrow (m_3, n_3) = (m_1, n_1) \pm (m_2, n_2)$$

3-wave cascade:

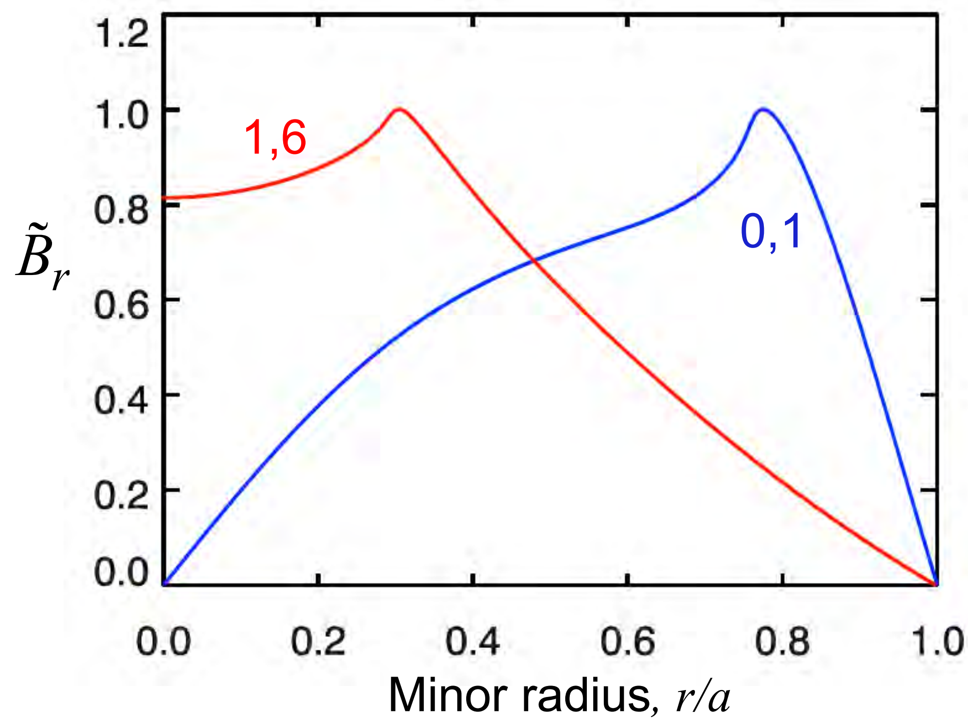


etc.

Same m=0 couples to many m=1

(m=2 branch important for cascade to smallest scales)

Linear Eigenfunctions for Tearing Modes



A dynamo-like process arrests the peaking tendency of the current profile—i.e., how tearing instability saturates

- Ohm's law: $\mathbf{E} - \eta \mathbf{J} = \frac{1}{en} \mathbf{J} \times \mathbf{B} - \mathbf{V} \times \mathbf{B}$

- Suppose there are non-axisymmetric quantities, i.e., from tearing instability:

$$\mathbf{B} = \langle \mathbf{B} \rangle + \tilde{\mathbf{B}}$$

toroidal surface average

spatial fluctuation

$$\tilde{B} \sim \tilde{b}(r) e^{i(m\theta - n\phi)}$$

$$\frac{\tilde{B}}{\langle B \rangle} << 1$$

- Then mean-field parallel Ohm's law becomes:

$$\langle E \rangle_{\|} - \eta \langle J \rangle_{\|} = \boxed{\frac{1}{en} \langle \tilde{\mathbf{J}} \times \tilde{\mathbf{B}} \rangle_{\|} - \langle \tilde{\mathbf{V}} \times \tilde{\mathbf{B}} \rangle_{\|}} = \langle \tilde{\mathbf{V}}_e \times \tilde{\mathbf{B}} \rangle_{\|}$$

dynamo-like effects from magnetic instability

Computational model for tearing-relaxation recently extended to include two-fluid effects

- Nonlinear multi-mode evolution solved using NIMROD

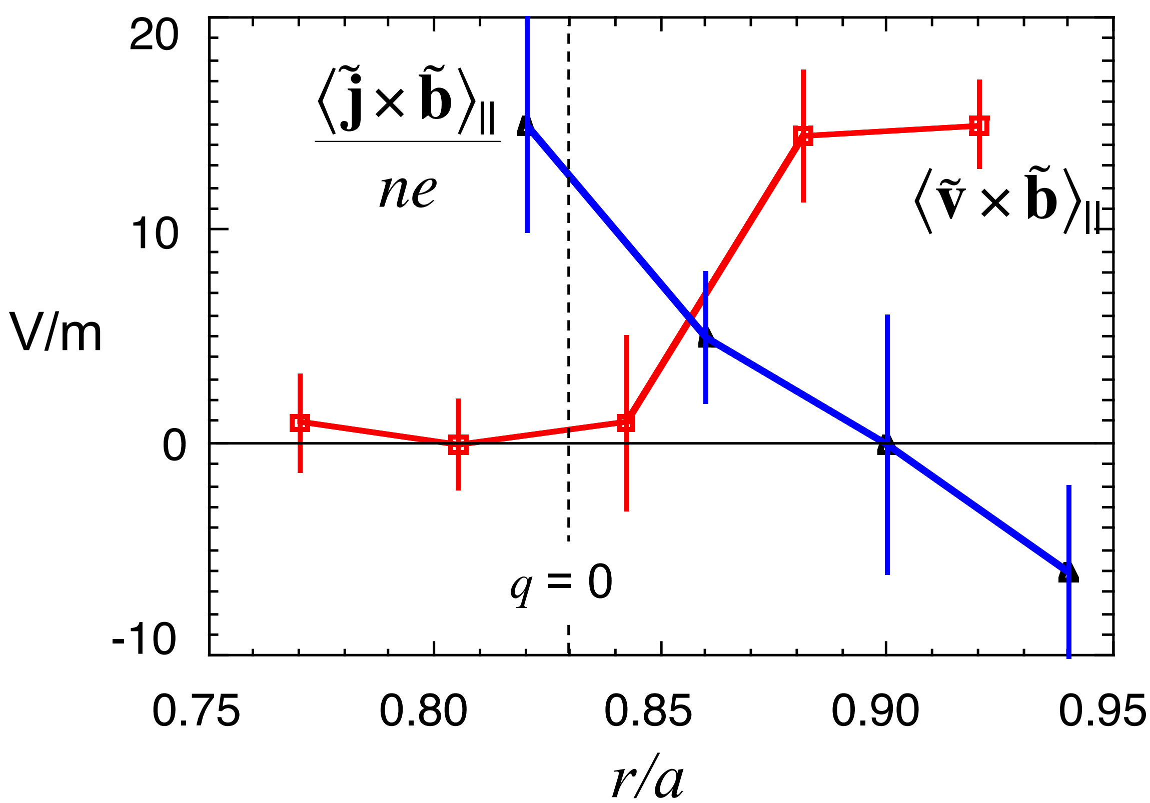
Ohm's law: $$\mathbf{E} = -\mathbf{V} \times \mathbf{B} + \frac{1}{ne}\mathbf{J} \times \mathbf{B} - \frac{1}{ne}\nabla p_e + h\mathbf{J} + \frac{m_e}{ne^2}\frac{\partial \mathbf{J}}{\partial t}$$

Momentum: $$nm_i \frac{d\mathbf{V}}{dt} = \mathbf{J} \times \mathbf{B} - \nabla p - \nabla \times P_{gyro} - \nabla \times n\, nm_i \mathbf{W}$$

New understanding: relaxation couples electron and ion momentum balance

Both MHD and Hall dynamo terms are measured large in MST

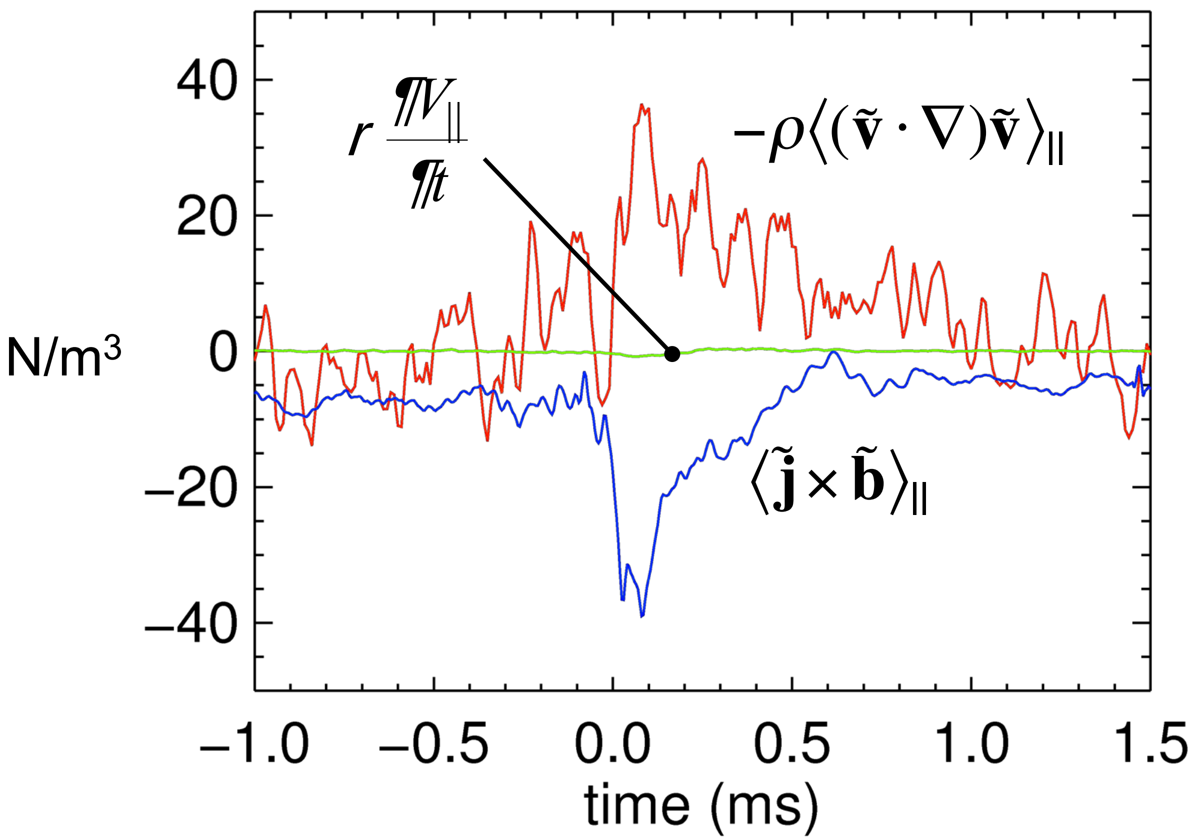
Turbulent emf during sawtooth relaxation event



Probe measurements in outer region

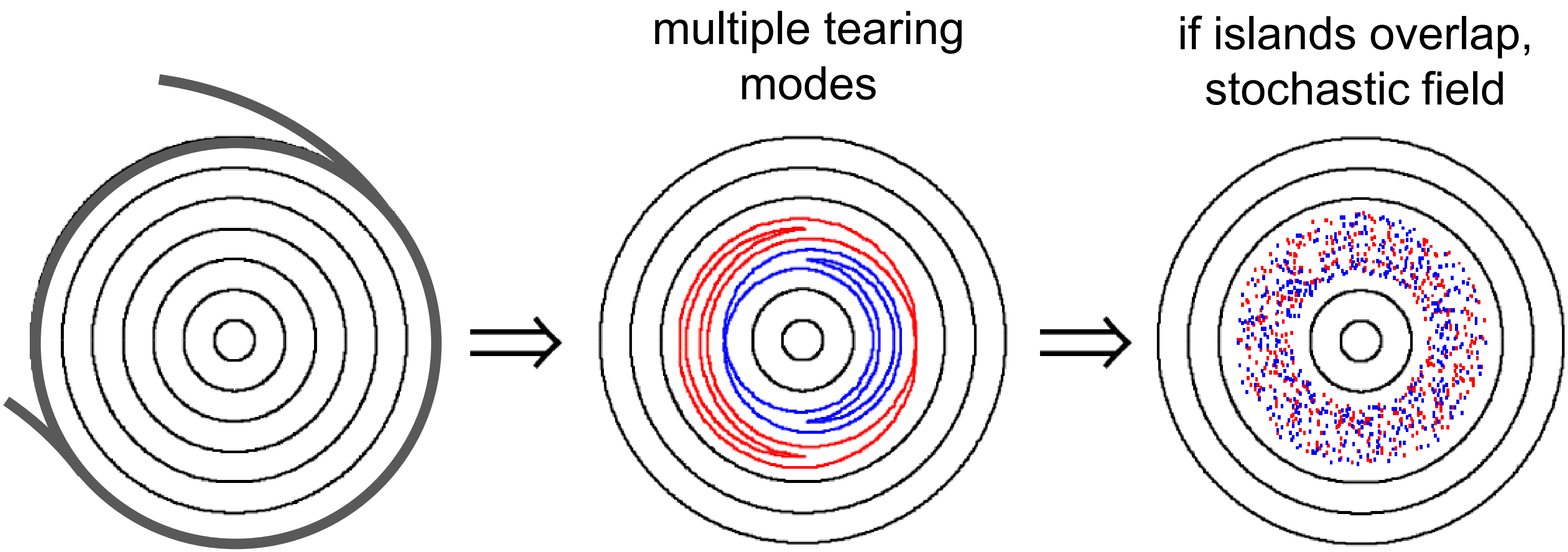
The Reynolds and Maxwell stresses burst to huge amplitudes during relaxation events, tending to cancel

Turbulent stresses measured during the sawtooth relaxation cycle



Stochastic Transport
and
Paths to Improved Confinement

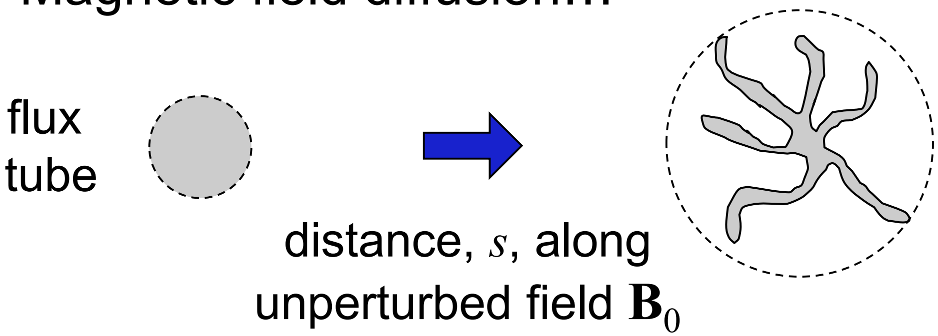
Multiple tearing modes cause the magnetic field to become stochastic



Stochastic transport in the collisionless limit

Consider a test particle streaming along a stochastic magnetic field

Magnetic field diffusion…



$$\langle(\Delta r)^2\rangle = D_m \Delta s$$

average radial displacement associated with field line diffusion

$$D_m = \frac{\langle(\Delta r)^2\rangle}{\Delta s} = \frac{\int_0^\infty \tilde{B}_r(0)\tilde{B}_r(s)ds}{B_0^2}$$

$$= L_{ac}\langle\tilde{B}_r^2\rangle / B_0^2 \quad \text{(units of length)}$$

… compounds collisional diffusion

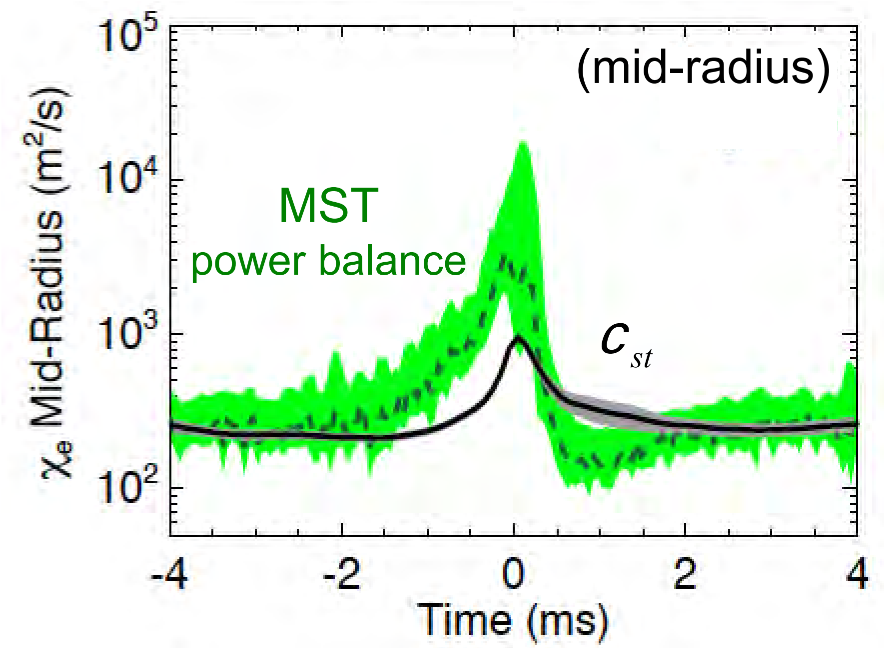
$$\chi_{st} = \frac{\langle(\Delta r)^2\rangle}{\Delta t} = \frac{D_m \lambda_{mfp}}{\tau_c} = D_m v_T$$

(case for $\lambda_{mfp} >> L_{ac}$)

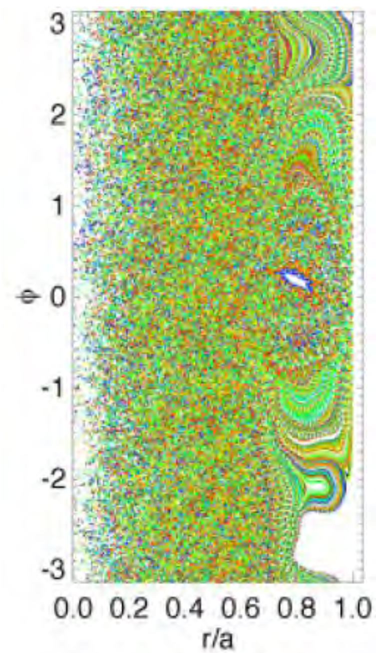
Rechester & Rosenbluth, '78
and others

Electron heat conduction in standard MST plasmas is close to expectations for stochastic transport

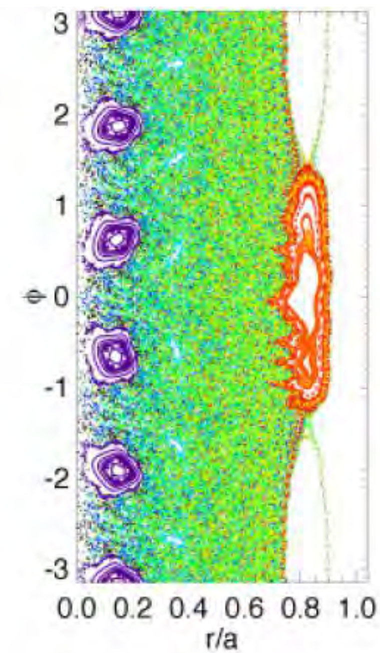
- Test particle picture works well when D_m is evaluated directly from field line trajectories



At Sawtooth

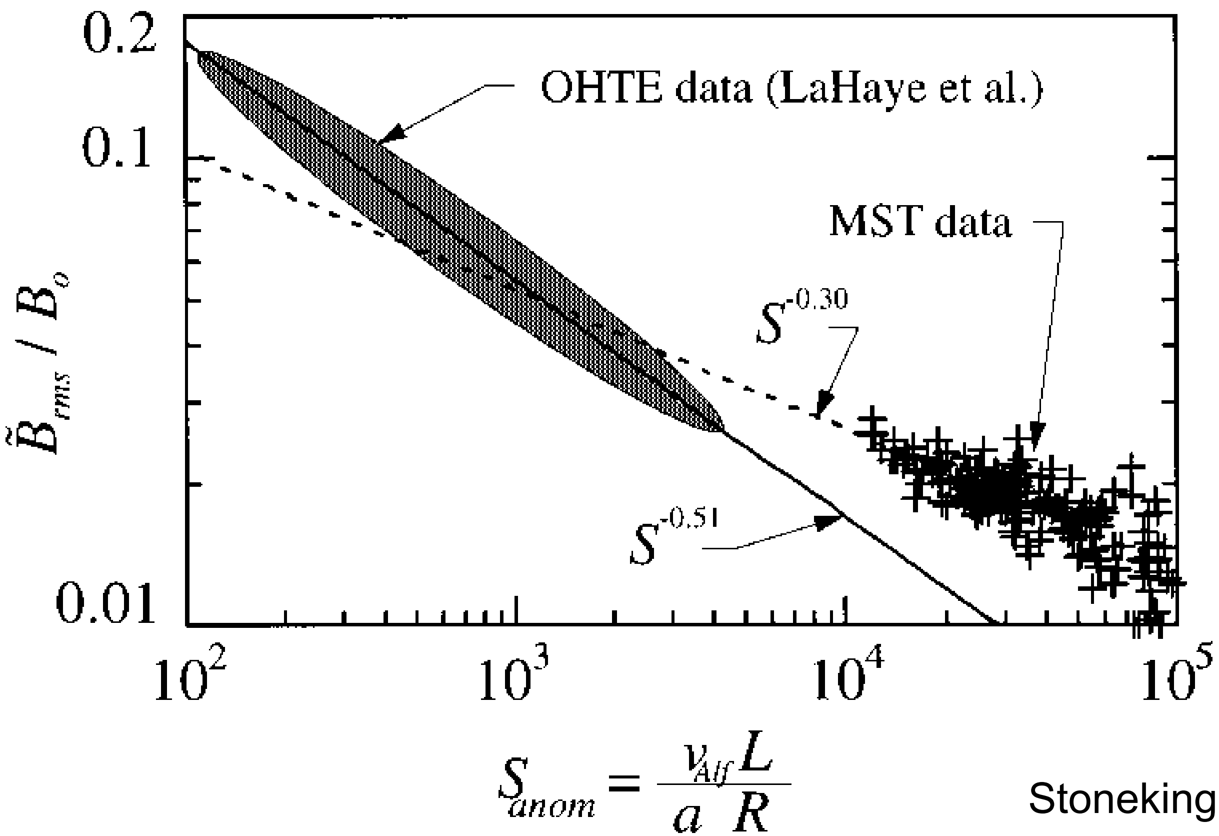


After Sawtooth



Keen interest in the scaling of the magnetic fluctuation amplitude with Lundquist number and magnetic Prandtl number

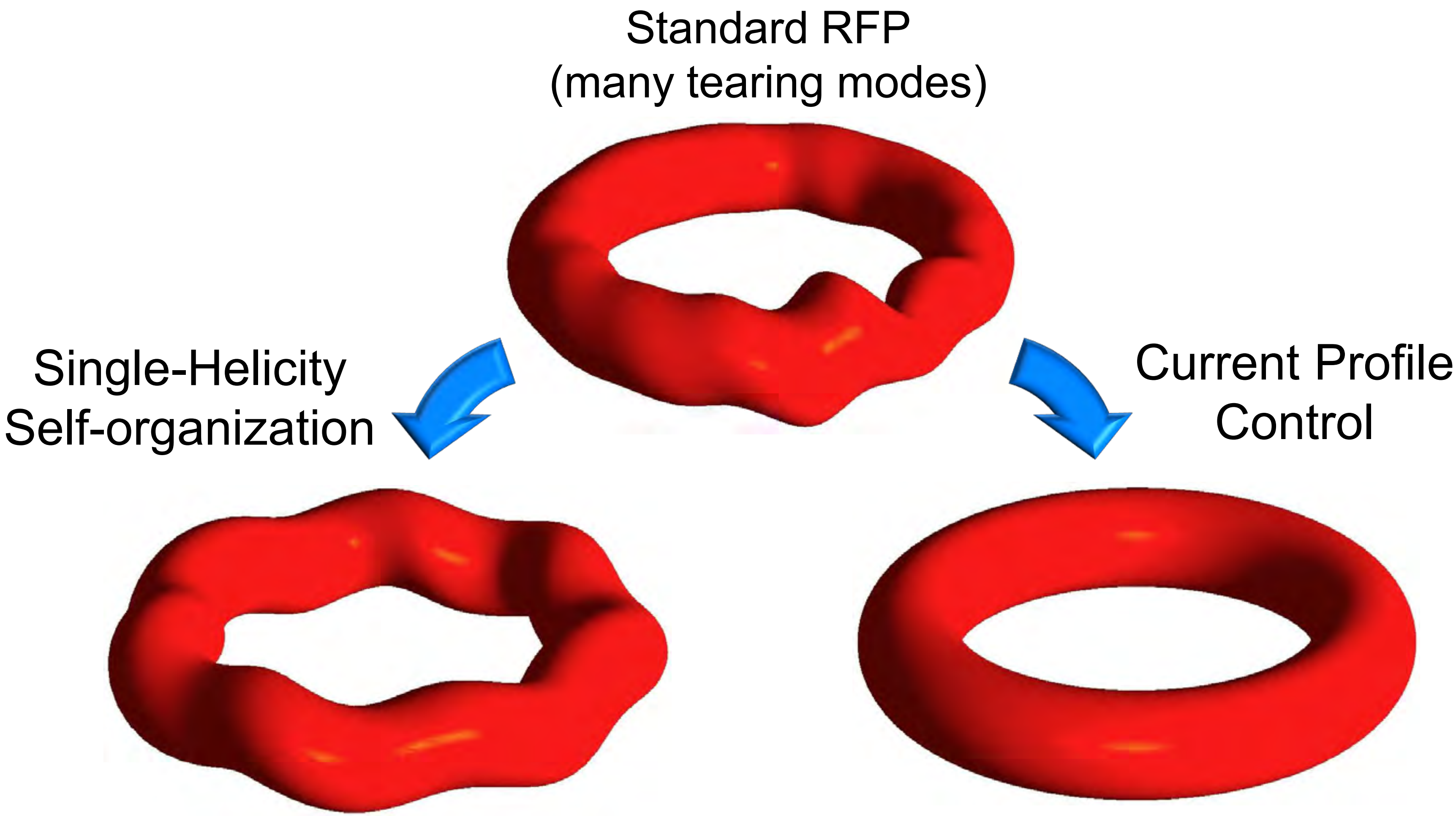
- Lundquist number, $S = \tau_R / \tau_A \sim I_p T_e^{3/2} / n^{1/2}$, and Prandtl number, $P = \nu/\eta$, are the key dimensionless parameters in visco-resistive MHD
- Renewed S-scaling studies are underway on MST, motivated by goals for validation studies and improved measurement capability, e.g., Z_{eff}



…but Z_{eff} in these measurements is known to be inaccurate

Stoneking et al, PoP ’98

Two paths to improved confinement in the RFP

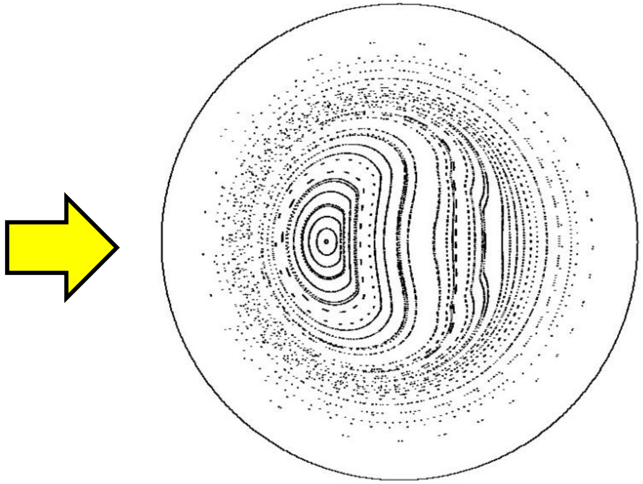


Spontaneous helical equilibrium creates stellarator-like plasmas in RFP experiments, studied extensively in RFX-Mod

- Tendency for quasi-single-helicity (QSH) appears at high current and high temperature

Magnetic Island

Helical Equilibrium



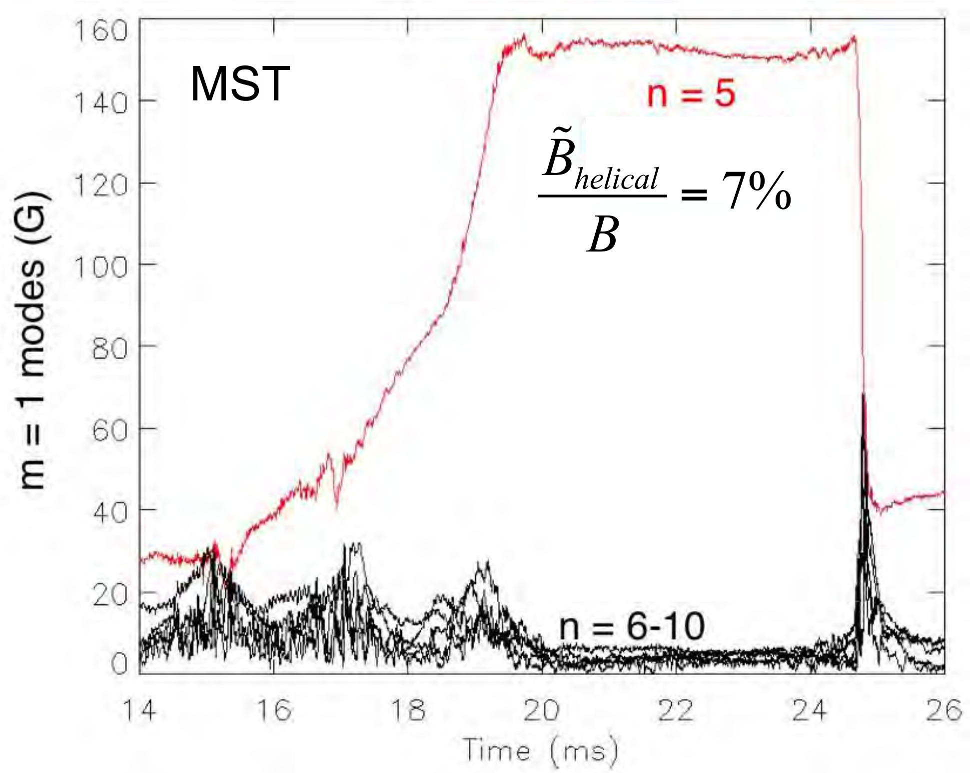
n=7 mode in RFX-Mod

$$\frac{\tilde{B}_{helical}}{B} = 2\%$$

$$\frac{\tilde{B}_{helical}}{B} = 5\%$$

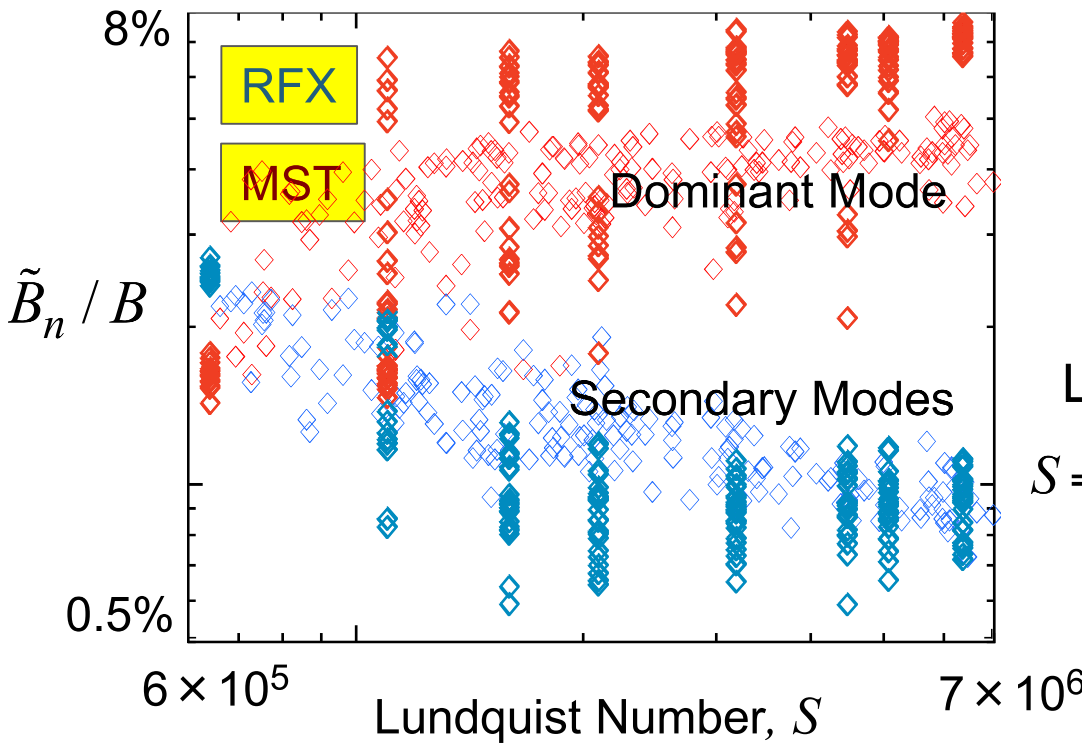
MST produces large helical modes and has capable diagnostic set to measure 3D effects

- Growing collaborations with the stellarator community, e.g., opportunity to develop 3D equilibrium reconstruction methods and tools such as V3FIT
- MST diagnostics for equilibrium reconstruction:
  - Interferometry/Faraday rotation, MSE, 2-color SXR, Thomson scattering



Understanding the dependence on plasma current in accessing the quasi-single-helicity (QSH) regime

- Lundquist number appears to be a unifying parameter for QSH transition



Lundquist Number

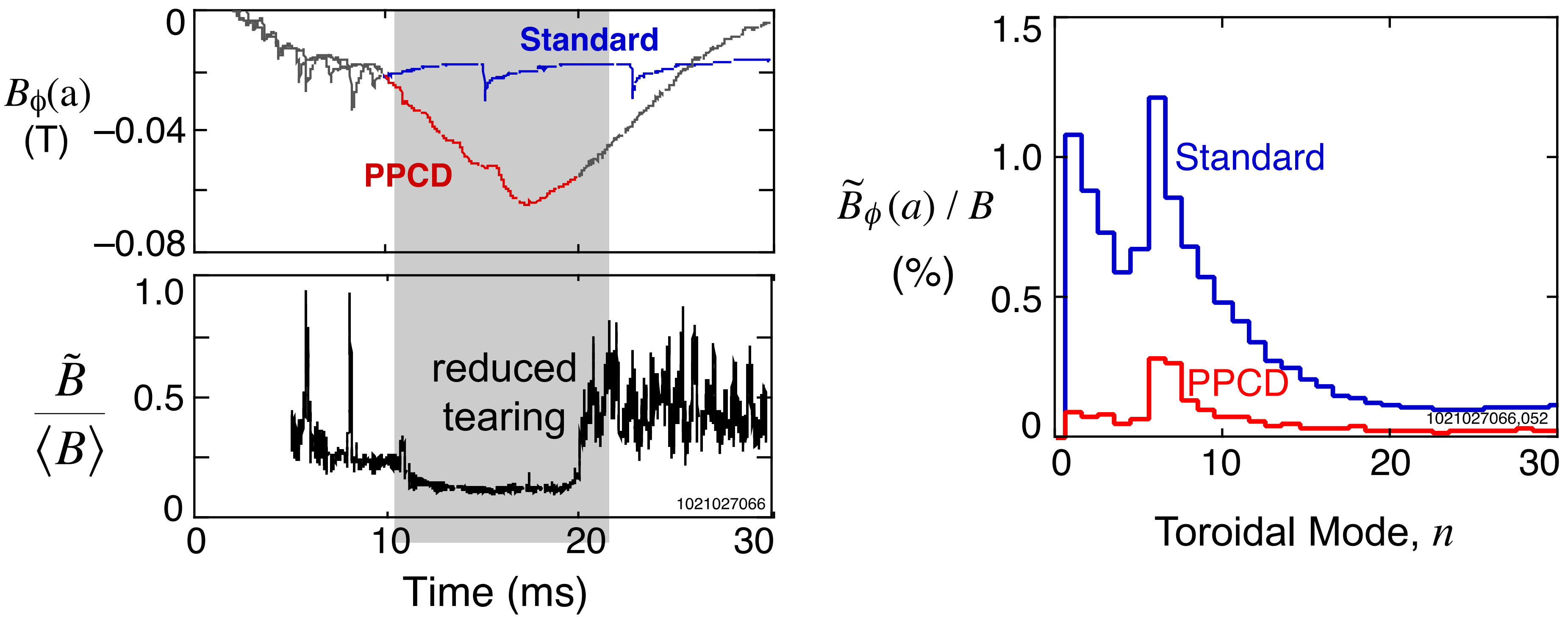
$$S = t_R / t_A \sim I_p T_e^{3/2} n^{-1/2}$$

I_P = 0.5 MA → RFX → 1.3 MA

0.2 MA → MST → 0.6 MA

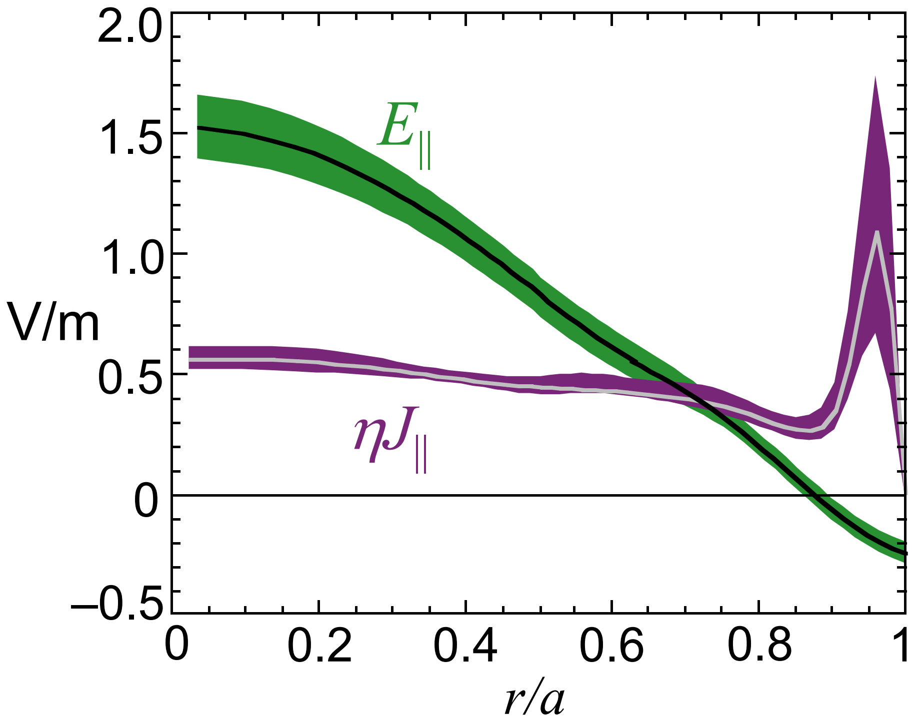
Inductive pulsed poloidal current drive (PPCD) provides simple current profile control of tearing

- 30-fold decrease in χ_e at mid-radius
- 10-fold increase in global energy confinement

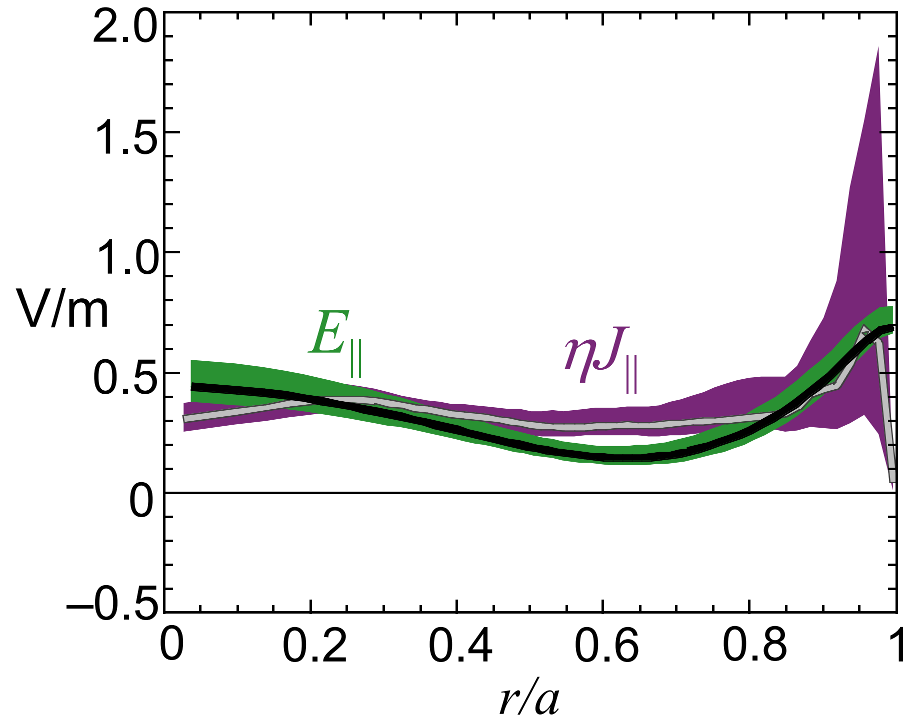


A simple Ohm's law is attained with inductive control

Standard Self-Organized



Current Profile Control



strong dynamo

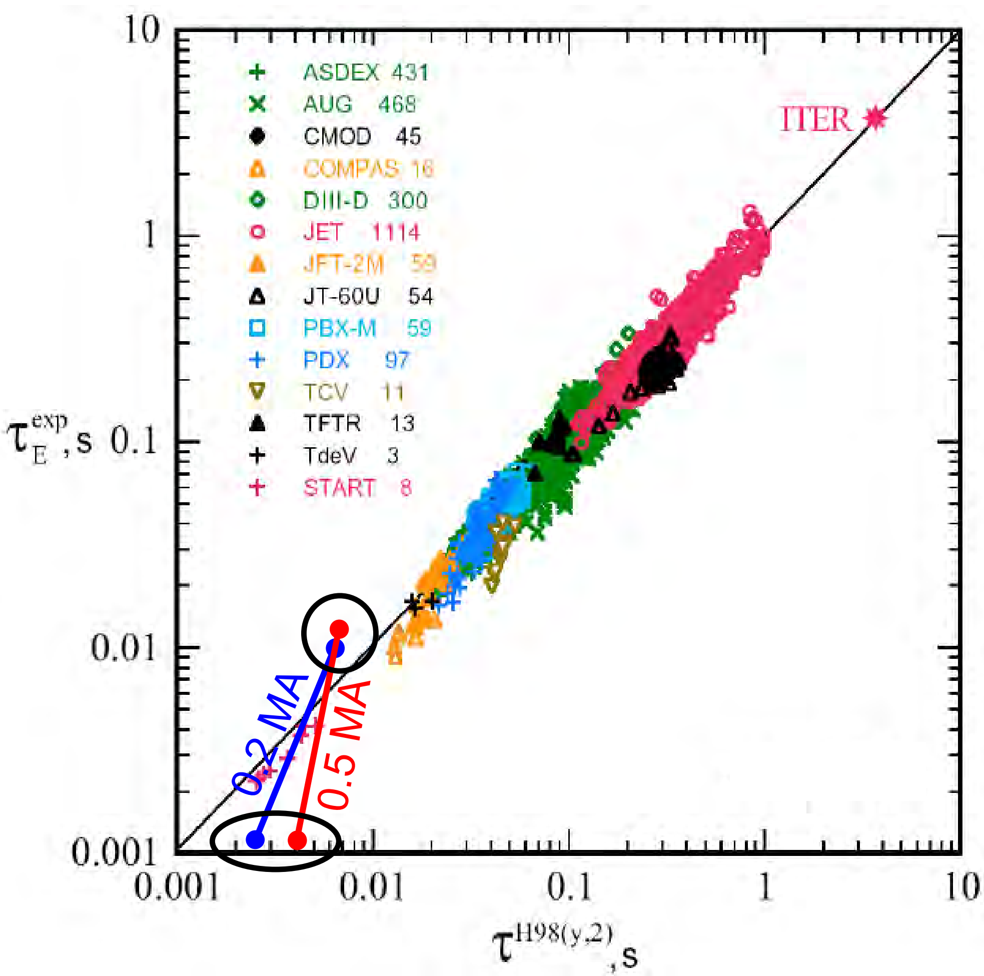
$E_{||}$ ¹ $hJ_{||}$

weak dynamo

$E_{||}$ » $hJ_{||}$

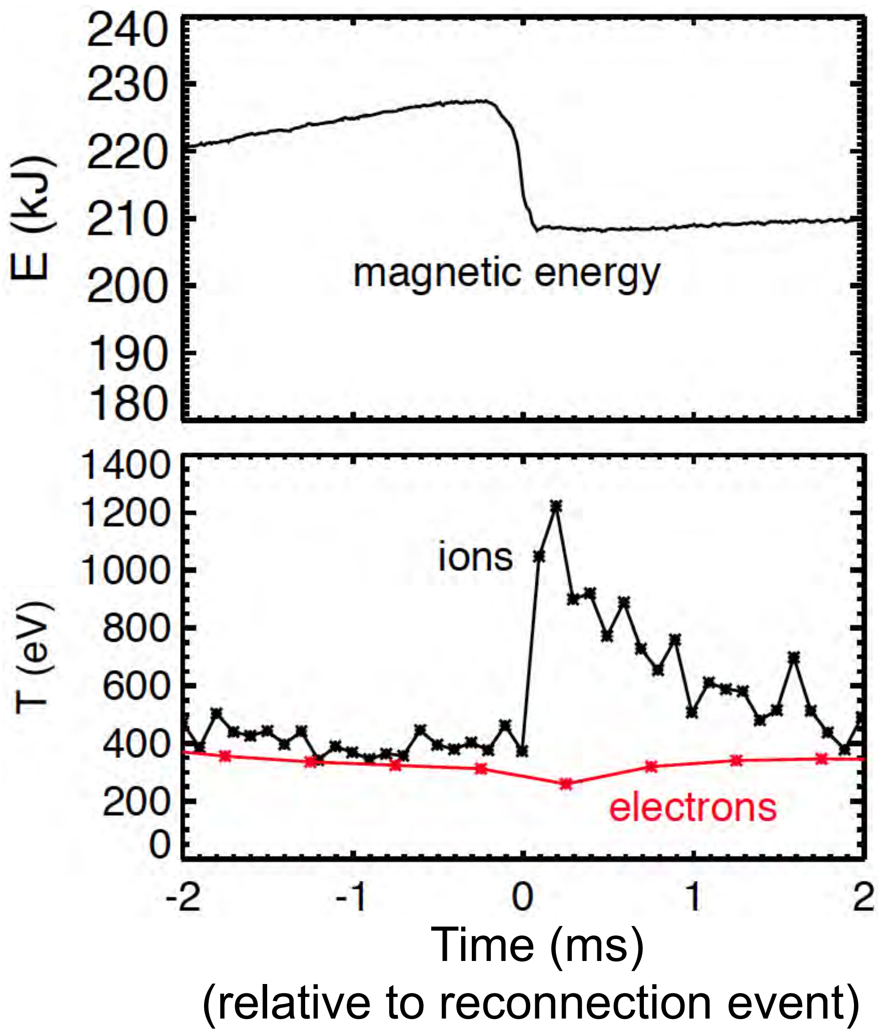
Confinement is (transiently) comparable to that for a tokamak with the same size and field strength

- Use same $\langle B \rangle$, n, P_{heat}, size, shape to define a tokamak reference via scaling



Ion Physics

Powerful ion heating occurs during the sawtooth crash



A large fraction of the magnetic energy released by reconnection is transferred to the ions

$T_i > T_e$

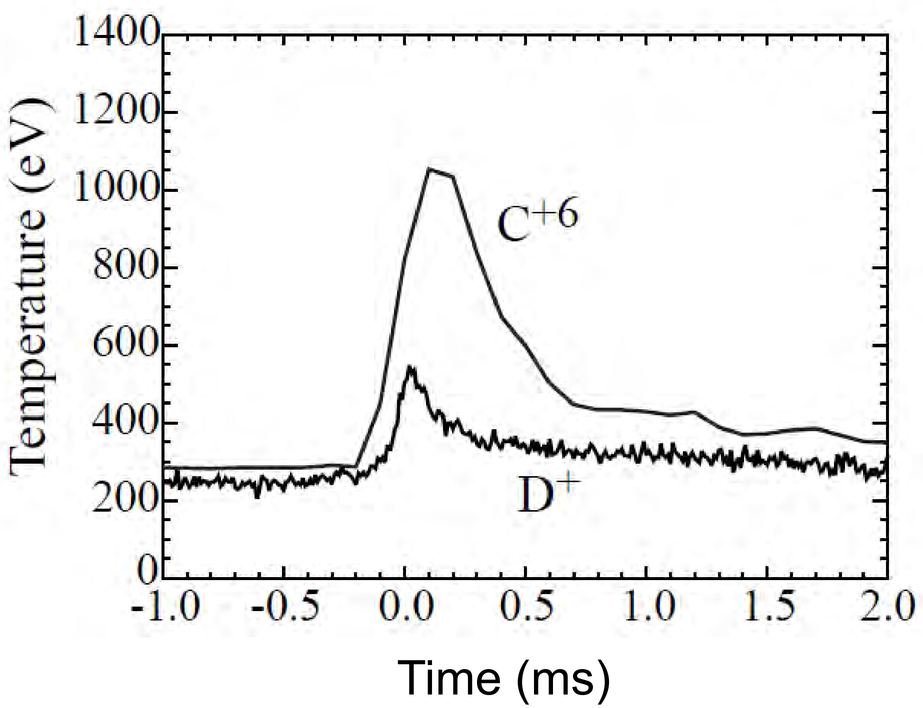
Δt ~100 μs $<<$ e-i equil. time

Clearly the ion heating mechanism is not collisional

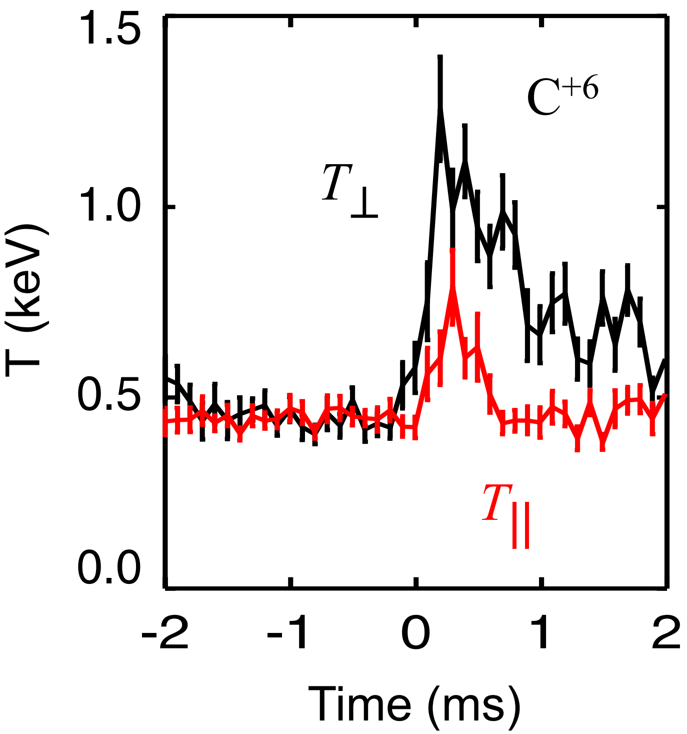
Impurity ions are hotter, and the heating is anisotropic

- Key diagnostics:
  - Rutherford scattering for majority ions
  - Charge-exchange recombination spectroscopy (CHERS) for minority ions
  - Neutral particle energy analyzers for ion distribution

Minority Ions Hotter than Majority Ions

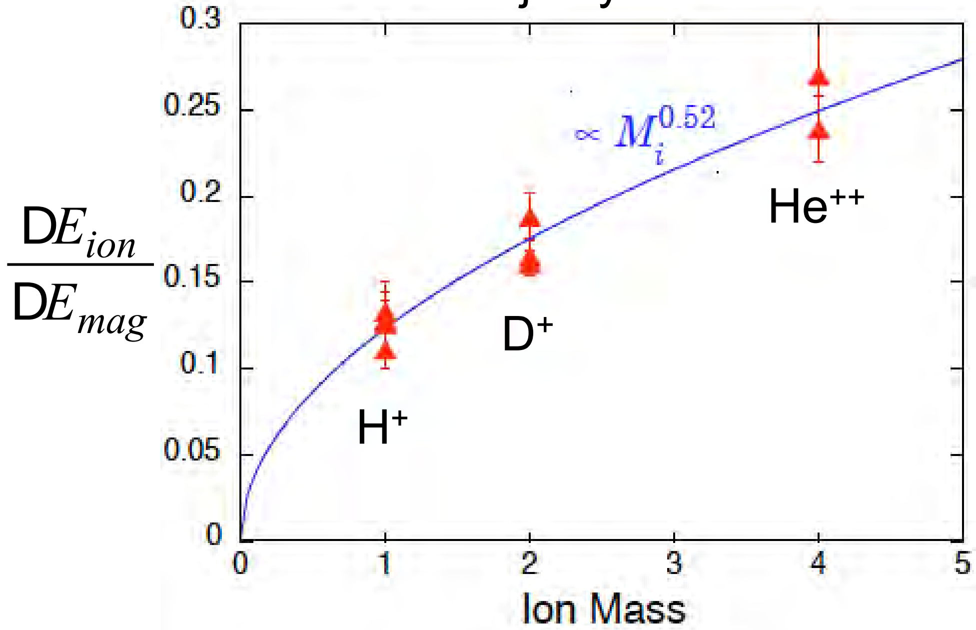


Heating is Anisotropic



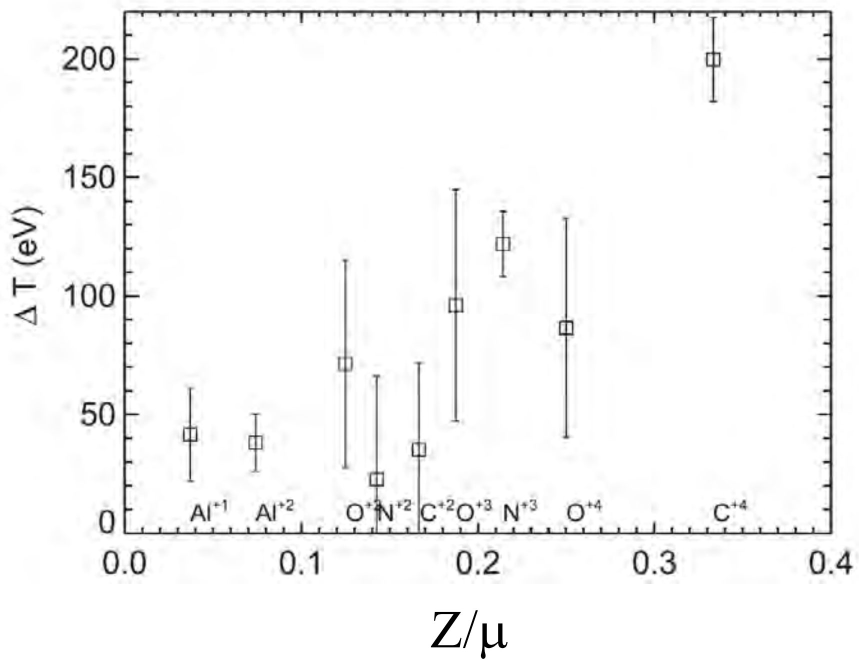
Heating depends on mass and charge

Majority Ions



(varied fueling gas)

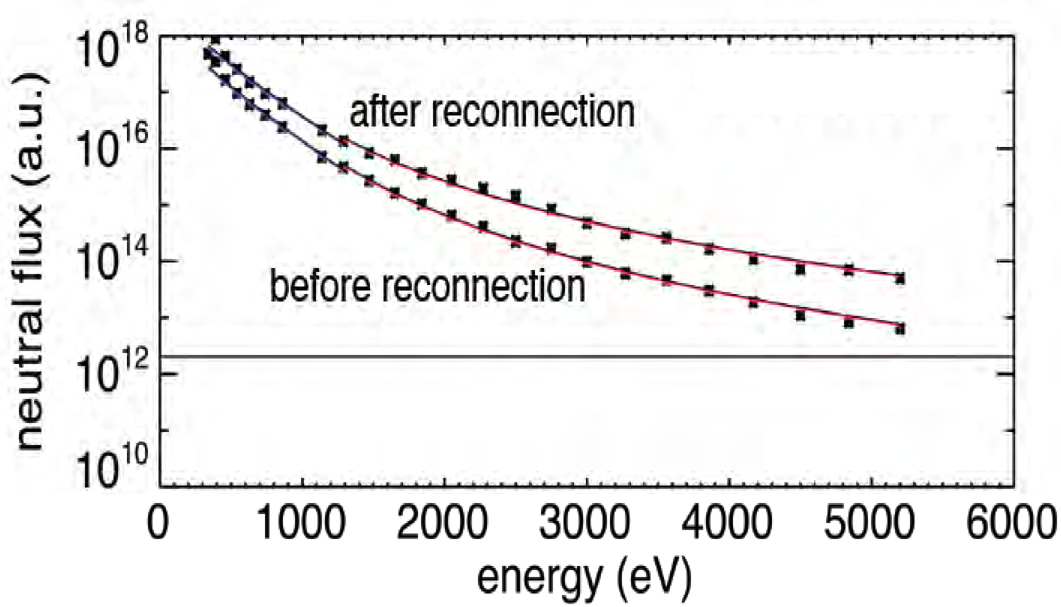
Minority Ions

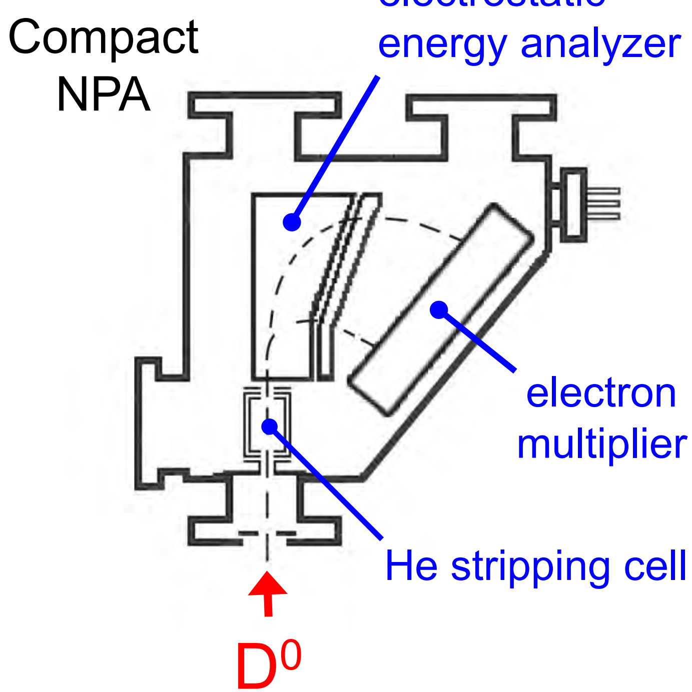


Spontaneous energetic ion tail is generated during the sawtooth crash

- Ion distribution is well-fit by a Maxwellian plus power-law tail
- D-D neutron measurements imply tail extends to >> 5 keV

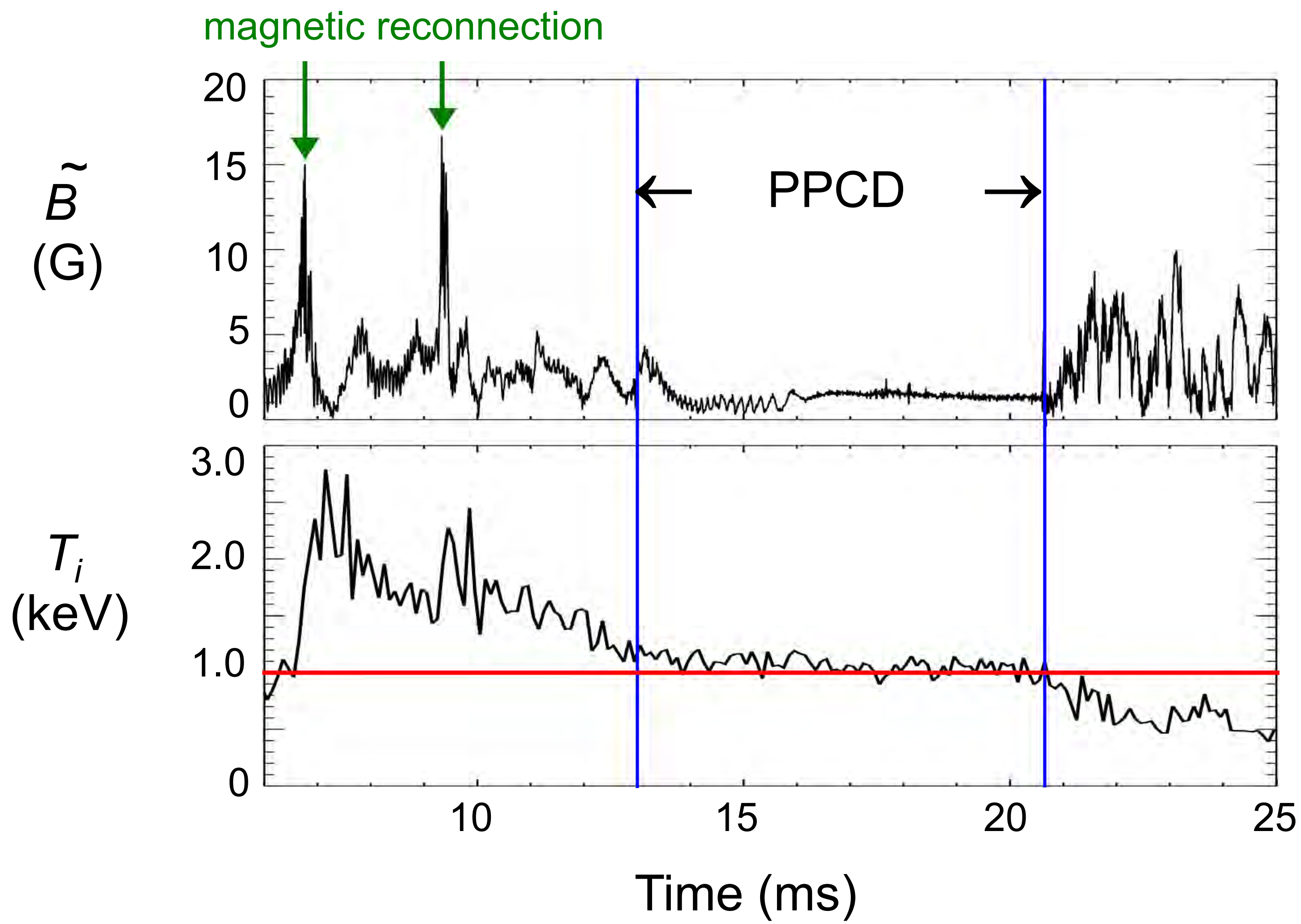
$$f_{D+}(E) = A\, e^{-E/kT} + B\, E^{-\gamma}$$





with Florida A&M Univ

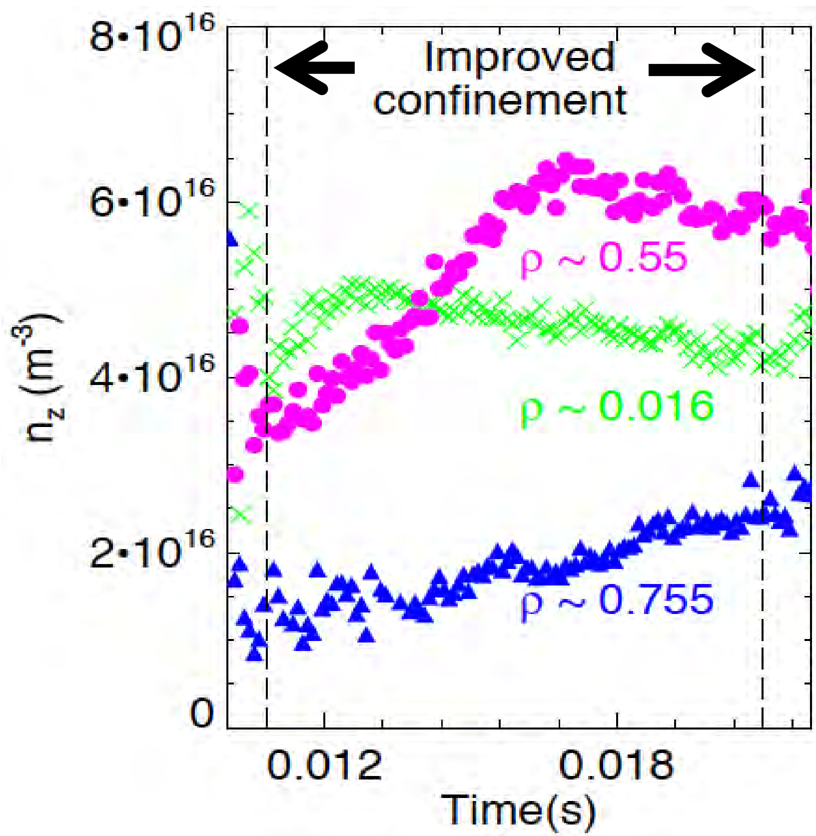
PPCD timed to immediately follow especially large sawteeth confines the ion heat and helps maximize beta



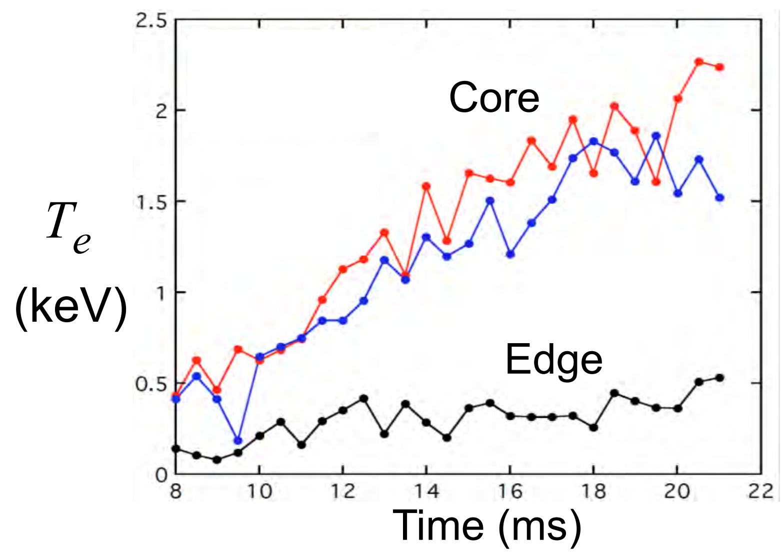
Impurity evolution reveals improved particle confinement and outward particle convection

- PPCD plasmas hot enough to burn through carbon in most of the plasma volume

Fully-stripped carbon



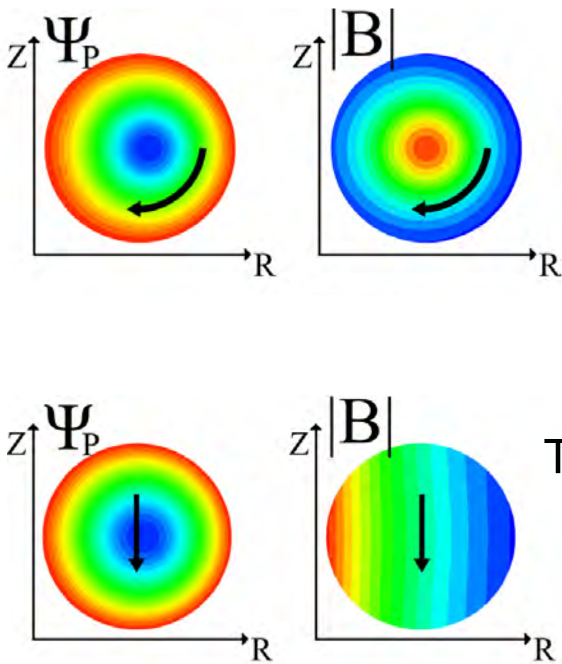
Fast Thomson measurements capture rapid increase in T_e



(this data from a higher current plasma)

The minimal perpendicular collisional diffusion is classical in the RFP, despite its toroidal geometry

Fig.3.8 (r, θ) coordinates



RFP

∇B points towards minor axis

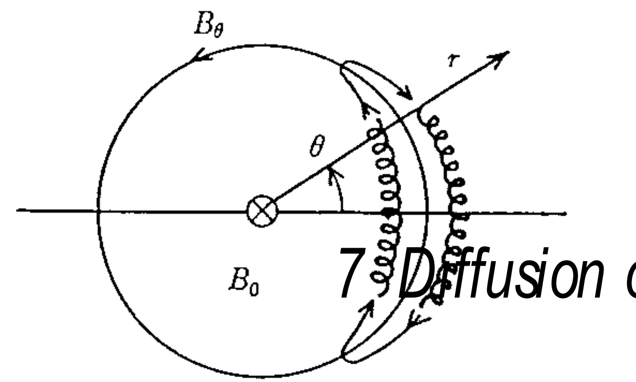
Tokamak

…note the length a
…e magnetic line of
…e following relatio

holds, we find

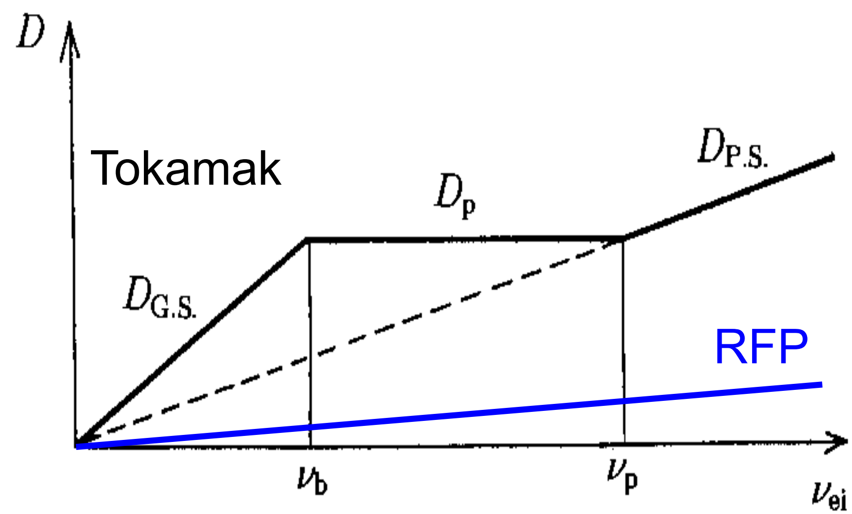
In the RFP:

$$\frac{D_{banana}}{r} < 1$$



7. Diffusion

"banana" orbit



Note: neoclassical modifications to *parallel* transport not negligible

$B = B_0 \left(1 - \frac{r}{R_0}\cos(\kappa l)\right)$.

Fig.7.3 Dependence of the diffusion coefficient on collision frequency in

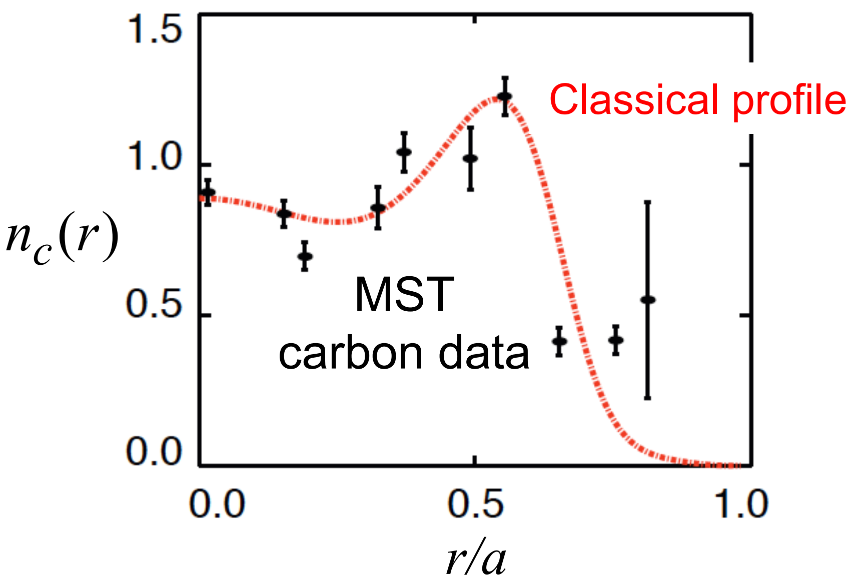
Classical confinement of impurity ions when tearing is suppressed in PPCD plasmas

- Modeling of collisional coupling of multiple ions used for detailed comparison with measurements

Classical Transport

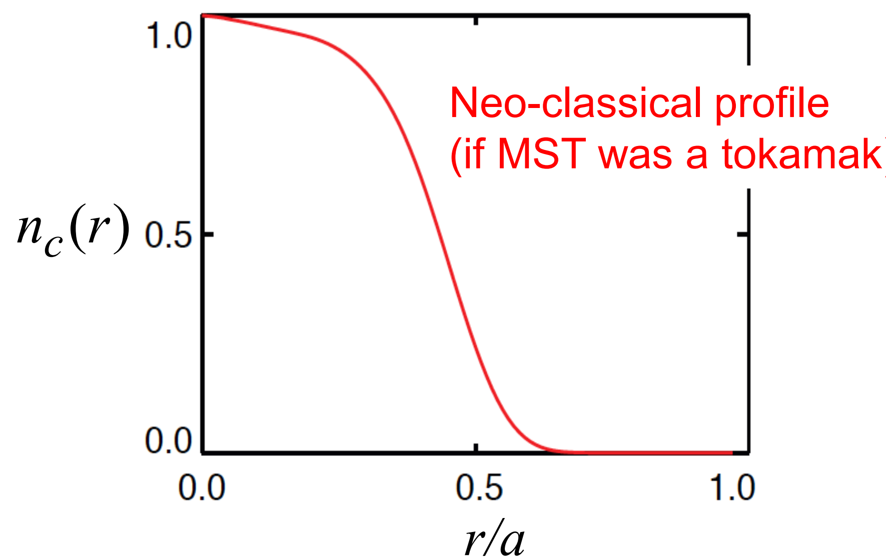
$$\frac{n_Z(r)}{n_Z(0)} = \left[\frac{n_i(r)}{n_i(0)}\right]^{Z} \left[\frac{T_i(r)}{T_i(0)}\right]^{-0.5Z-1}$$

"temperature screening"



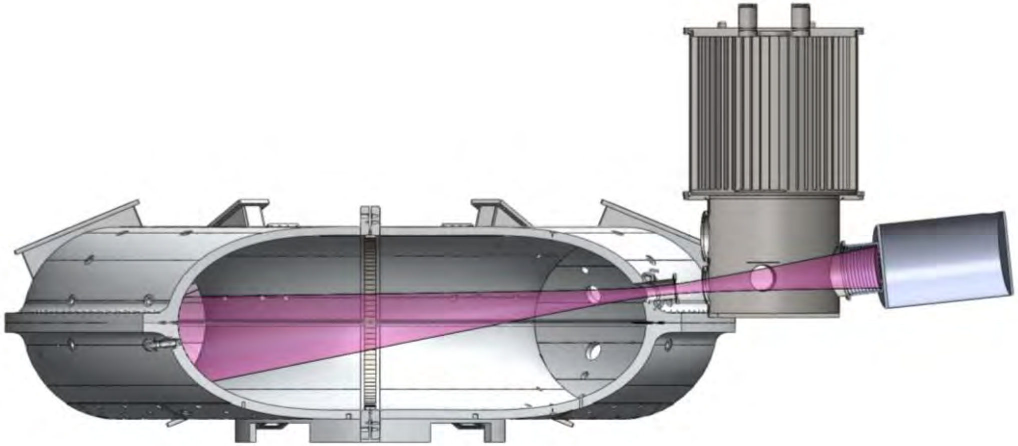
Neo-classical Transport

$$\frac{n_Z(r)}{n_Z(0)} = \left[\frac{n_i(r)}{n_i(0)}\right]^{Z} \left[\frac{T_i(r)}{T_i(0)}\right]^{1.5Z}$$



Energetic Ion Physics
and
Ion Runaway During Sawteeth

1 MW neutral beam injection on MST (unique for RFPs)

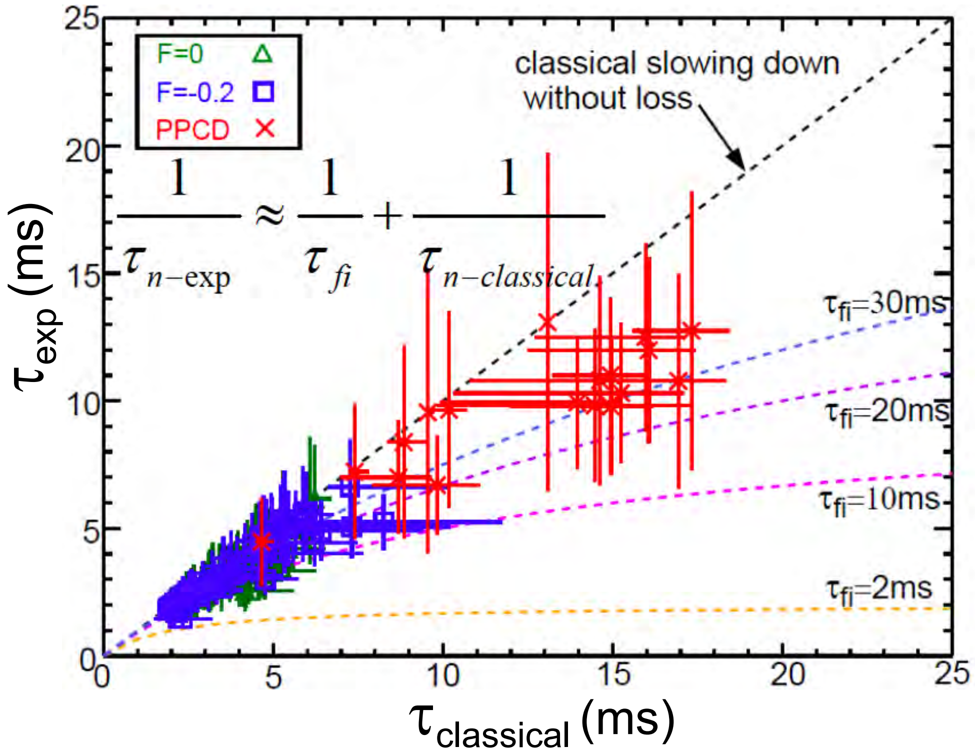


NBI Parameter	Specification
Beam energy	25 keV
Beam power	1 MW
Pulse length	20 ms
Composition	97% H, 3% D
Energy fraction (E:E/2E/3:E/18)	86%:10%:2%:2%

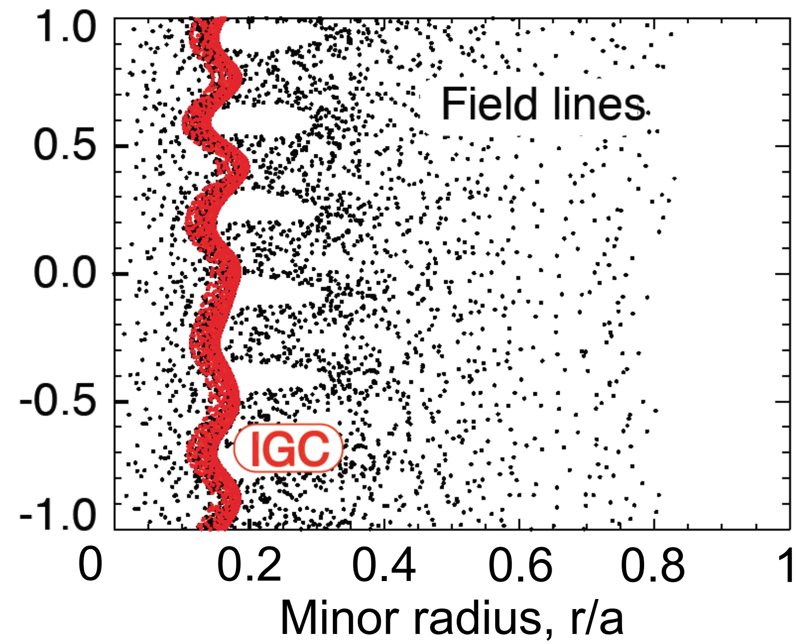
Energetic ion confinement is near-classical, even in standard RFP plasmas despite stochastic field

- Ion motion deviates from electron motion due to $\nabla\mathbf{B}$ and curvature drifts, and possibly orbit-averaging of fluctuations ($\rho_{\mathrm{fi,\,25keV}}/a \sim 0.2$)

Fast Ion Confinement

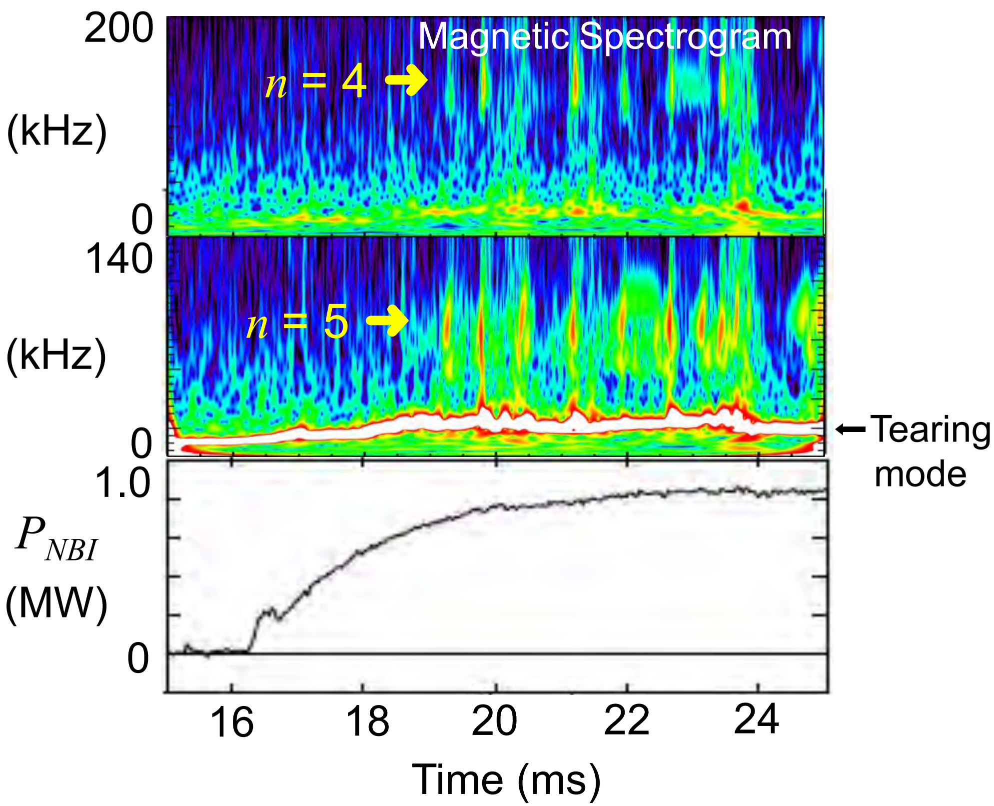


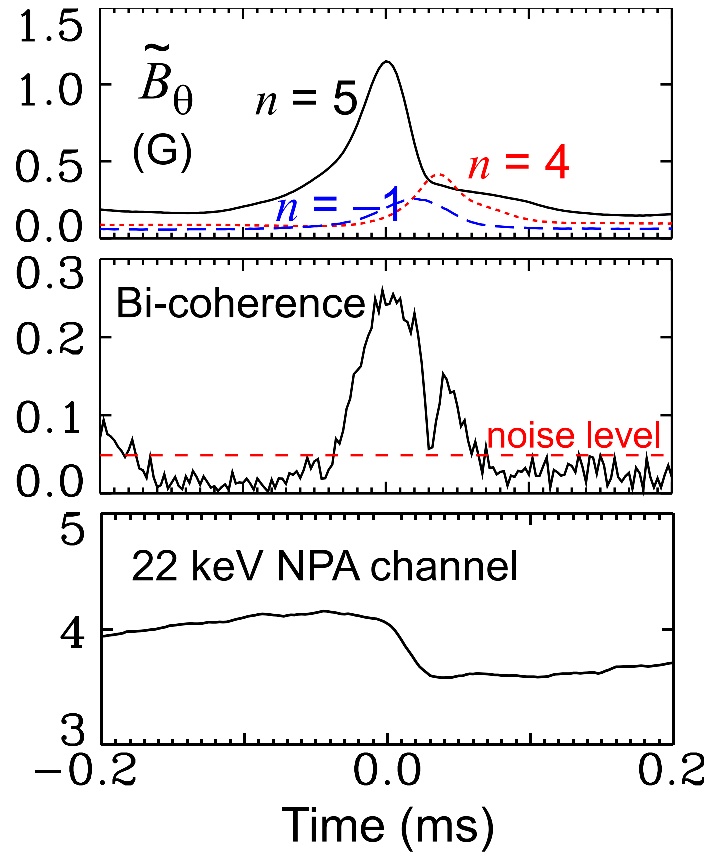
Full-orbit analysis of a test fast ion



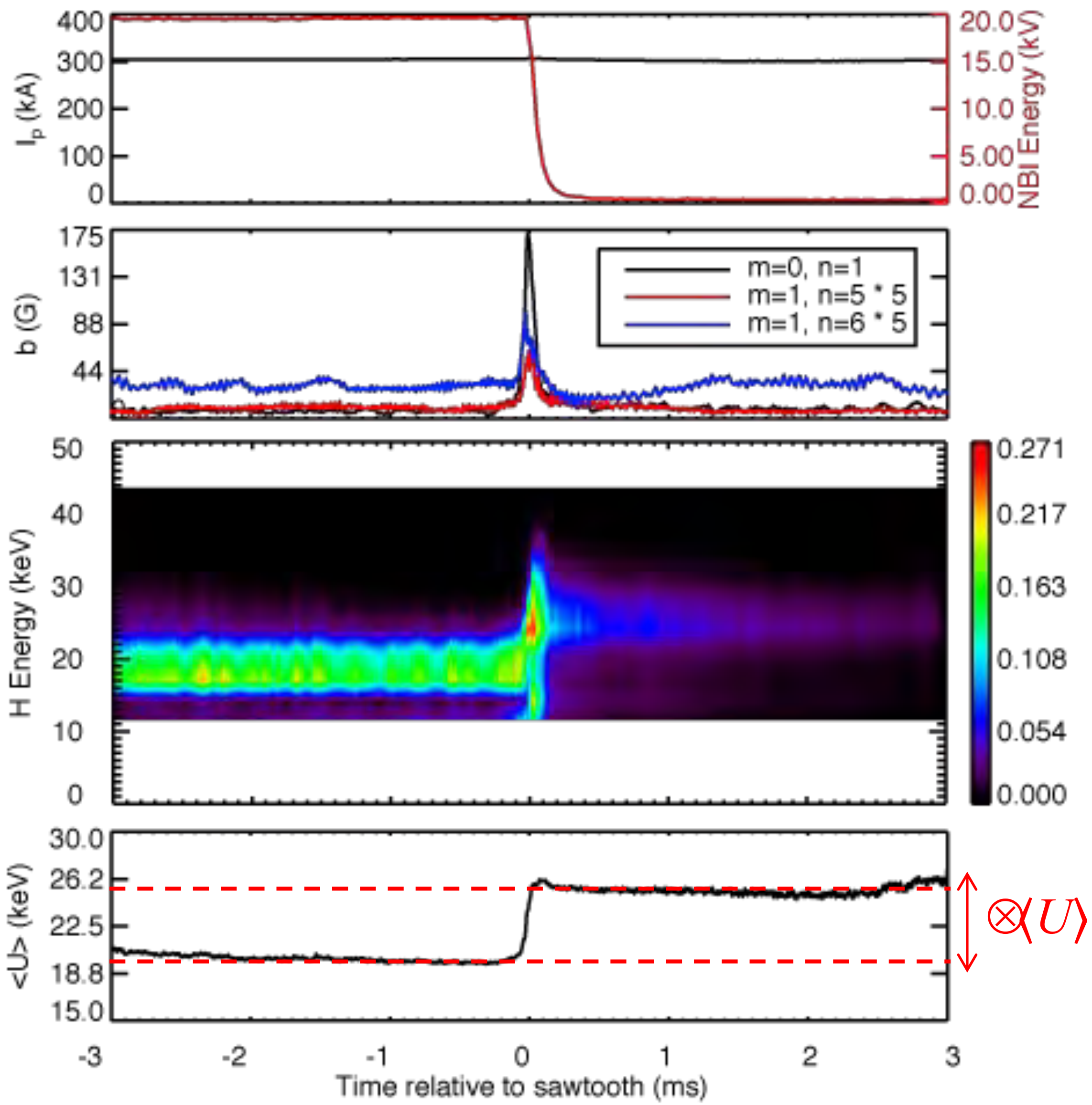
First observations of energetic ion modes by NBI in an RFP, and transport from multiple-mode interactions

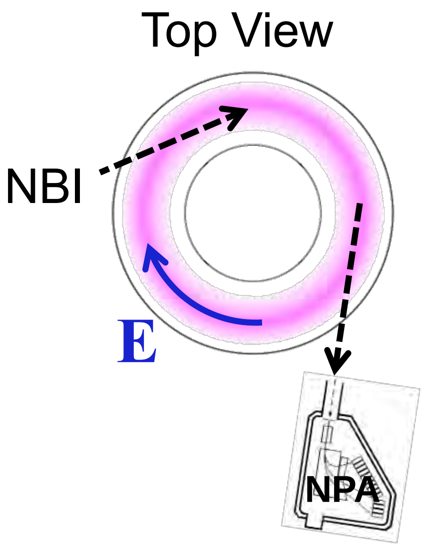
- Several bursting modes observed, that are nonlinearly coupled



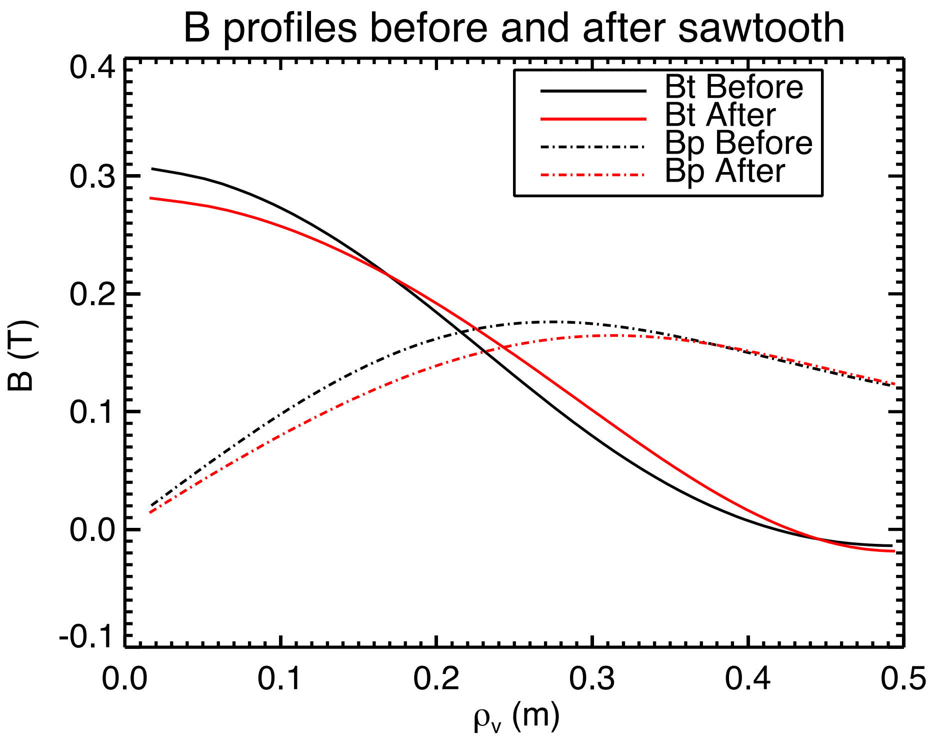


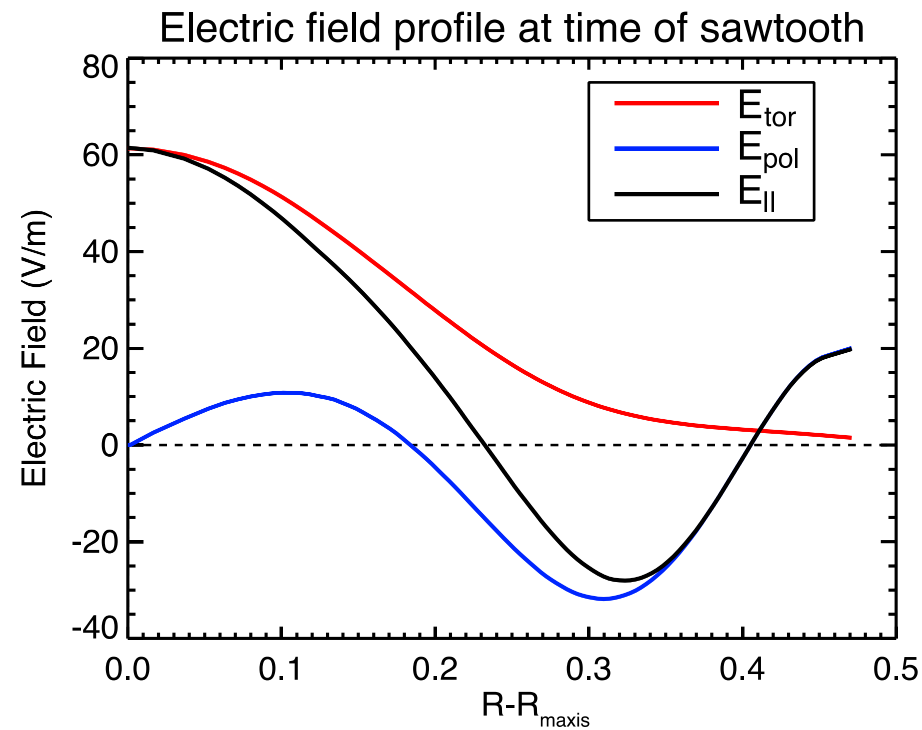
The fast ions are energized during the sawtooth reconnection event





Abrupt change in the equilibrium magnetic field induces a large-scale inductive electric field

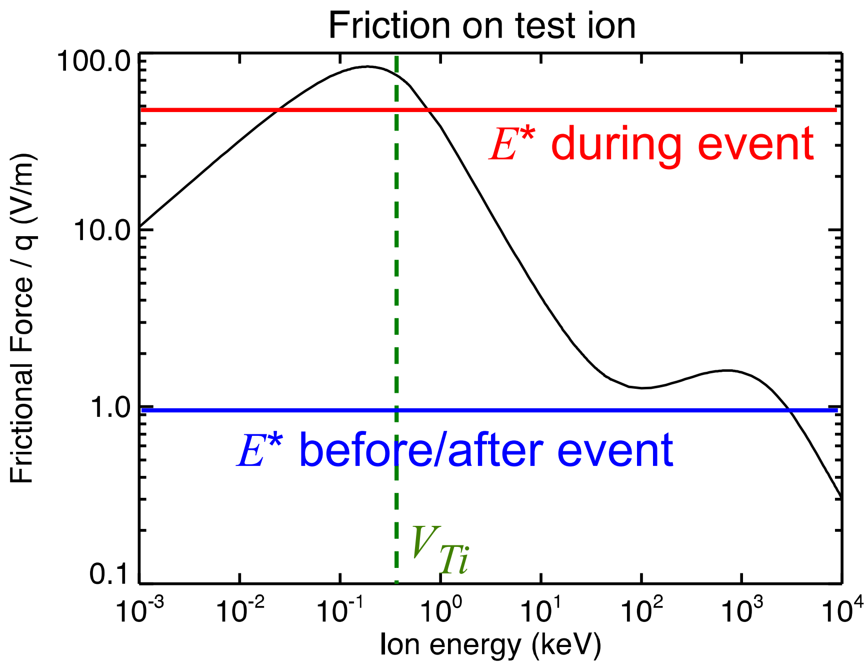




The inductive electric field (~ 50 V/m) is sufficiently large to consider runaway process

- Test particle analysis (Furth and Rutherford, PRL '72):

$$m\frac{dv}{dt} = qE^* - \mathcal{F} \qquad E^* = E \cdot \left(1 - \frac{Z}{Z_{eff}}\right) \cdot f\left(\frac{n_e^{tr}}{n_e}\right) \qquad \frac{E^*}{E} \simeq 80\% \text{ in MST}$$



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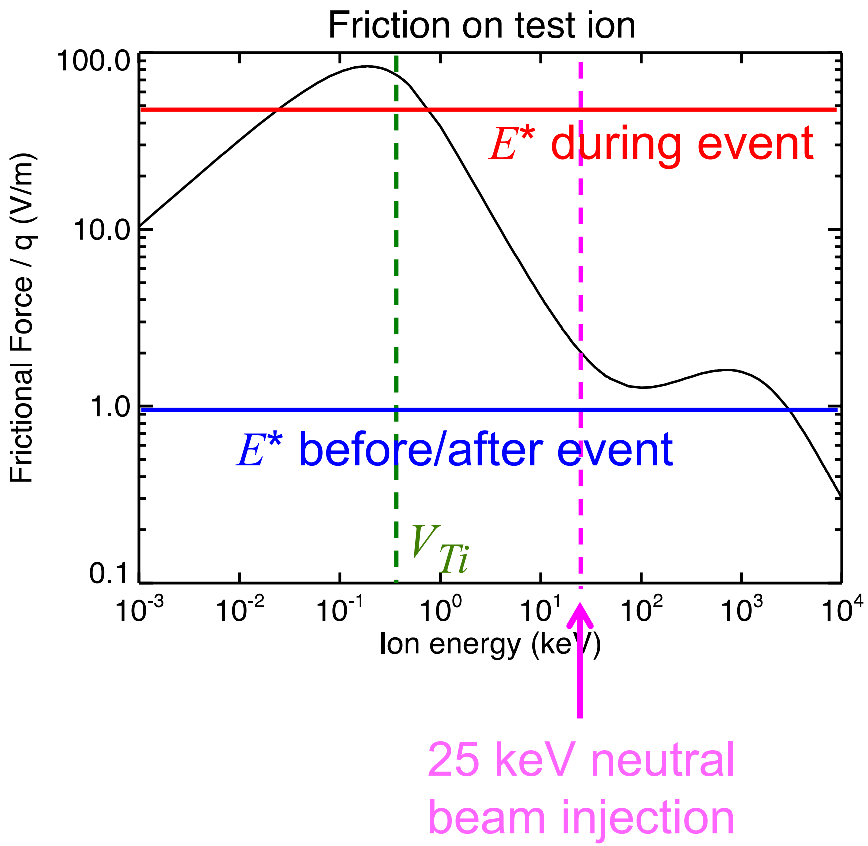
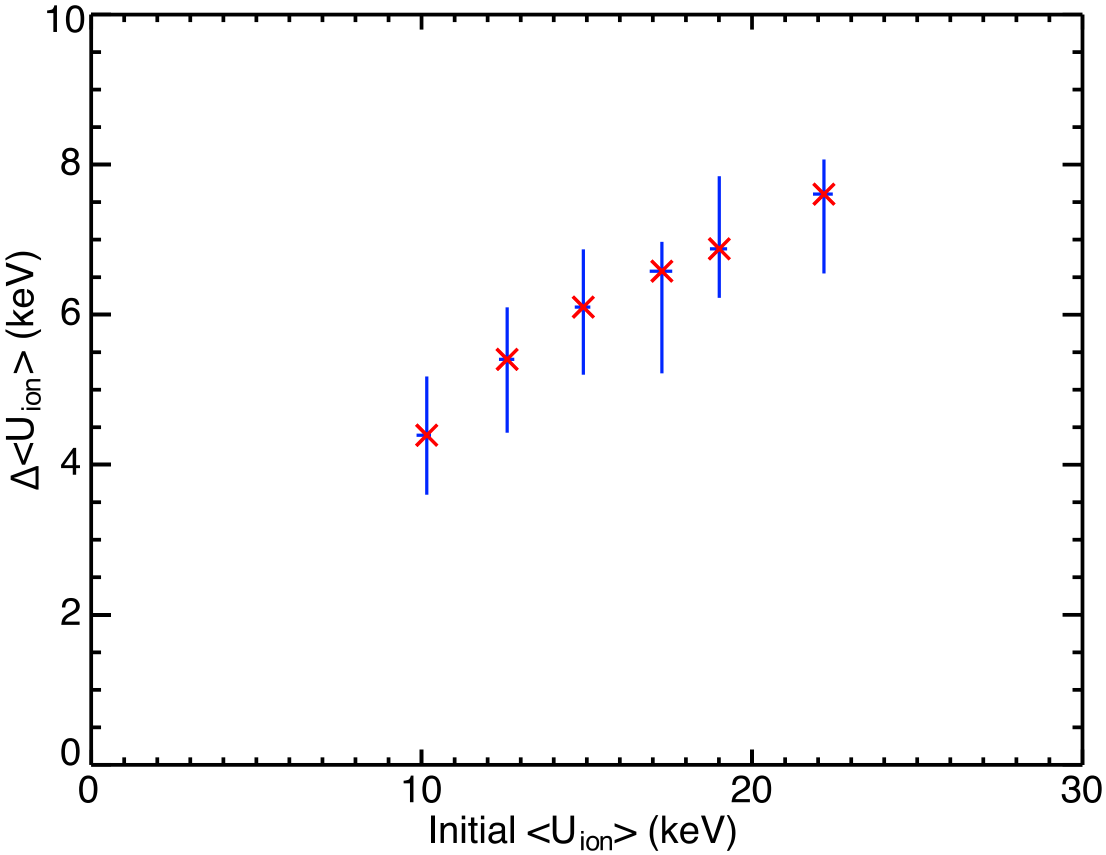
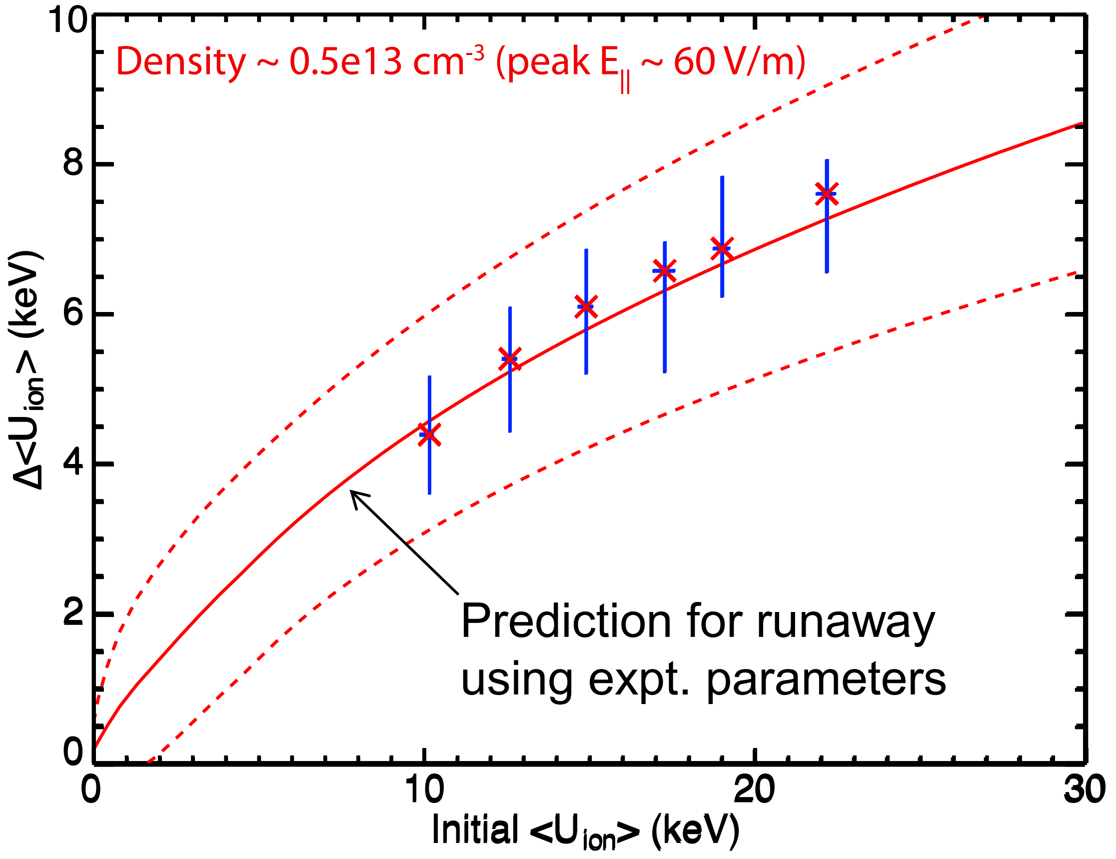


Energy gained increases as the fast ion's initial energy is increased (adjusted by the NBI voltage)



Energy gain is consistent with runaway acceleration



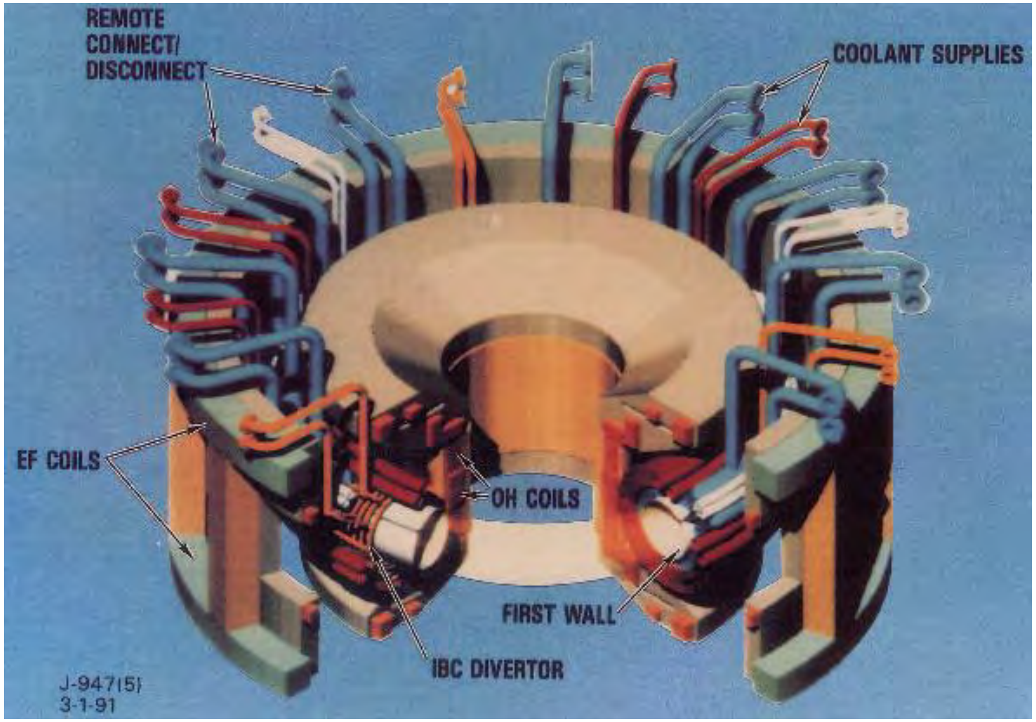
RFP Fusion Perspectives

In the magnetic fusion context, the RFP is best known as a high beta, compact fusion reactor option

- Since 1990, the TITAN system study has provided an RFP development target
- TITAN concept has attractive/novel features:
  - Steady-state, via oscillating field current drive
  - High core radiation for power handling, uniform on first wall surface
  - Single-piece maintenance
  - Low cost-of-electricity (COE) projection

TITAN Parameters

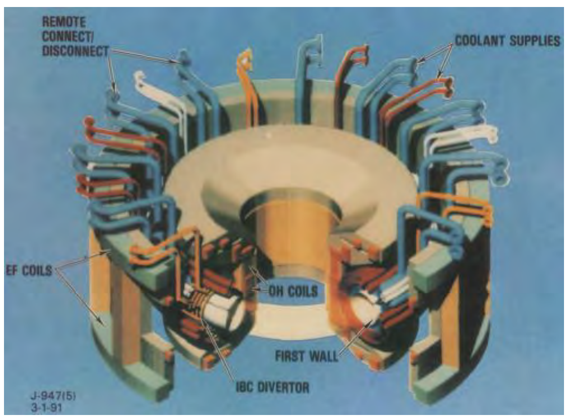
- a = 0.6 m
- R = 3.9 m
- I = 18 MA
- β_θ = 23%
- τ_E = 0.15 s
- 2300 MW fusion
- 970 MW net elec.

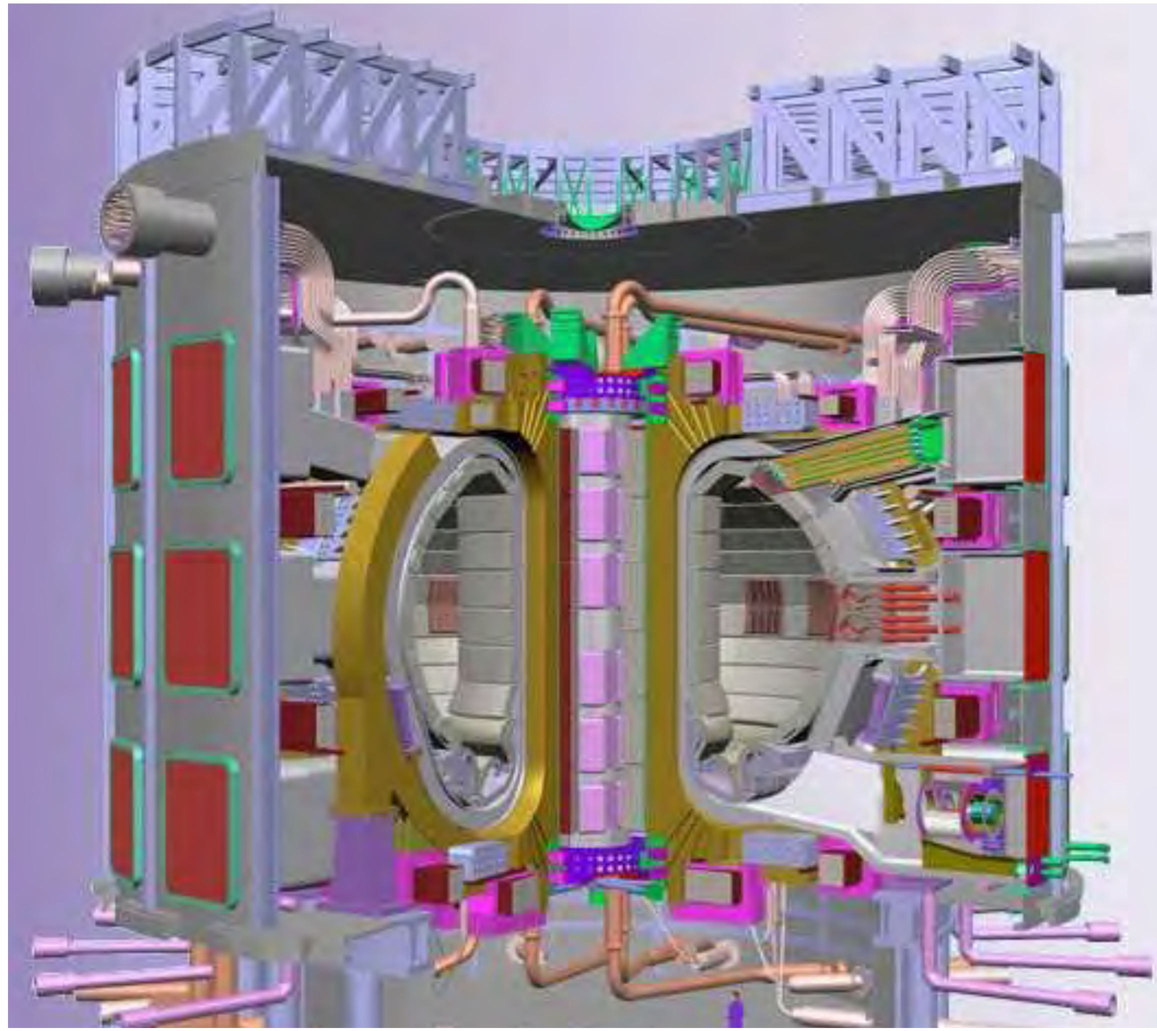


TITAN is very compact … on purpose, to assess the potential benefits of a compact fusion power core

TITAN wall loading:

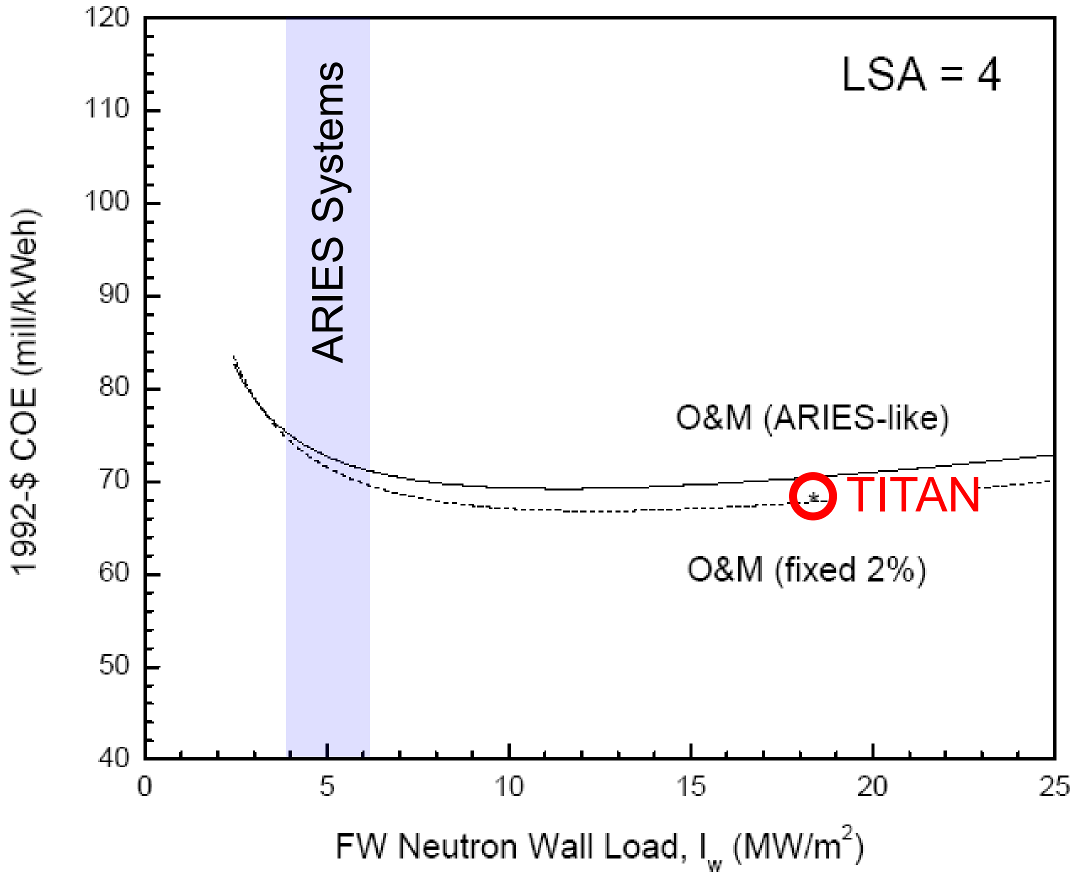
- 18 MW/m^2 neutrons
- 4.6 MW/m^2 radiation





System analysis indicates cost-of-electricity (COE) varies weakly above a threshold wall loading P_n / A ~ 5 MW/m^2

- COE levels out when the plasma minor radius is about the same as the blanket thickness (~ 1 m), a generic result for MFE



COE update in 2007 by Ron Miller, Decysive Systems

Scaling TITAN to an ARIES-like size with P_n/A ~ 5 MW/m²

- Power scalings, taking T fixed (~ 10-15 keV)

$$P_{fusion} \sim n^2 V \sim n^2 a^2 R \sim b^2 B^4 a^2 R \sim b^2 I^4 R / a^2 \sim \underbrace{\left(\frac{b^2 I^4}{a^3} \frac{R}{a} \right)}_{P_{fusion}/Area} a^2$$

$$P_{\Omega} \sim I^2 R / a^2 \quad \text{(ohmic heating)}$$

- Scale with fixed P_{fusion} and fixed temperature to ARIES-like wall load (5 MW/m²)

	R / a	$P_n / 4\pi^2 aR$	I_p	$B(a)$	n / n_G	β_θ	P_Ω
TITAN	3.9 / 0.6 = 6.5	20 MW/m²	18 MA	6.0 T	0.63	22%	30 MW
ARIES-like $R/a = 4$ $P/A = 5$ MW/m²	6.1 / 1.53 = 4.0	5 MW/m²	30 MA	3.9 T	0.5 T=13 keV	15%	20 MW

Consistent with analysis using TITAN system code

A comment on “ fusion power goes like B^4 ”

- A common phrase in MFE, just the simple point:

$$P_{fusion} \sim n^2 V \sim \left(\frac{n^2}{B^4}\right) B^4 V \sim \beta^2 B^4 V$$

- B^4 really defines a threshold magnetic field $\langle B \rangle$ ~ 5 T for a reactor plasma
  - a little smaller ➡ not enough reactivity
  - a little bigger ➡ rapid increase in fusion power density & wall loading
  - Importantly, higher B permits lower β (an option for robust stability)

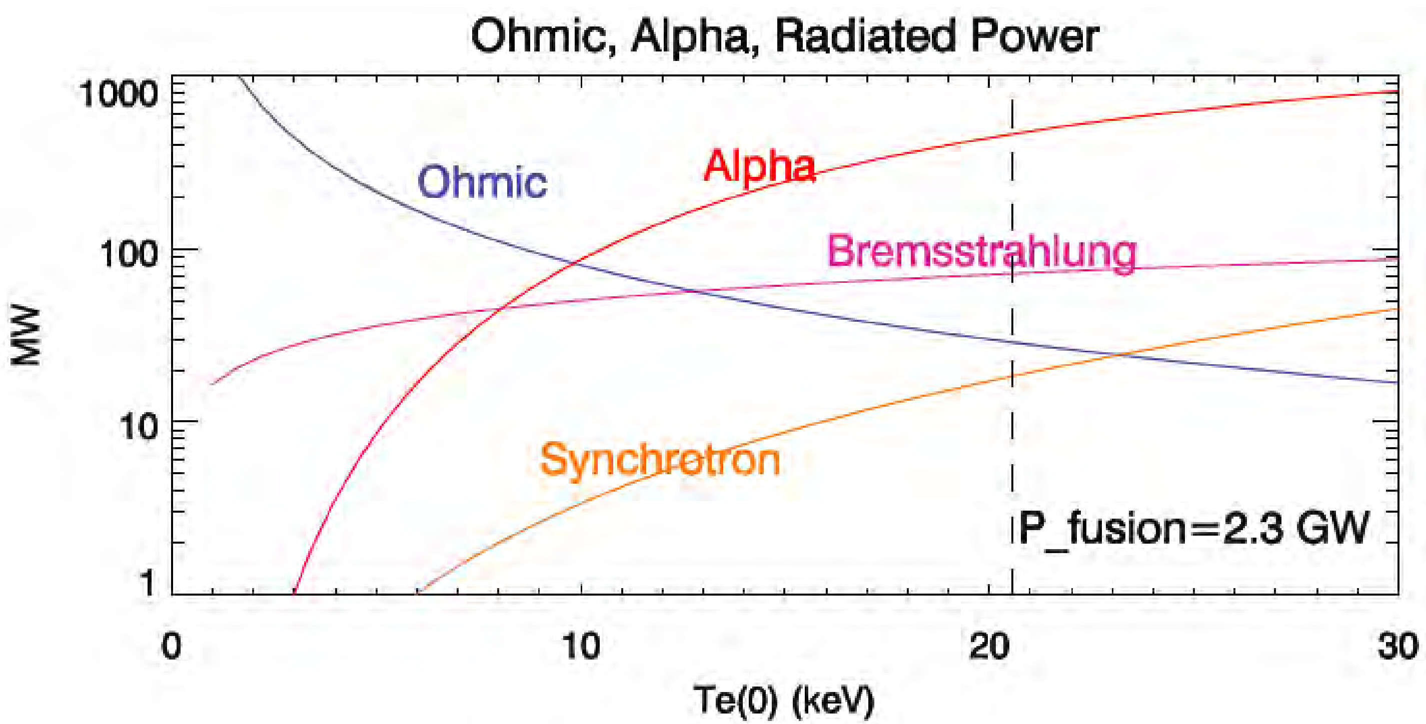
$$B^4 \sim \frac{P_{fusion}}{\beta^2 V} \sim \left(\frac{P_{fusion}}{A}\right)\frac{1}{\beta^2 a} \quad \Rightarrow \quad B \sim \left(\frac{P_{fusion}}{A}\right)^{1/4} \beta^{-1/2} a^{-1/4}$$

Hence $\langle B \rangle$ ~ 5 T for any magnetic configuration, c.f., ARIES

The RFP’s advantage is $B_{coil} / \langle B \rangle < 1$

Ohmic heating (Lawson-like) power balance for ARIES-like RFP

- 1D profiles similar to present-day RFP assumed
- a =1.5 m, R/a=4, I_p = 30 MA, $\langle B \rangle$=5.6 T, B_{coil} ~ 3 T, P_n/A ~ 5 MW/m^2, P_{fusion}=2.3 GW

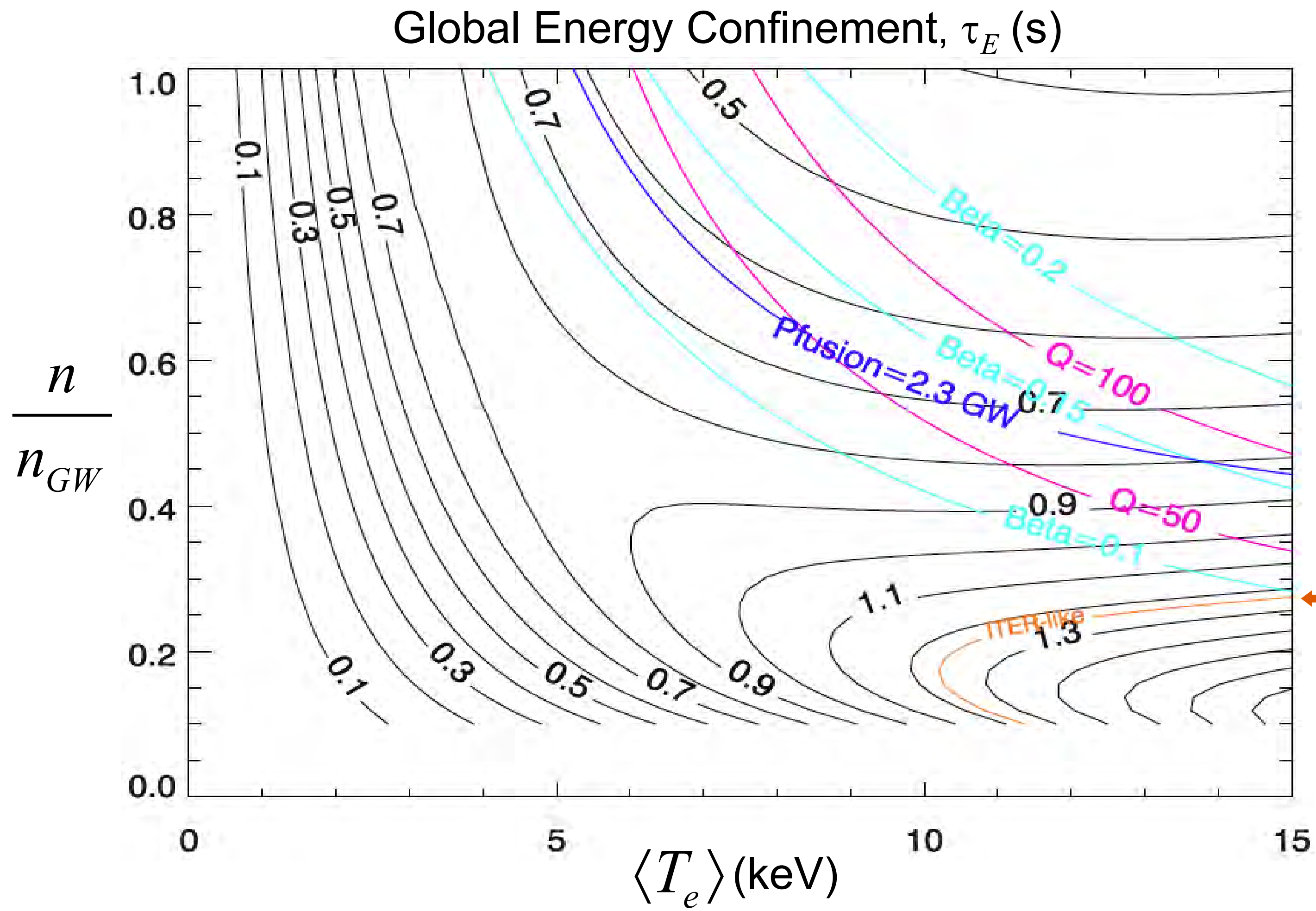


$$J/B \sim 1 - r^4$$

$$T \sim 1 - r^2, \quad n \sim 1 - r^8$$

$$Z_{eff} = 2, \quad Z_{neo} = 2$$

Ohmic ignition is possible in the RFP if energy confinement is similar to that of a tokamak



← "ITER-like" means scaled as $\tau_E \sim \kappa a^2$

Summary

- MST program – a synergy of efforts that
  - Advance the RFP
  - Advance the science of toroidal confinement
  - Advance basic plasma physics

- Excellent progress in understanding key physics
  - Tearing and magnetic relaxation
  - Stochastic transport physics
  - Multiple paths to improved confinement
  - Emerging story on drift-waves, not covered here

- Understanding of ion physics is maturing
  - Heating characteristics more detailed than ever, but underlying mechanism(s) still elusive
  - Classical impurity ion confinement when tearing is reduced
  - Classical energetic ion confinement allows building fast ion pressure
  - NBI-driven EP modes observed for the first time in an RFP, which appear to limit the local fast ion pressure

MST and the world RFP program have achieved the Proof-of-Principle metrics that were established in 1998

- Proof-of-principle initiated in 2000, with a ramp of funding over several years

As proposed 1998

RFP Proof-of-Principle Research Program

ISSUE	ACTIVITY	CRITERIA
Confinement	• Current Scaling *(RFX, TPE-RX)* • $J(r)$-Control *(MST, others?)*	$\tau_E > 10$ ms $T > 1$ keV
Beta Limit	• Auxiliary Heating *(MST)*	$\beta \sim 15\%$
Resistive Shell	• Diagnose Instabilities *(T2)* • Control *(exploratory concept expts.)*	stabilize kinks
Current Sustainment	• Test Oscillating Field Current Drive *(MST, others?)*	~0.1 A/W with τ & T above
Optimized RFP Configurations	• Optimize Profiles: $\mathbf{J}, p, \mathbf{V}$ *(MST, others?)* • Optimize Geometry: shape, *R/a*, etc. *(exploratory concept expts.)*	
System Studies	• Renew with Advanced RFP Concepts	

DECISION

2004

High Current / Long Pulse Experiment

and/or

Advanced Control Experiments

or

Reduced RFP Research

Supporting Plasma Physics Theory & Expt.

Figure 6. World-integrated RFP proof-of-principle research program.

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System Studies	• Renew with Advanced RFP Concepts	

Achieved:

12 ms
T_e ~ 2 keV, T_i ~1-2 keV

26%, with pellet injection

Stable by active control

0.1 A/W, 10% drive
standard confinement

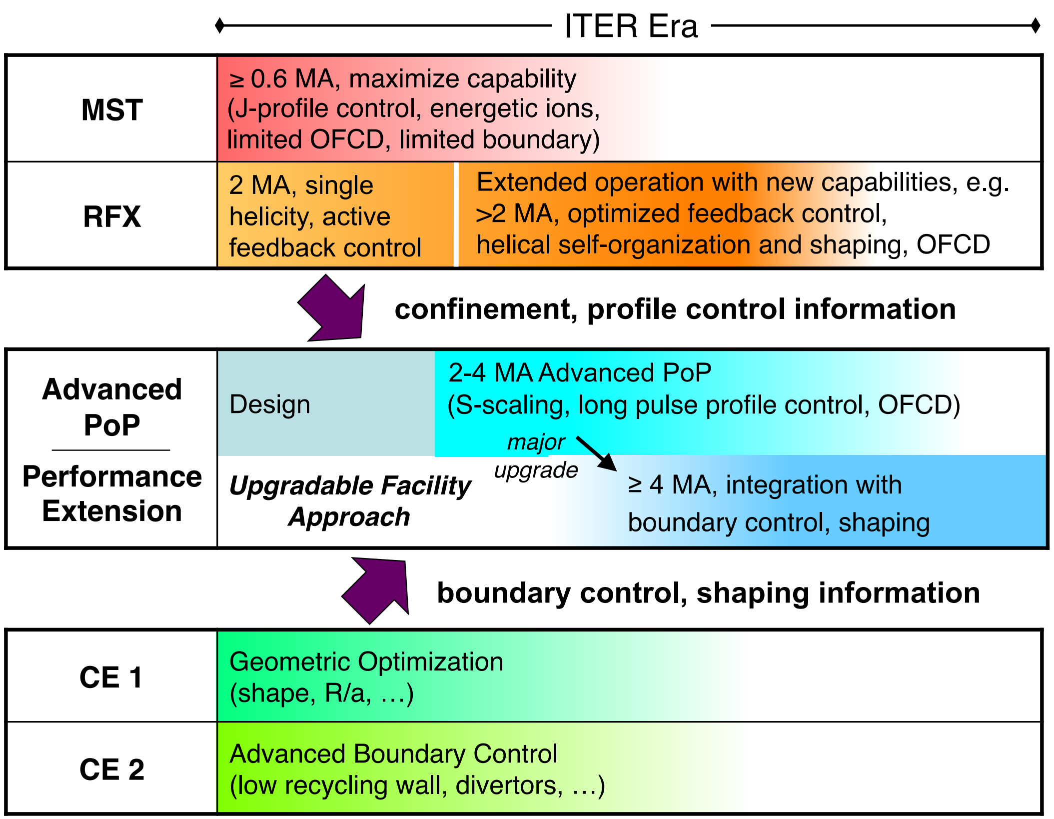
New experiments estab.

(near zero effort)

Figure 6. World-integrated RFP proof-of-principle research program.

Excellent progress warrants planning for a next-step RFP

- Build on progress in last decade in improved confinement, control of resistive wall modes, and demonstrated high beta.
- MST is not saturated in its capabilities, while RFX shows transformational new trends at high current.
- Resolution of high priority issues in confinement and sustainment requires higher current, longer pulse length.



Conceptual development path for the RFP, output from TAP/ReNeW.

RELAX, …

Extrap-T2R, now KTX