3D Equilibrium Reconstruction for Stellarators and Tokamaks

Samuel A. Lazerson[1]

D. Gates[1], N. Pablant[1], J. Geiger[2], Y. Suzuki[3], T. Strait[4]

[1] Princeton Plasma Physics Laboratory

[2] Institute for Plasma Physics

[3] National Institute for Fusion Science

[4] General Atomics

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Accurate modeling of experimental conditions requires 3D equilibrium reconstruction

- Three-dimensional equilibrium effects are significant for both stellarators and tokamaks
  - Internal Helical States
  - RMP Application
  - Error Fields
- Bootstrap current and finite beta effects are significant for stellarators
  - Measurement and control will be an issue for diverter design

Outline

- What is equilibrium reconstruction?
- What tools do we use?
- Examples
    - LHD - stellarator reconstruction
    - DIII-D - 3D tokamak reconstruction
    - W7-X - forward modeling for stellarators
    - ITER - forward modeling for 3D tokamaks
- Concluding remarks

What is equilibrium reconstruction?

It is the goal of equilibrium reconstruction to fit an equilibrium model to a set of experimental measurements within defined error bars.

- Profiles (Ne, Te, Ti, etc.)
- External Magnetics
- Plasma Shape

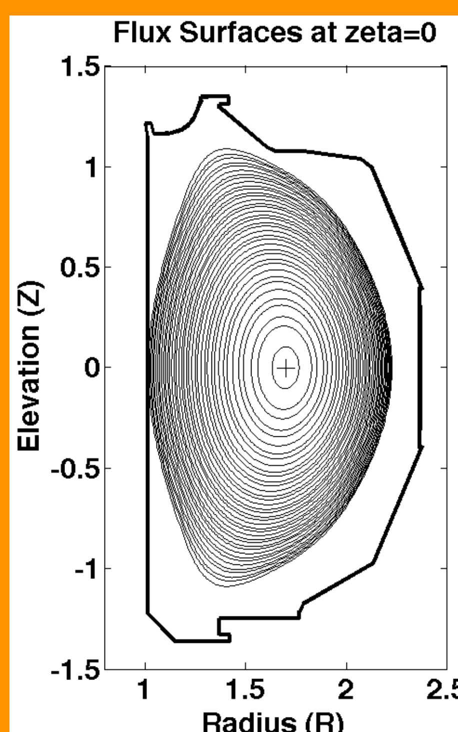
"Remember that all models are wrong; the practical question is how wrong do they have to be to not be useful?"
- George E.P. Box

Schematically what are we doing?

Input Parameters
- Currents
- Vacuum Fields
- Pressure, etc.

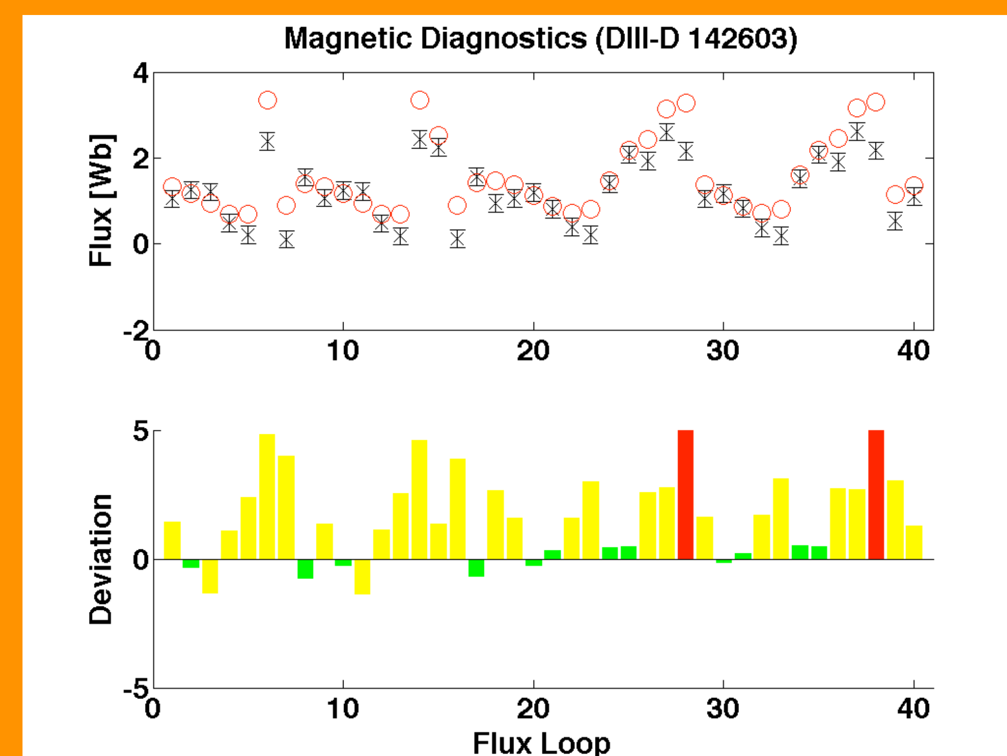
These need to result in a good match to these.

Equilibrium Model



Synthetic Signals

Measurements



What do we need to consider before we start?

- What are we measuring?
- Do these measurements constrain our equilibrium model or any model?
- Is there a simpler equilibrium model?
- Is the equilibrium model robust?

How do we fit the equilibrium to data?

- Direct data interpretation
    - Robust but often impossible
- Searching Methods
    - Versatile but can be misled by bad data
- Statistical Methods
    - Most robust but difficult to implement

How do we fit the equilibrium to data?

- Searching Methods
  - Versatile but can be misled by bad data

Equilibrium Reconstruction as Curve Fitting

Equilibrium is defined by a set of input parameters (p0). Reconstruction seeks to find a perturbation to those parameters (h) which provides a set of input parameters (pf) which are a better fit to experimental measurements.

$$\vec{p}_f = \vec{p}_0 + \vec{h}$$

The quality of the set of a set of inputs (p) is evaluated by

$$\chi^2(\vec{p}) = \sum \left| \frac{\vec{y}_{target} - \vec{y}(\vec{p})}{\vec{\sigma}_{target}} \right|^2$$

The Levenberg-Marquardt method is used to find (h)

$$\vec{h}_{LEV-MAR} = [\underbrace{\tilde{J}^{\dagger}\tilde{W}\tilde{J}}_{\text{Jacobian}} + \lambda diag(\tilde{J}^{\dagger}\tilde{W}\tilde{J})]^{-1}\tilde{J}^{\dagger}\underbrace{\tilde{W}}_{\text{Weight Matrix}}(\vec{y}_{target} - \vec{y}(\vec{p}))$$

STELLOPT fits parameters using various elements

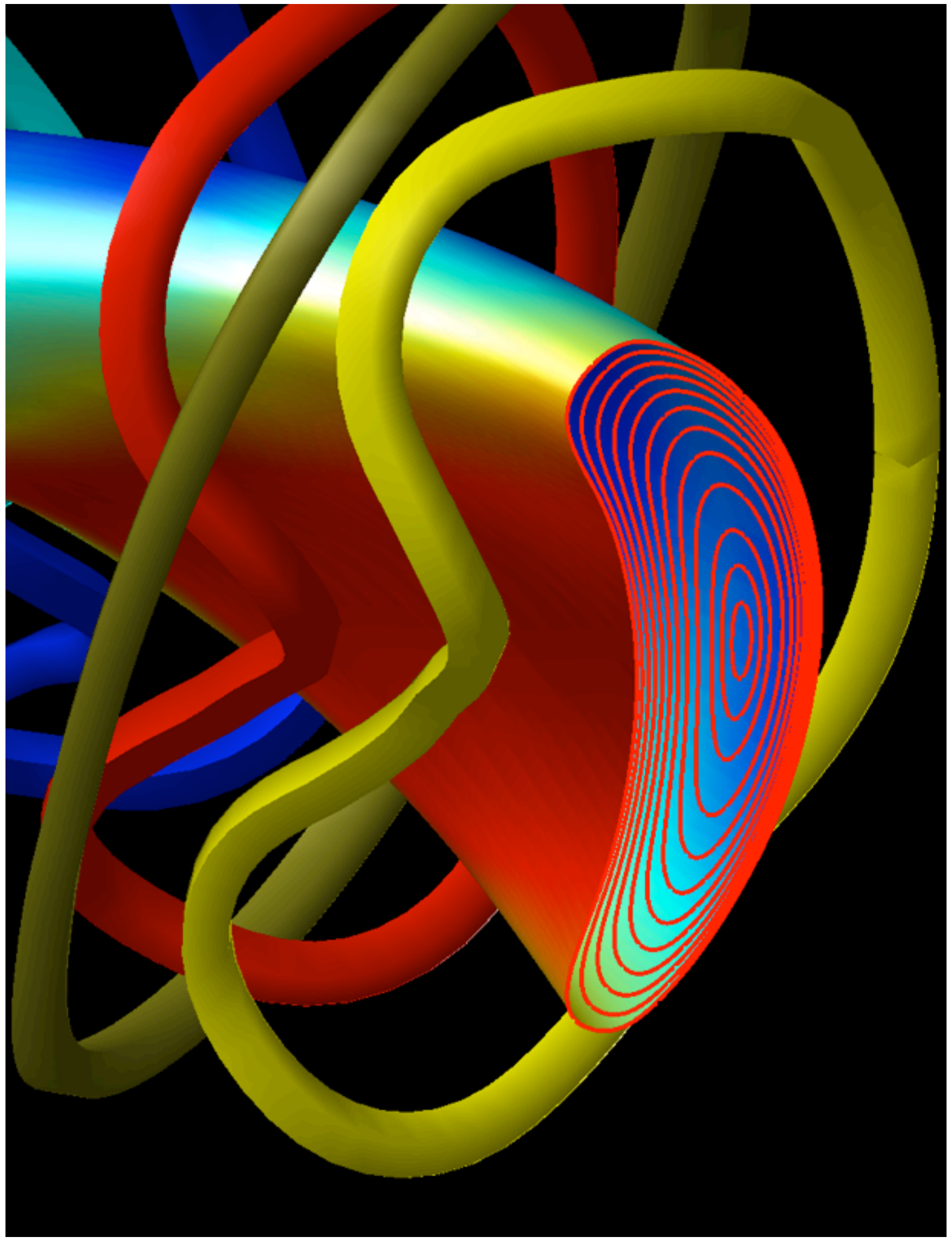
- Equilibrium Model: VMEC
  - Stepped Pressure Equilibrium Code (SPEC) under development
- Fitting Method: Levenberg-Marquardt
- Synthetic Diagnostics:
  - Magnetics
  - Charge Exchange
  - Motional Stark Effect
  - Thomson Scattering
  - Interferometry

VMEC provides a 3D equilibrium model

- Ideal 3D MHD equilibrium model
- Constraint of nested flux surfaces

$$W = \int \left(\frac{|B|^2}{2\mu_0} + \frac{p}{\gamma - 1} \right) d^3x$$

$$\vec{J} \times \vec{B} - \nabla p = 0$$

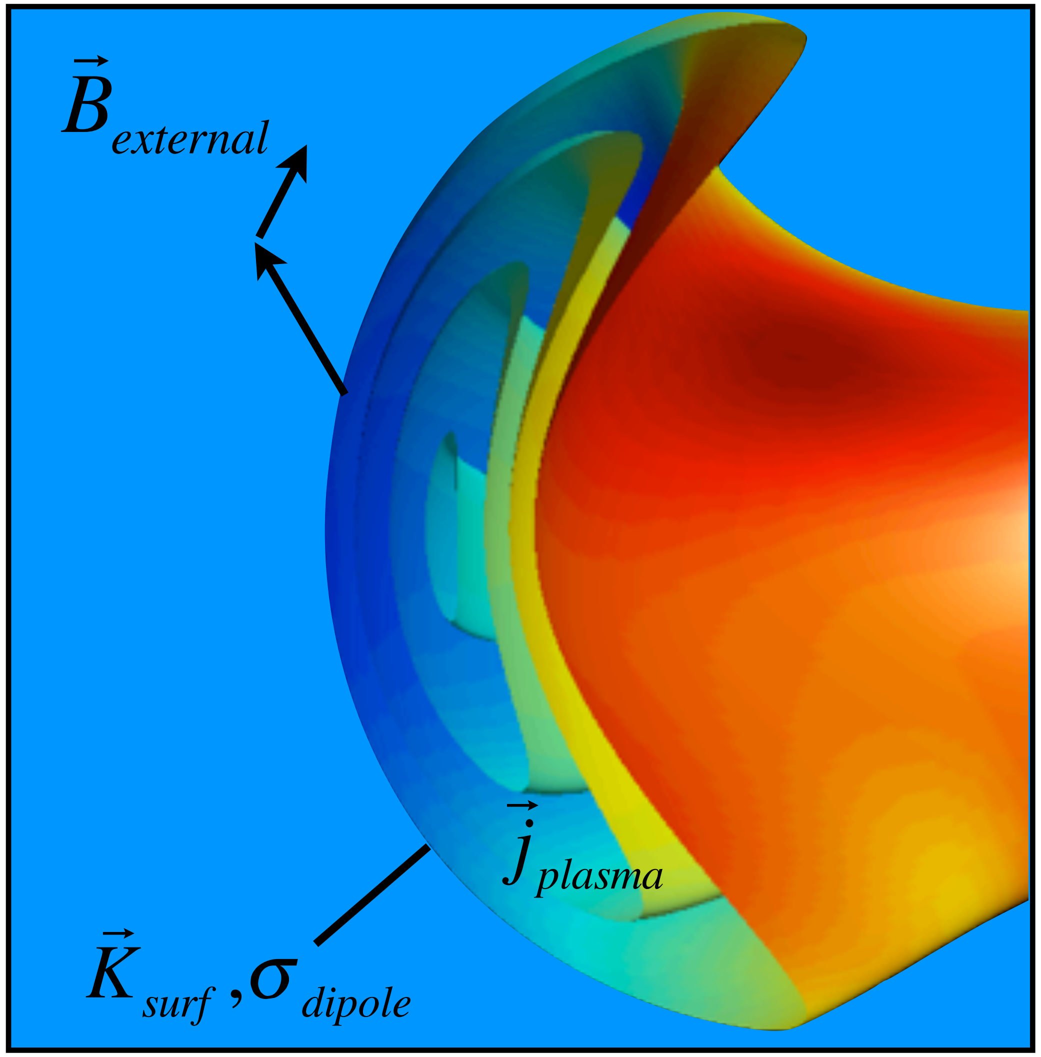


Free boundary W7-X equilibria as calculated by VMEC

S.P Hirshman and J.C.Whitson, Phys. Fluids 26 (12), 1983

DIAGNO2 calculates magnetic diagnostic response using the virtual casing principle in 3D

- Utilizes virtual casing principle (Shafranov et al.)
  - Surface integral is fast to compute
  - B on a surface is general (no code dependence)
- Calculates diagnostic responses



Surface Representation

$$\hat{n} \times \left(\vec{B}_{out} - \vec{B}_{in} \right) = \mu_0 \vec{K}_{surf}$$

$$\left(\vec{B}_{out} - \vec{B}_{in} \right) \cdot \hat{n} = \mu_0 \sigma_{dipole}$$

External Fields

$$\vec{B}(\vec{x}) = \frac{\mu_0}{4\pi} \int \frac{\vec{K}' \times (\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3} dA' + \frac{\mu_0}{4\pi} \int \frac{\sigma'_{dipole} (\vec{x} - \vec{x}')}{|\vec{x} - \vec{x}'|^3} dA'$$

H.J Gardner, Nuclear Fusion 30 1417-1424 (1990)

M. Drevlak et al. Nuclear Fusion 45 (2005)

S. Lazerson et al., Plasma Phys. Control. Fusion 55 025014 (http://dx.doi.org/10.1088/0741-3335/54/12/122002)

STELLOPT optimizes VMEC input parameters to match experimental measures

- Algorithms
  - Levenberg-Marquadt
  - Differential Evolution
  - Particle Swarm
  - Mapping
- Features
  - MPI Parallelization
  - Modular Target Functionals
  - Various profile parameterizations

$$\chi^2(\vec{p}) = \sum \left| \frac{\vec{y}_{target} - \vec{y}(\vec{p})}{\vec{\sigma}_{target}} \right|^2$$

Variables (p)	Targets (y_target)
Enclosed Flux	Flux Loops
Toroidal Current	Magnetic Probes
Pressure Scaling	Thomson (Te, Ne)
Te Profile	Charge Exchange (Ti)
Ne Profile	Line Integrated Density
Ti Profile	Motional Stark Effect
Current Profile	Vacuum Field
Vacuum Fields	Toroidal Current

Species profiles parameterize pressure profile

- STELLOPT parameterizes temperature and density as flux surface functions

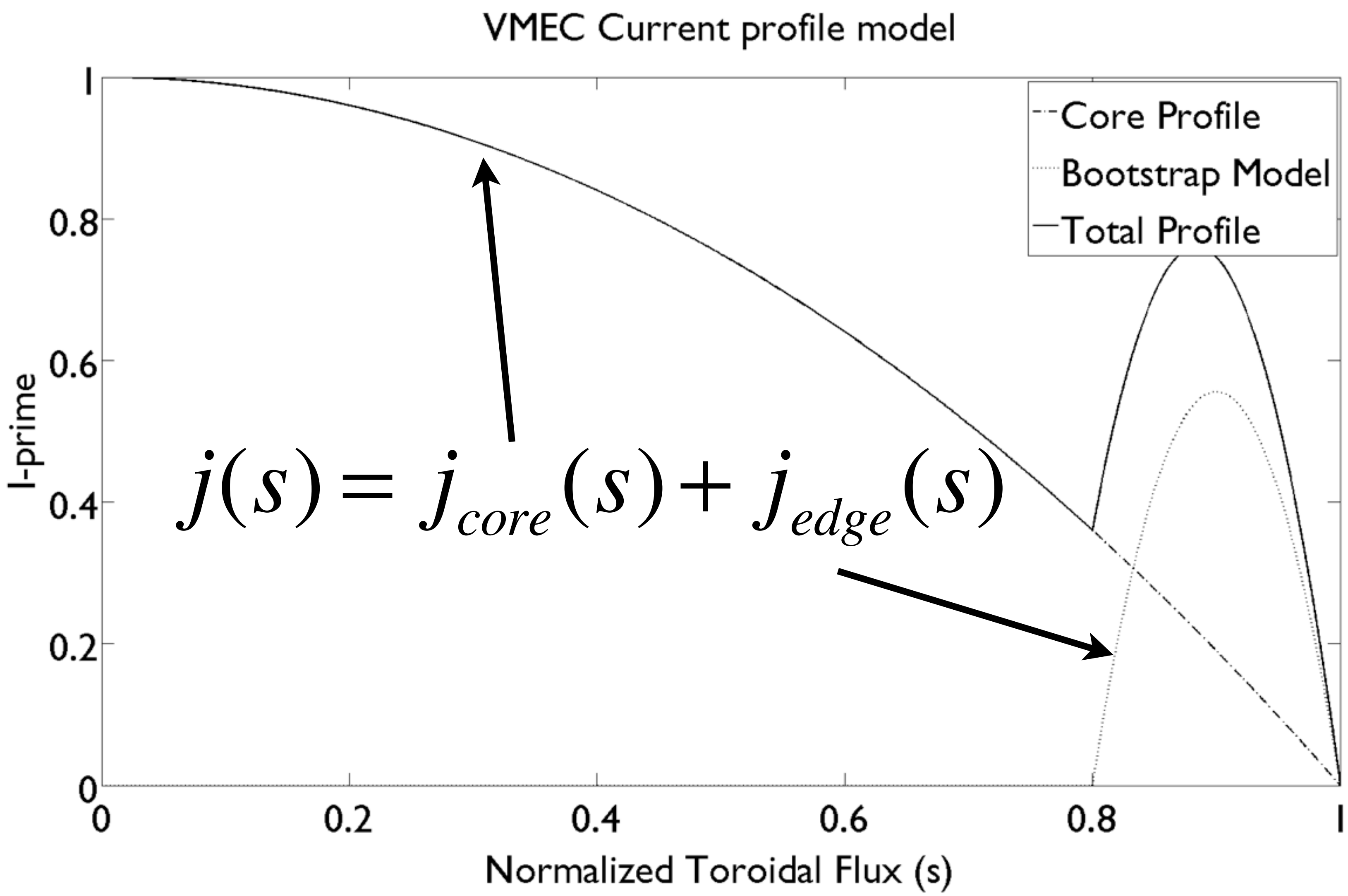
$$p(s) = n_e(s) k_B [T_e(s) + T_i(s)]$$

s: normalized toroidal flux

- VMEC profile parameterizations are implemented
  - Power Series, Two Power, Spline, Akima Spline, Two Lorentz, Pedestal, etc.

Current density divided into edge and core profiles

- Current density may also be treated as a sum of two currents

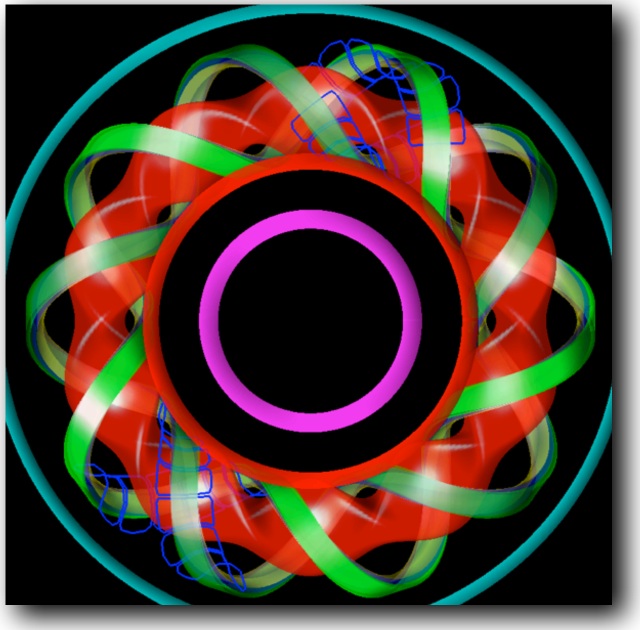


Present 3D reconstruction efforts

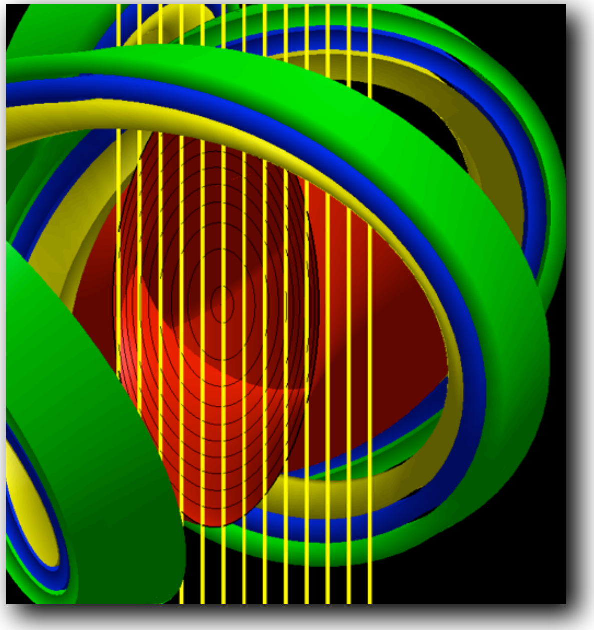
- **<u>STELLOPT</u>**
    - W7-AS, **<u>LHD</u>**, **<u>DIII-D</u>**, **<u>W7-X</u>**, **<u>ITER</u>**, NSTX
- V3FIT
    - CTH, HSX, RFX, DIII-D, MAST, JET

The Large Helical Device requires a 3D reconstruction capability

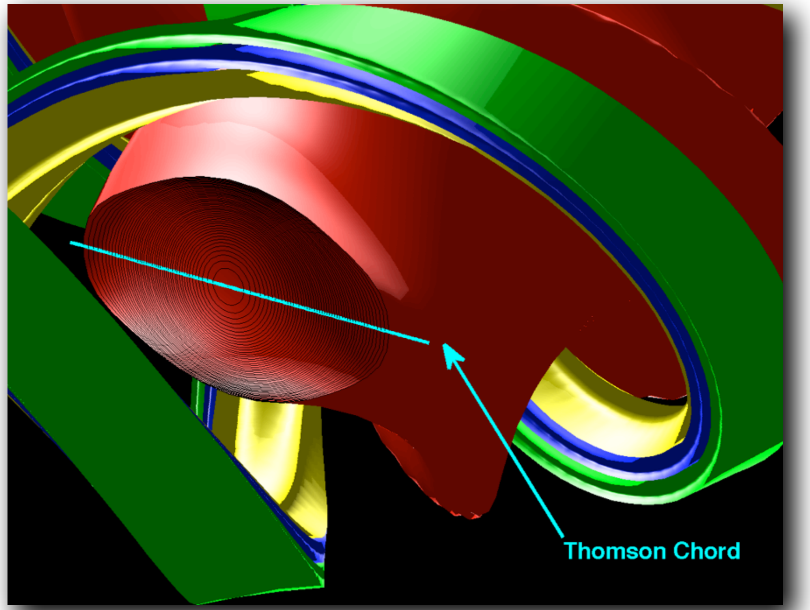
- Finite beta effects, beam driven current drive, and island healing modify the equilibrium away from vacuum configuration.



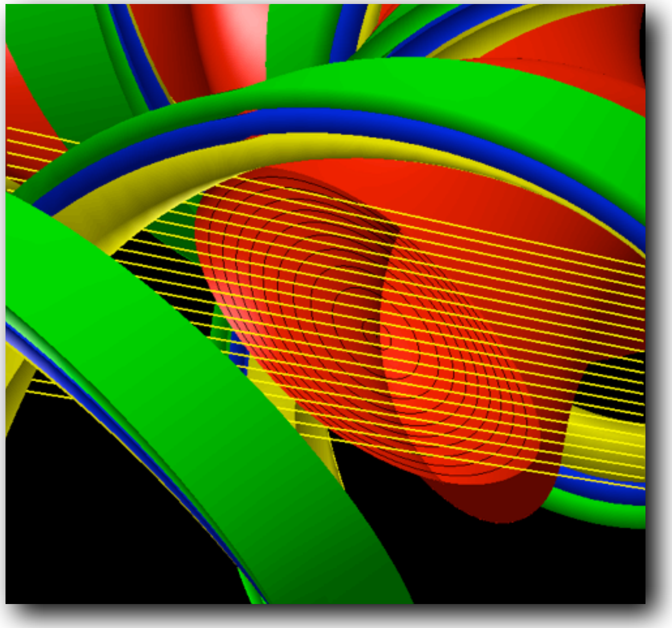
Magnetics



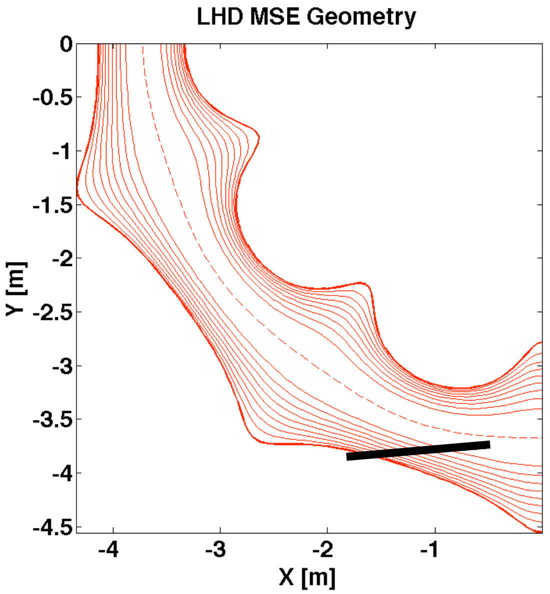
Interferrometry



Thomson

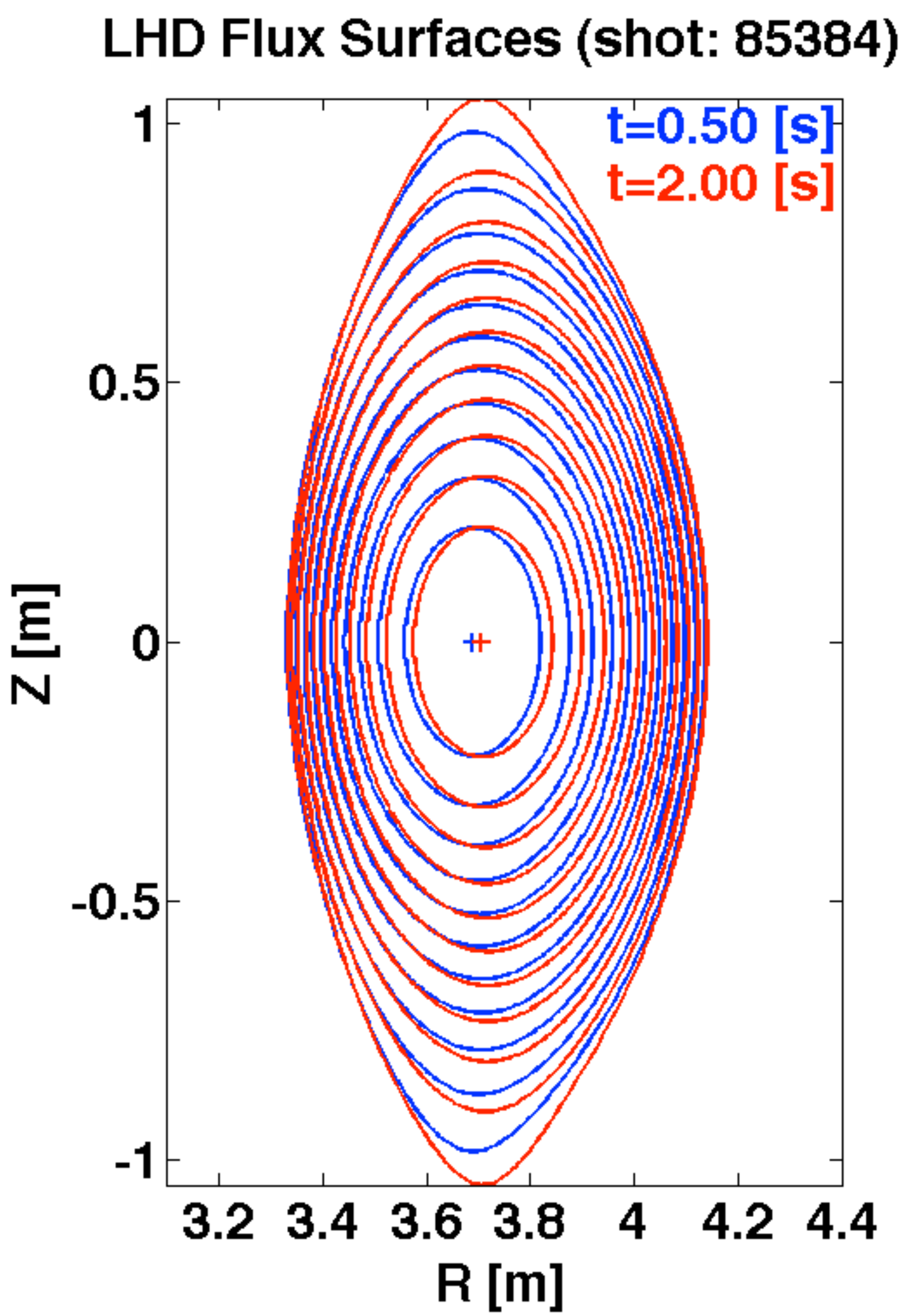


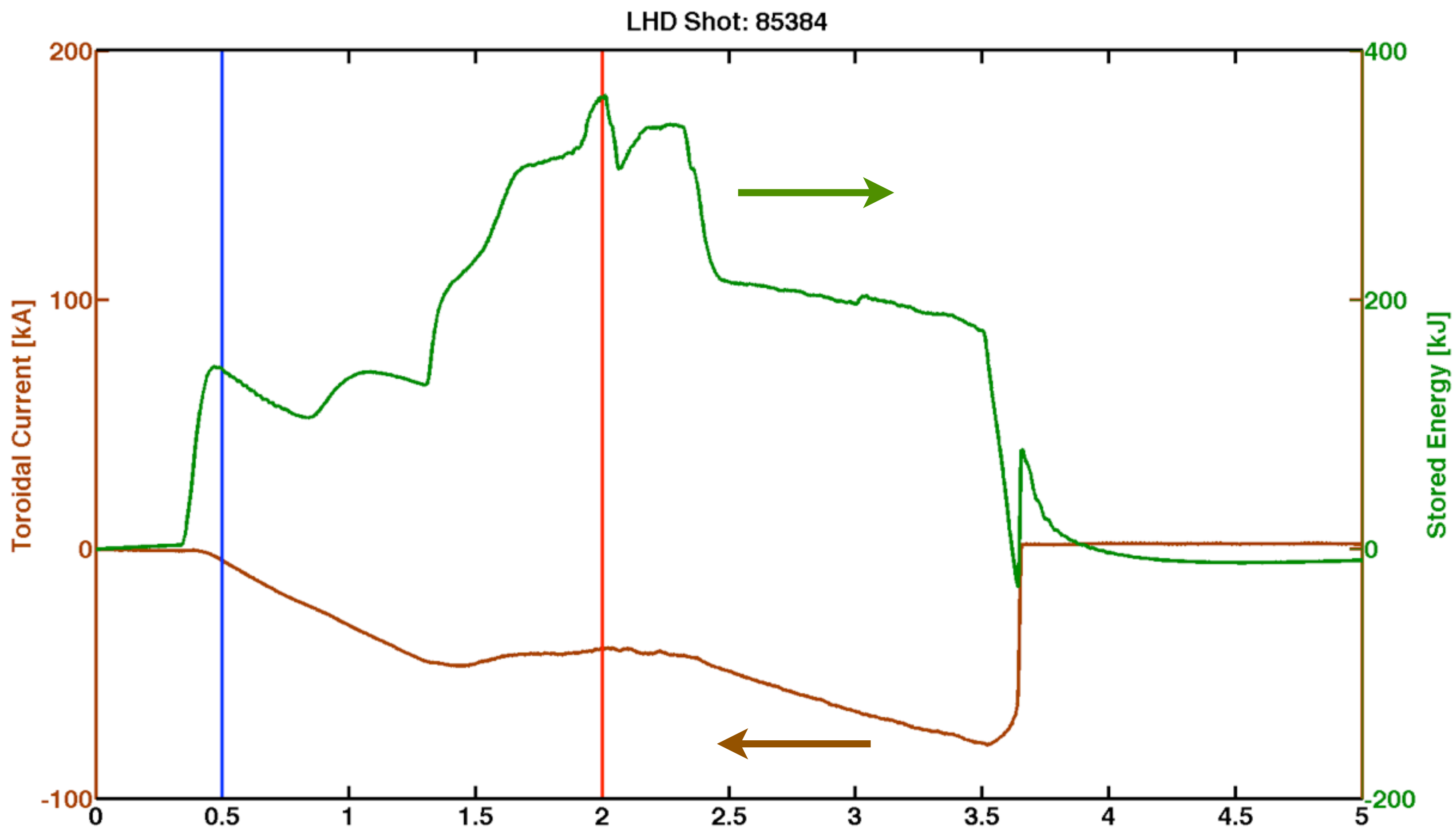
XICS



MSE

LHD has a significant axis shift at finite beta



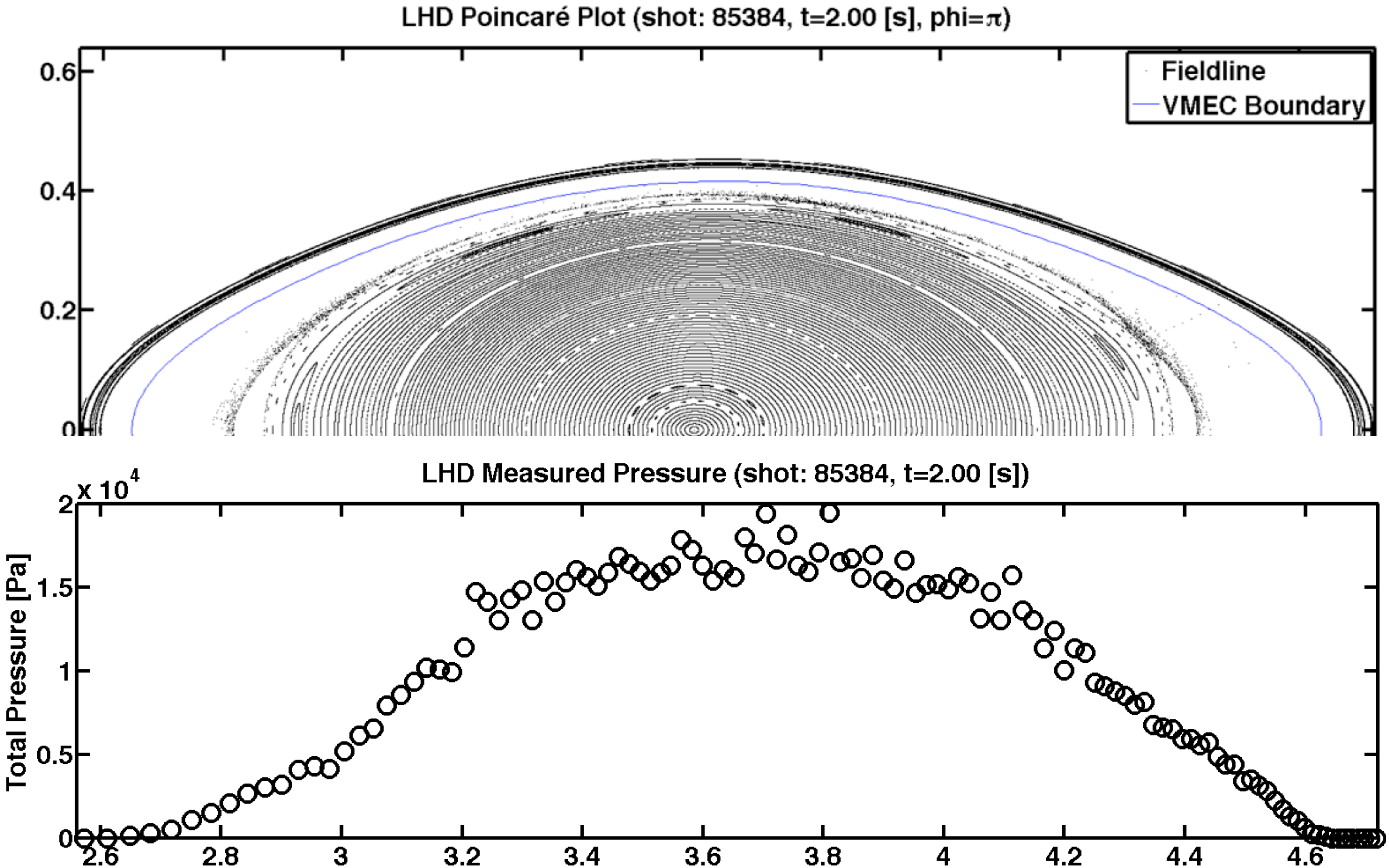


Raxis = 3.6034 [m] (vac)

Raxis = 3.6552 [m] (0.50 [s])

Raxis = 3.6794 [m] (2.00 [s])

Thomson-Vacuum mismatch highlights need for 3D reconstruction

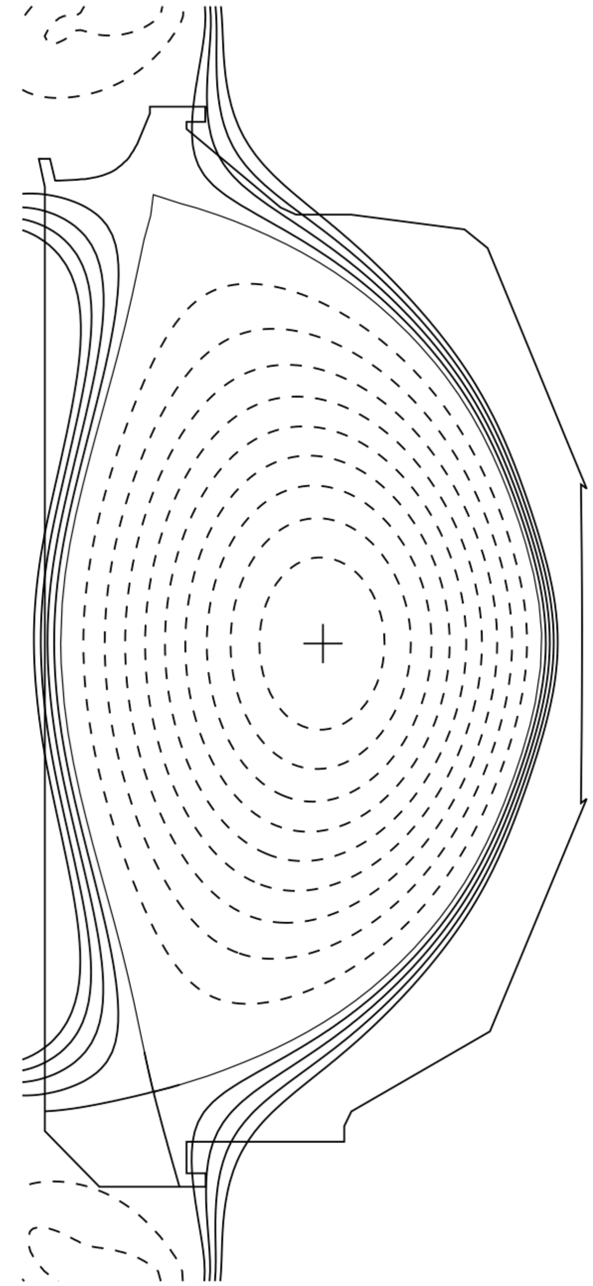


LHD reconstruction work is ongoing

- Have identified need for equilibrium reconstruction and has been incorporated into transport analysis codes (TASK3D)
- Actively used for XICS data inversion
- Implementation of MSE and Ti measurements allow usage on modern Tokamaks such as DIII-D

STELLOPT is being applied to 3D Tokamaks

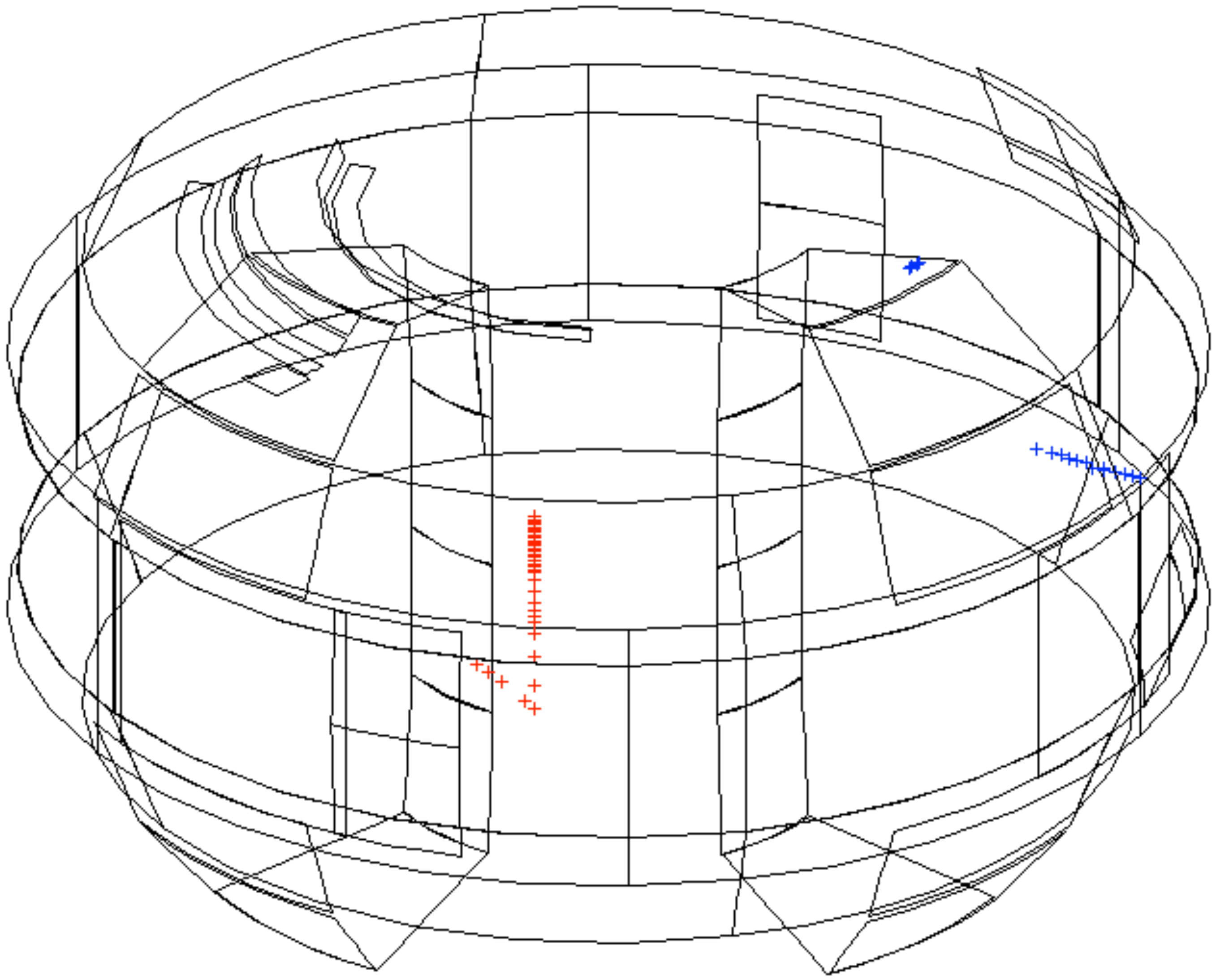
- A double null shot was conducted on DIII-D for use in stellarator codes (up/down symmetry constraint)
  - n=3 RMPs applied
  - No ELM suppression
- 3D equilibrium reconstructions provide accurate models of the plasma response as measured in the experiment



EFIT Reconstruction

The DIII-D diagnostics constrain 3D equilibrium

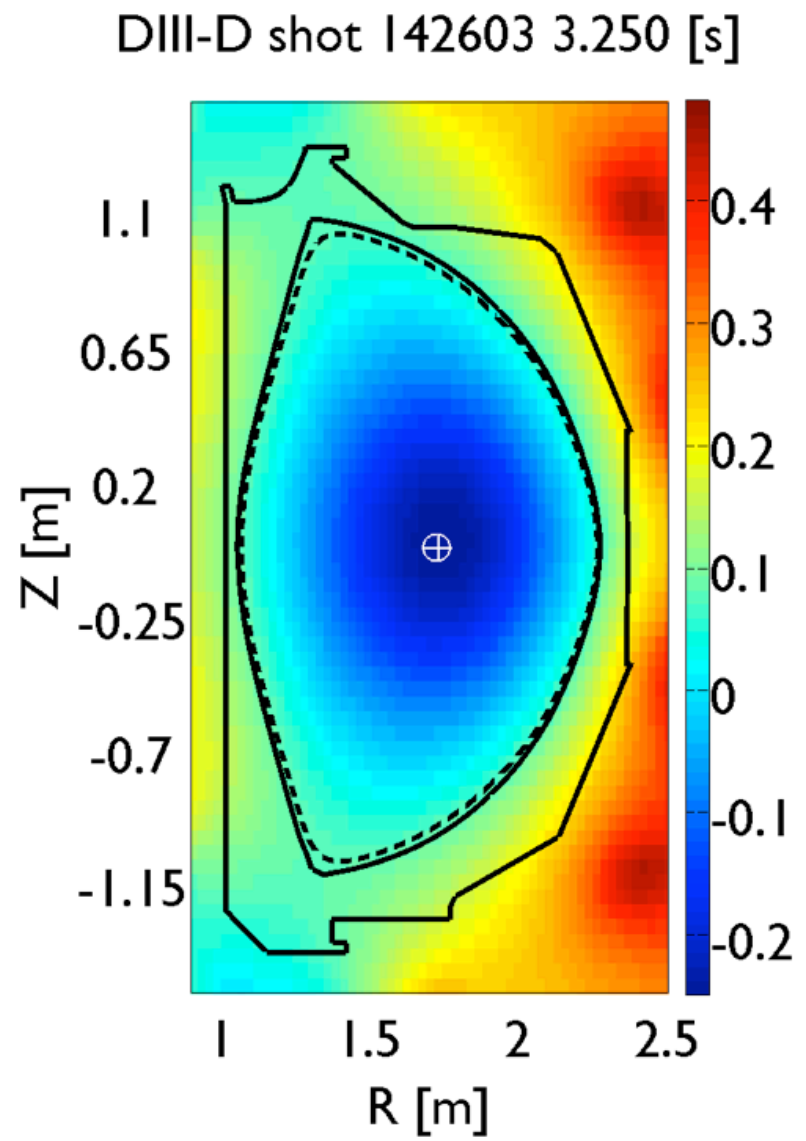
- Measurements
  - B-Probes (74)
  - Flux Loops (104)
  - Rogowski (1)
  - Interferometer (4)
  - Thomson (36)
  - CXRS (17)
  - MSE (32)

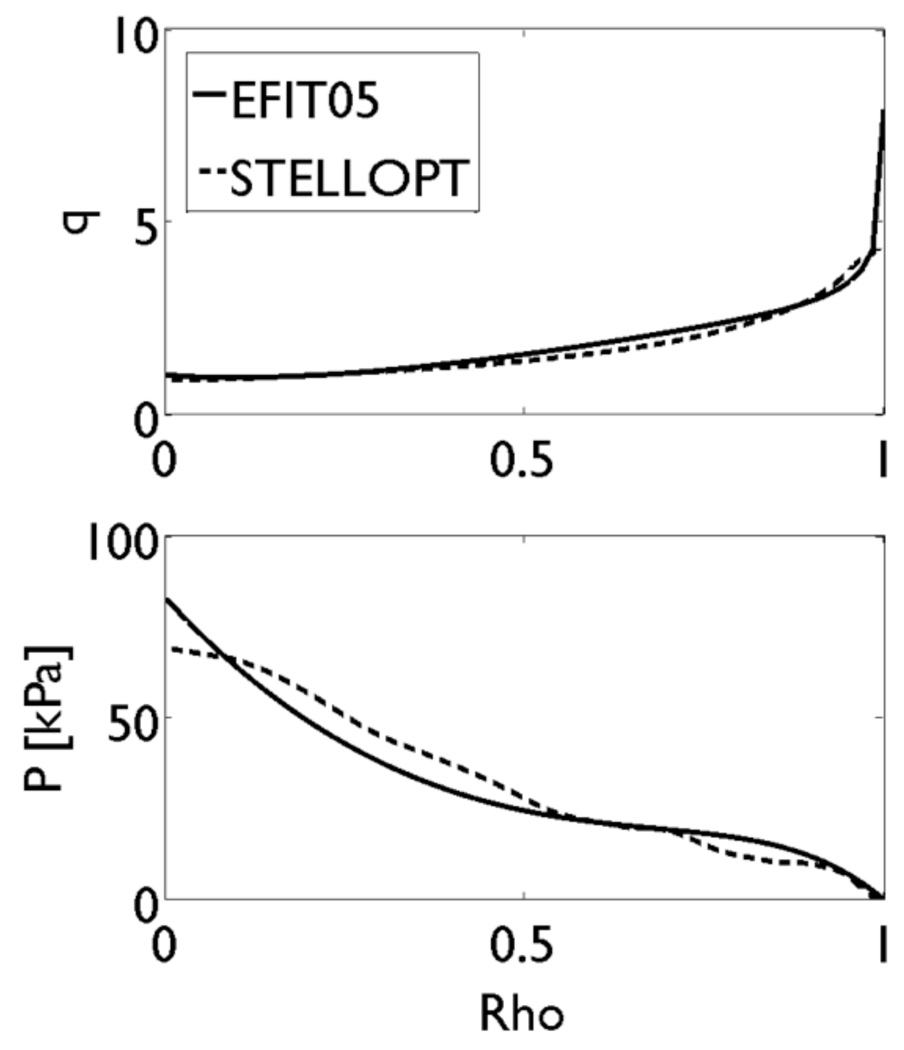


DIII-D non-axisymmetric flux loops, Thomson (red), and charge exchange (CXRS, blue)

2D STELLOPT compares well with EFIT

- VMEC boundary compares well with EFIT
- q profiles agree well





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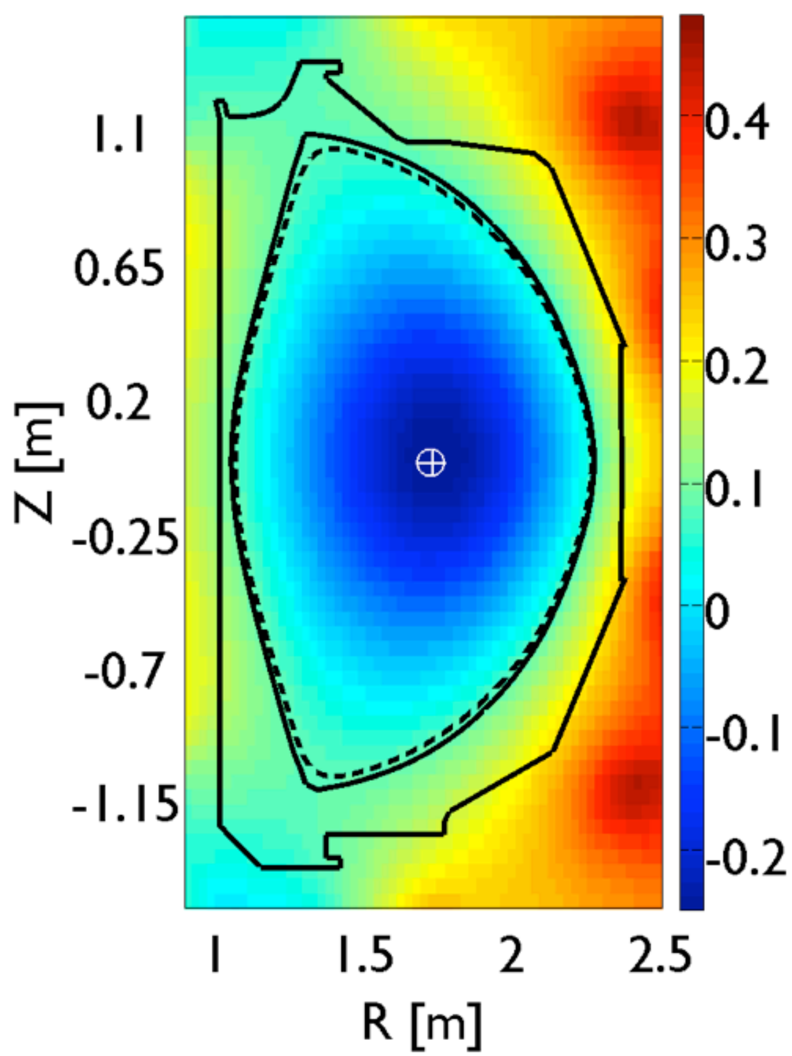

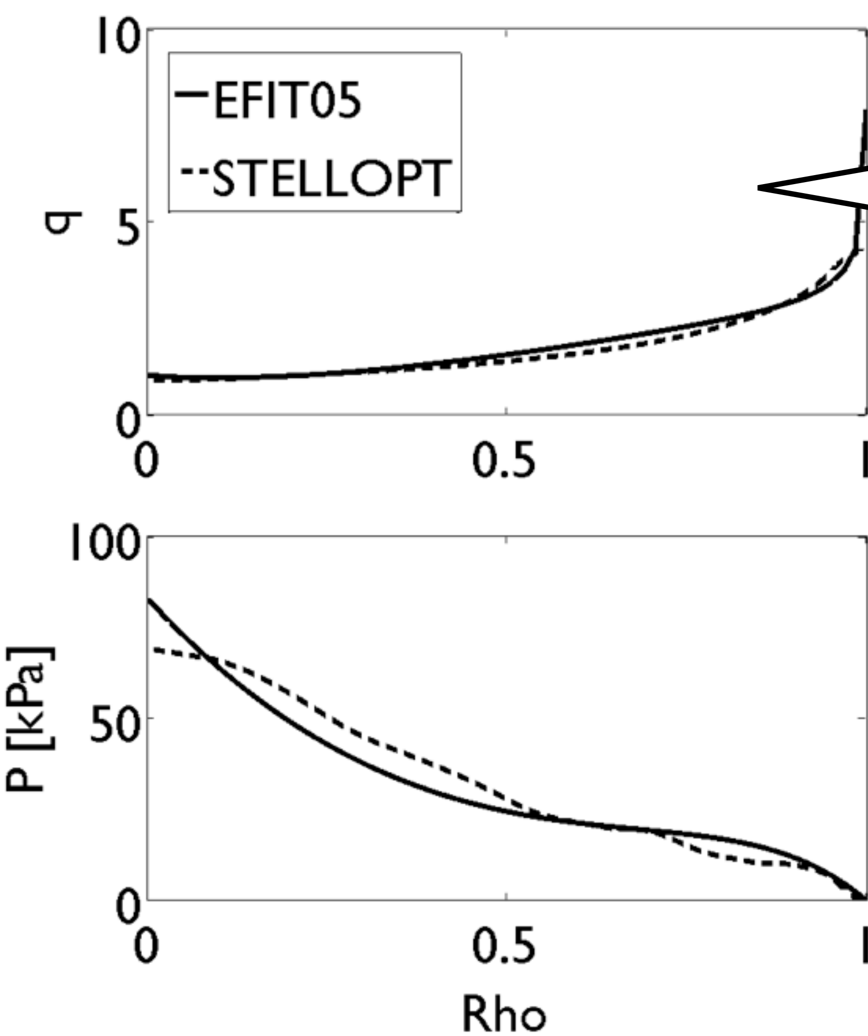

Edge discrepancy associated with VMEC inverse representation

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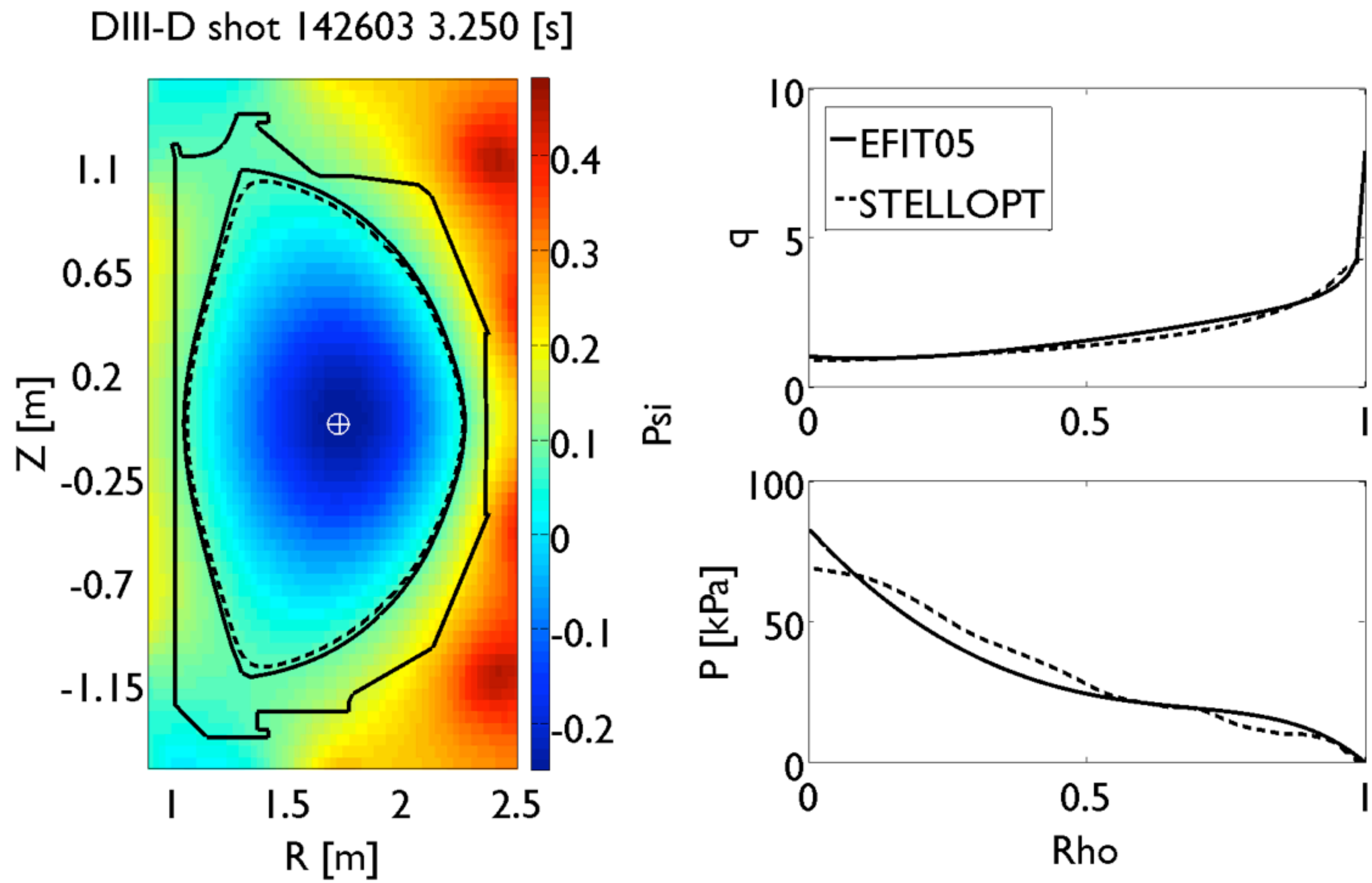

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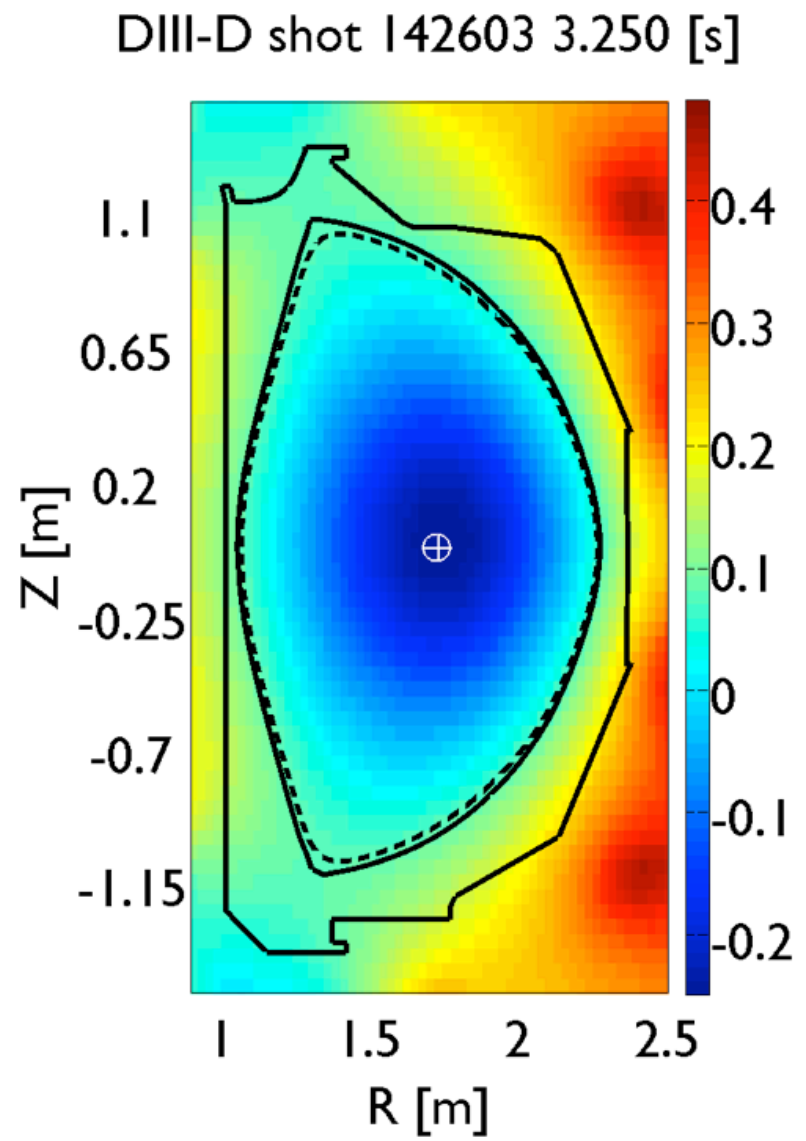
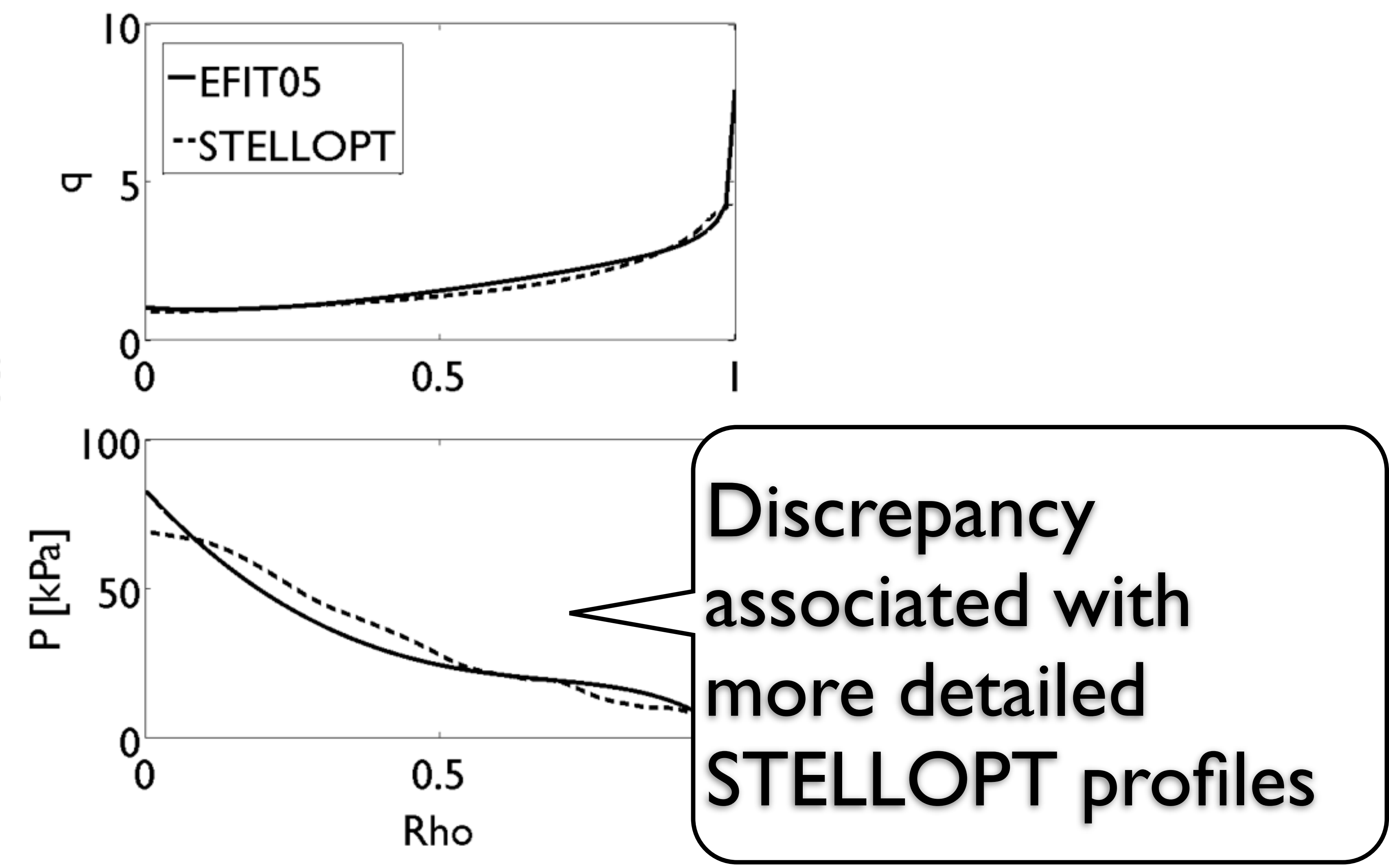
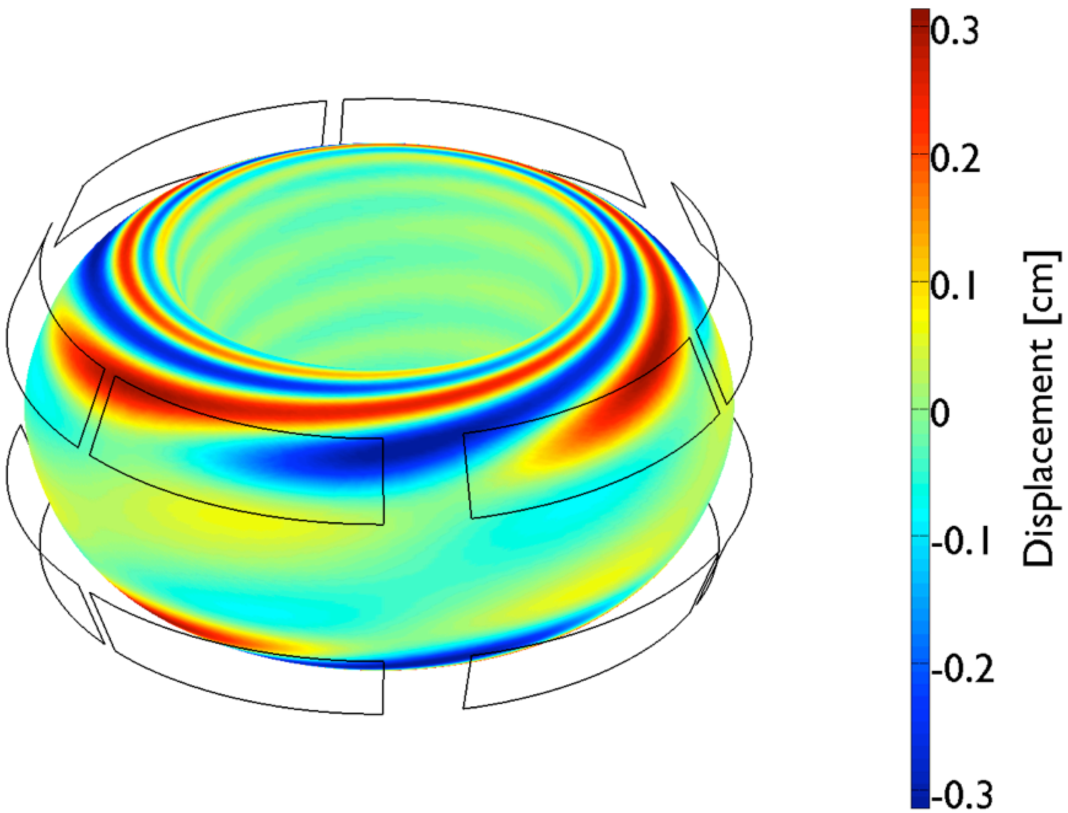




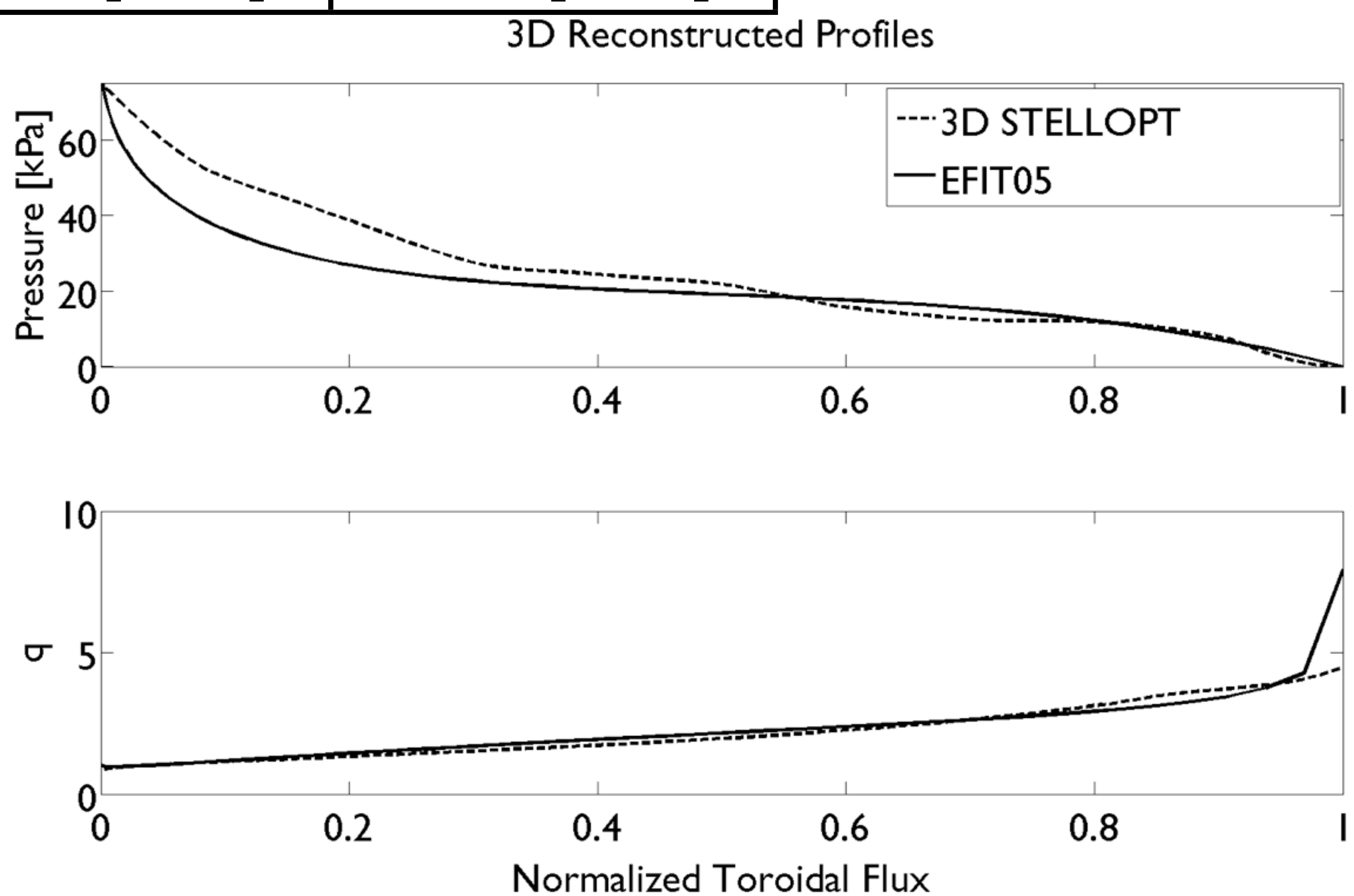
RMP effects are evaluated through 3D reconstruction

- +/- 3mm Boundary variation agrees with M3D-C1

	STELLOPT	EFIT05
Current	1.40 [MA]	1.39 [MA]
Beta	1.50%	1.61%
Volume	18.79 [m^-3]	19.39 [m^-3]

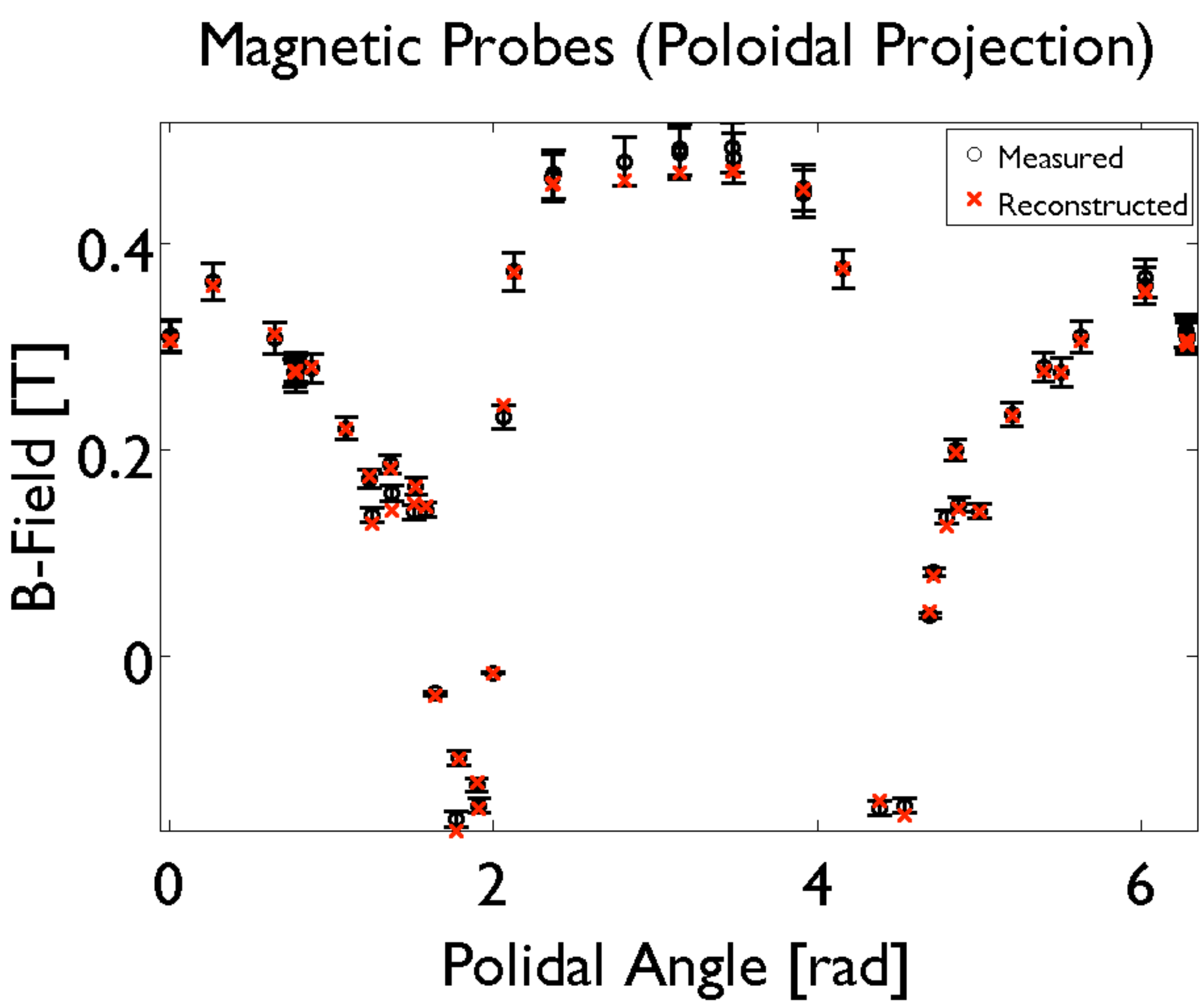


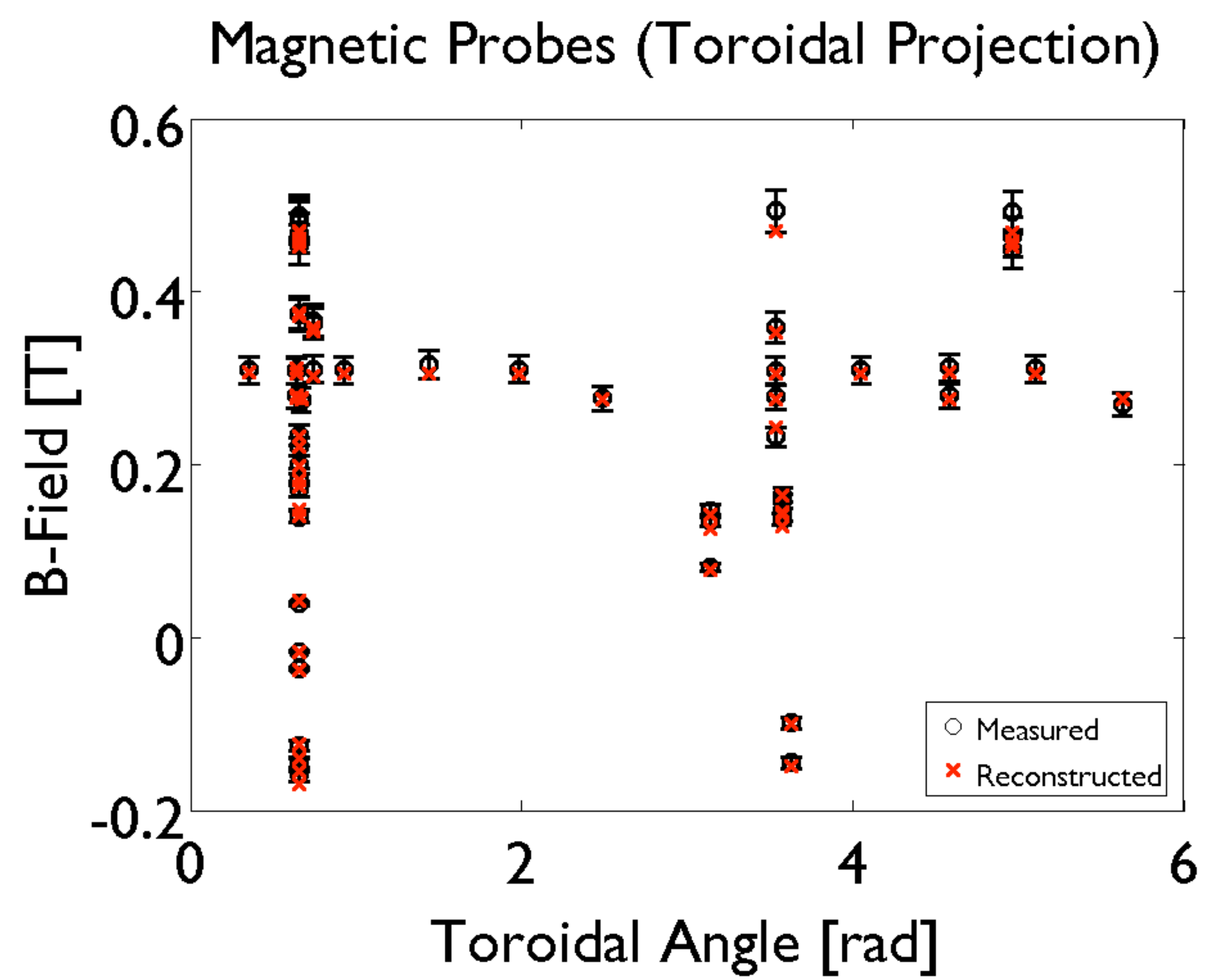
Reconstructed boundary displacements



Magnetic field probes indicate largest discrepancy near diverter region

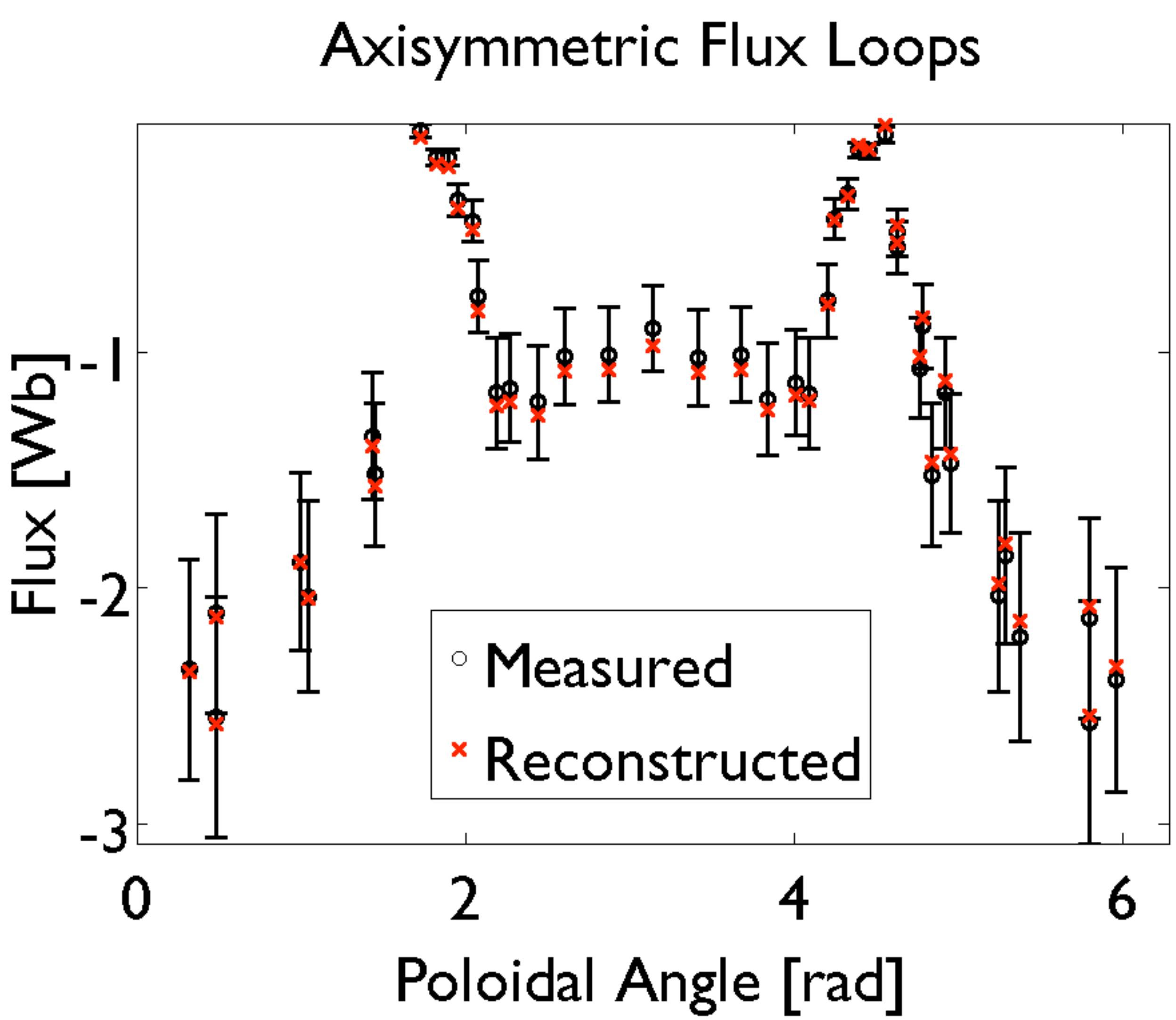
- Overall good agreement with B-Field Probes
- Diverter errors associated with representation

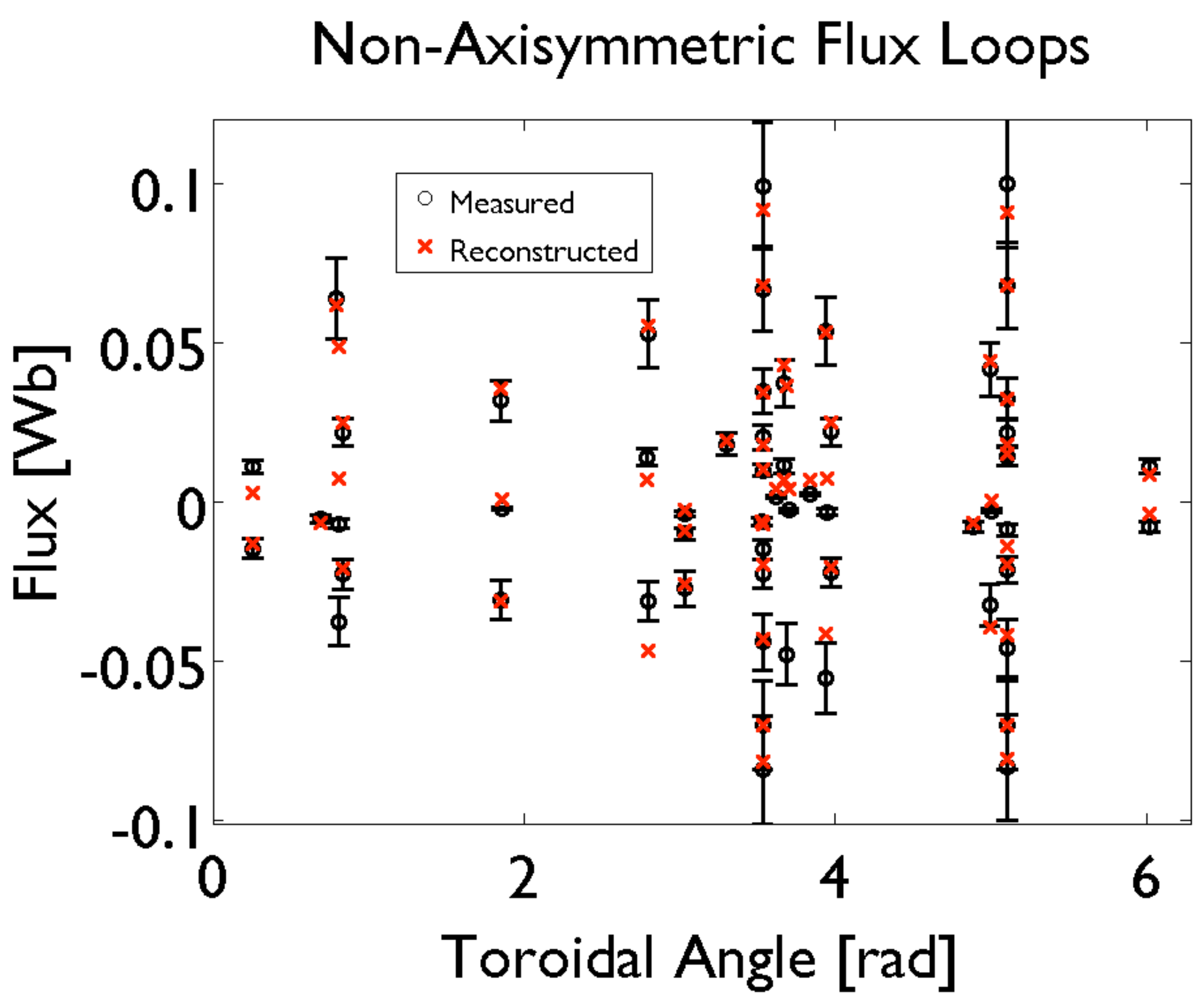




Flux loops show relatively good agreement

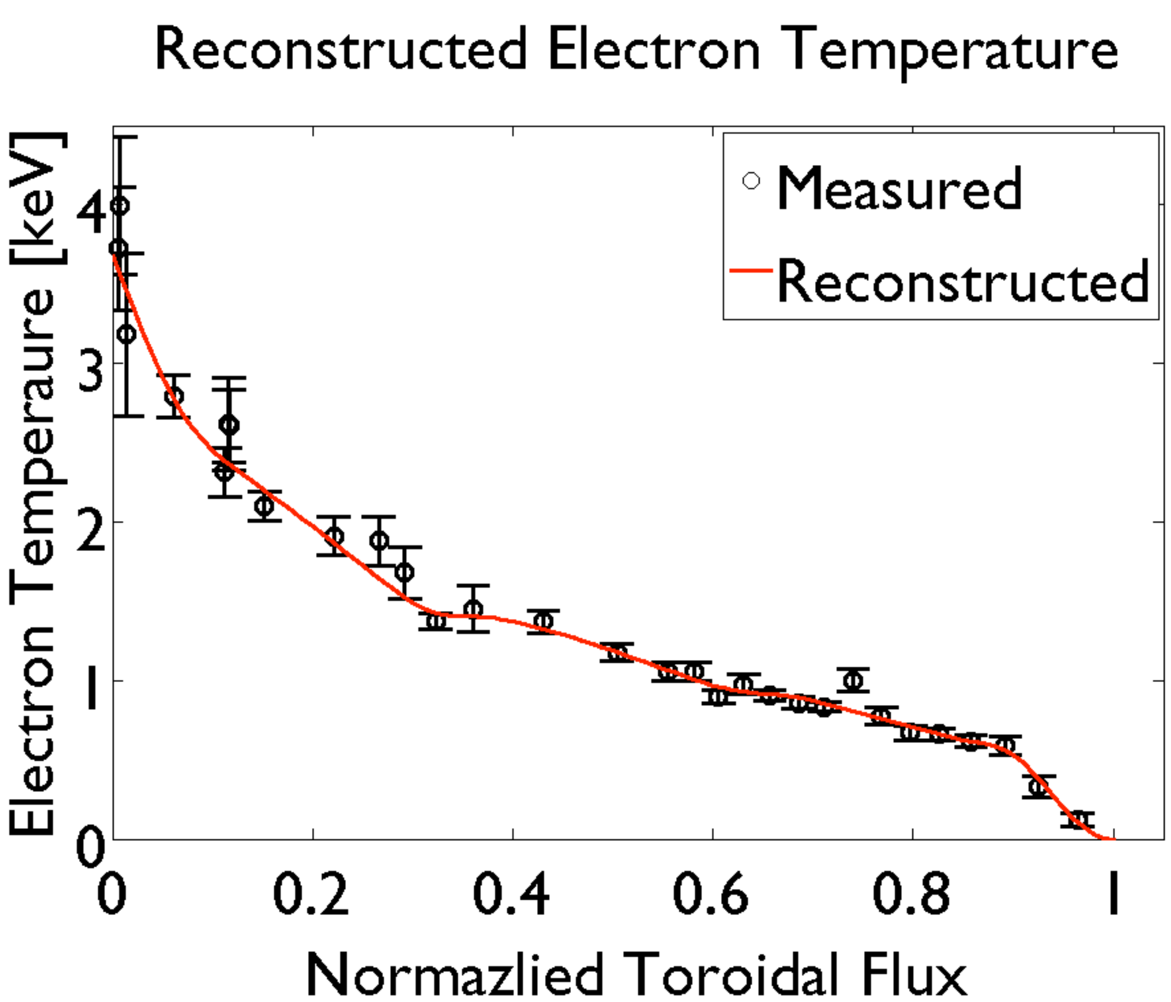
- Axisymmetric flux loops show similar agreement
- Non-axisymmetric loops lack C-Coil response

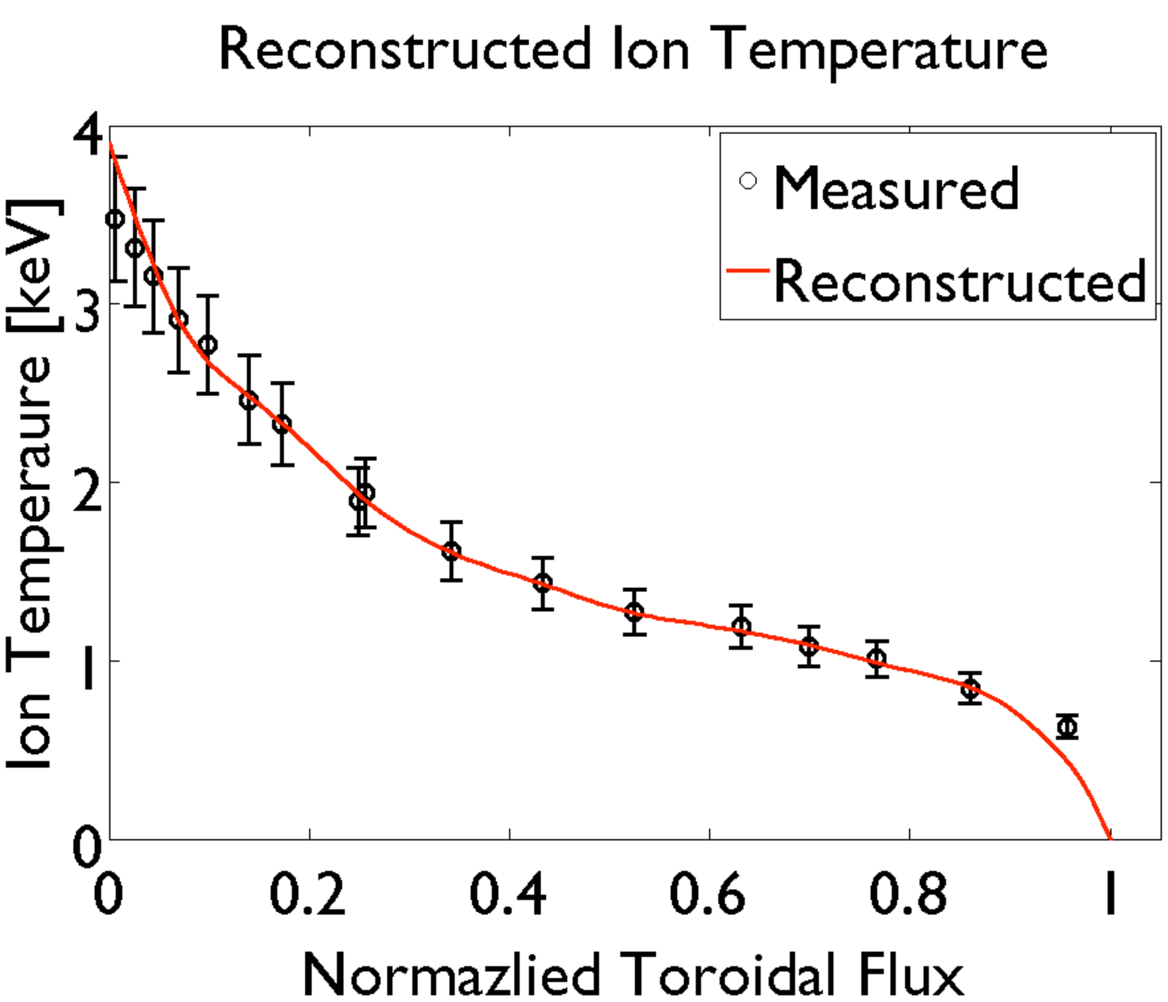




Profile fits attributed flattening to electrons

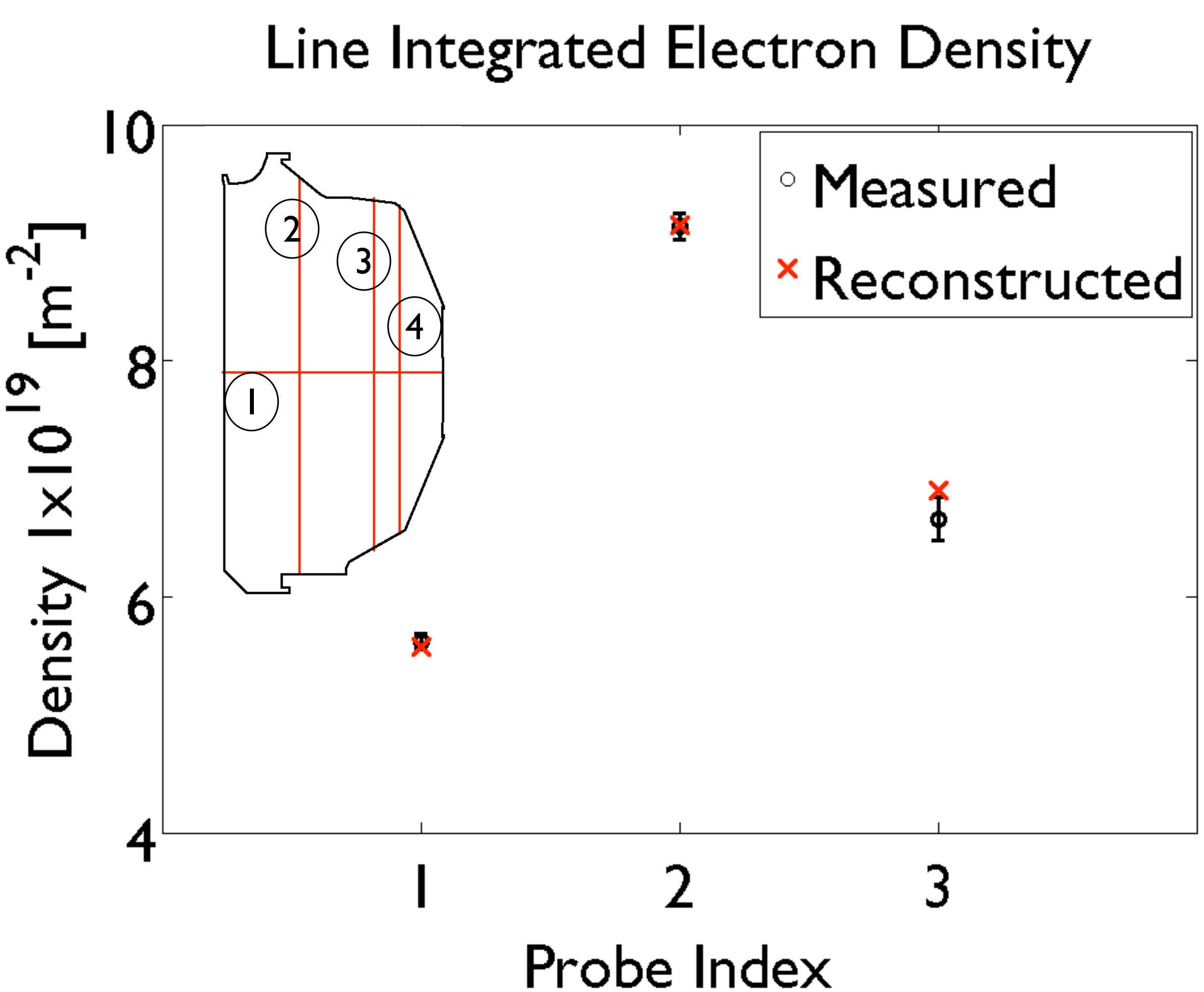
- Te temperature pedestal
- Ti temperature data lacking at edge

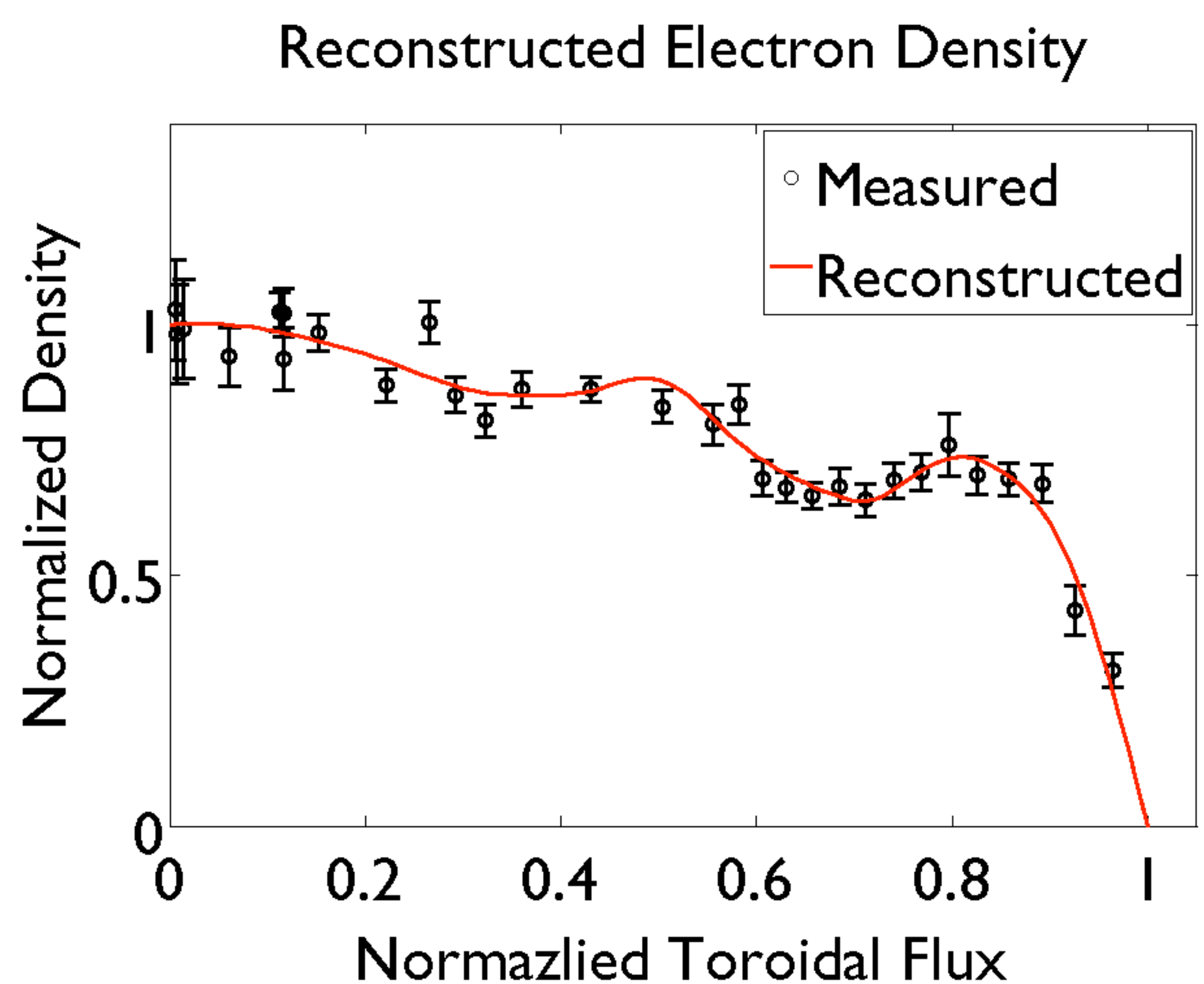




Electron density reconstruction indicates a flat profile

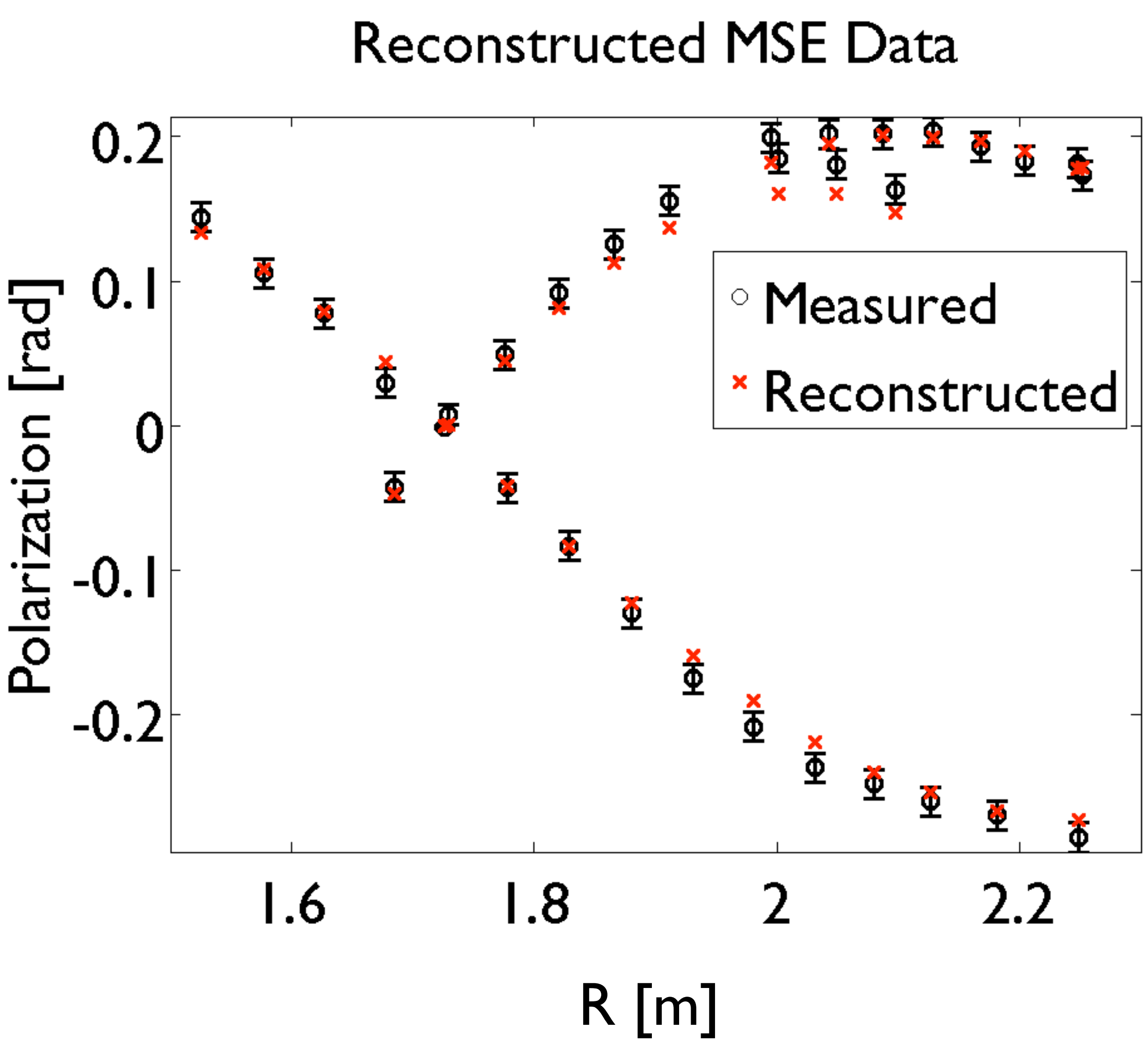
- Line integrate densities are well matched
- Ne relatively flat with some edge structure

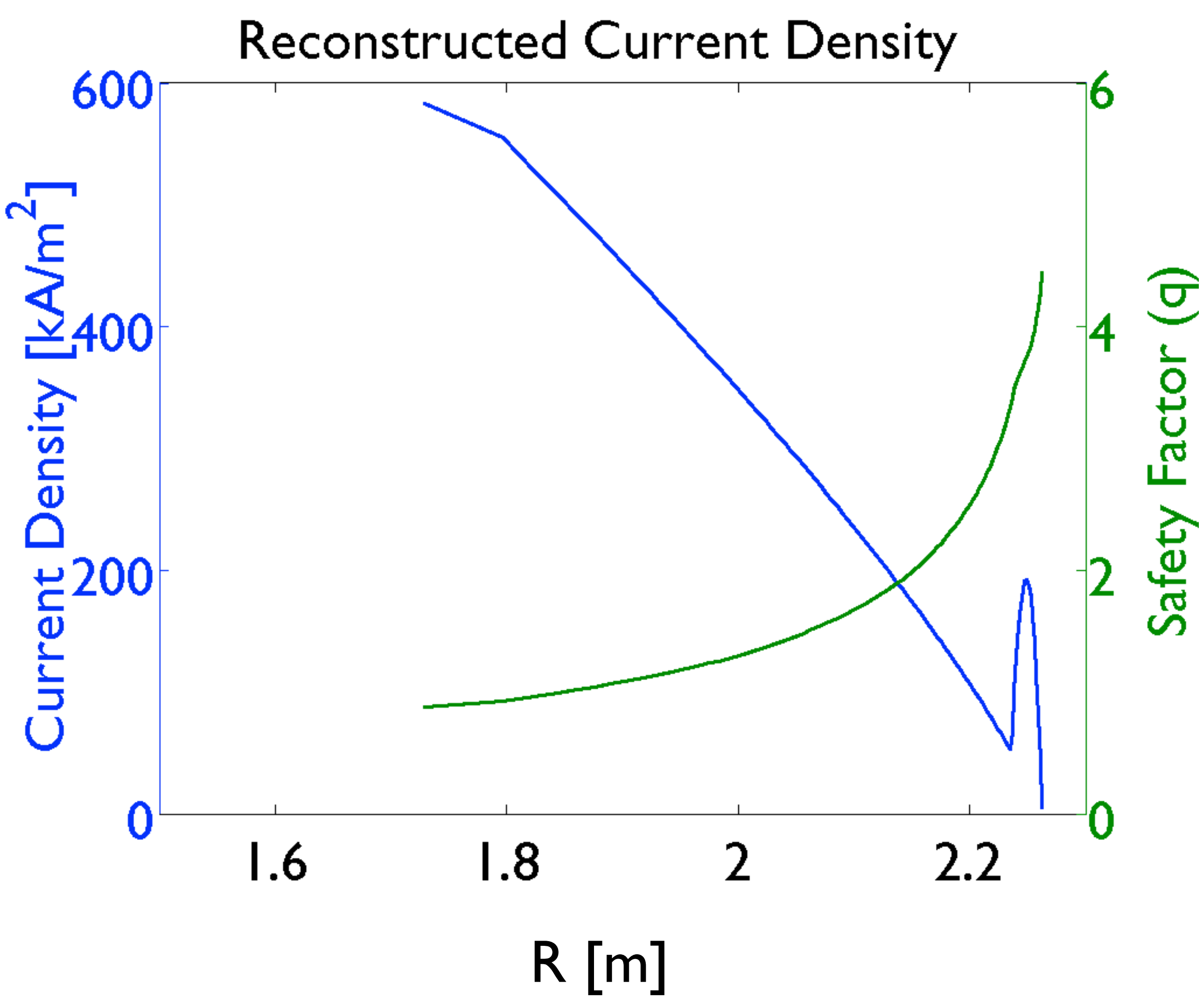




MSE constrains bulk current profile

- Only 3 data points constrain edge bootstrap





DIII-D 3D reconstructions provide useful analysis

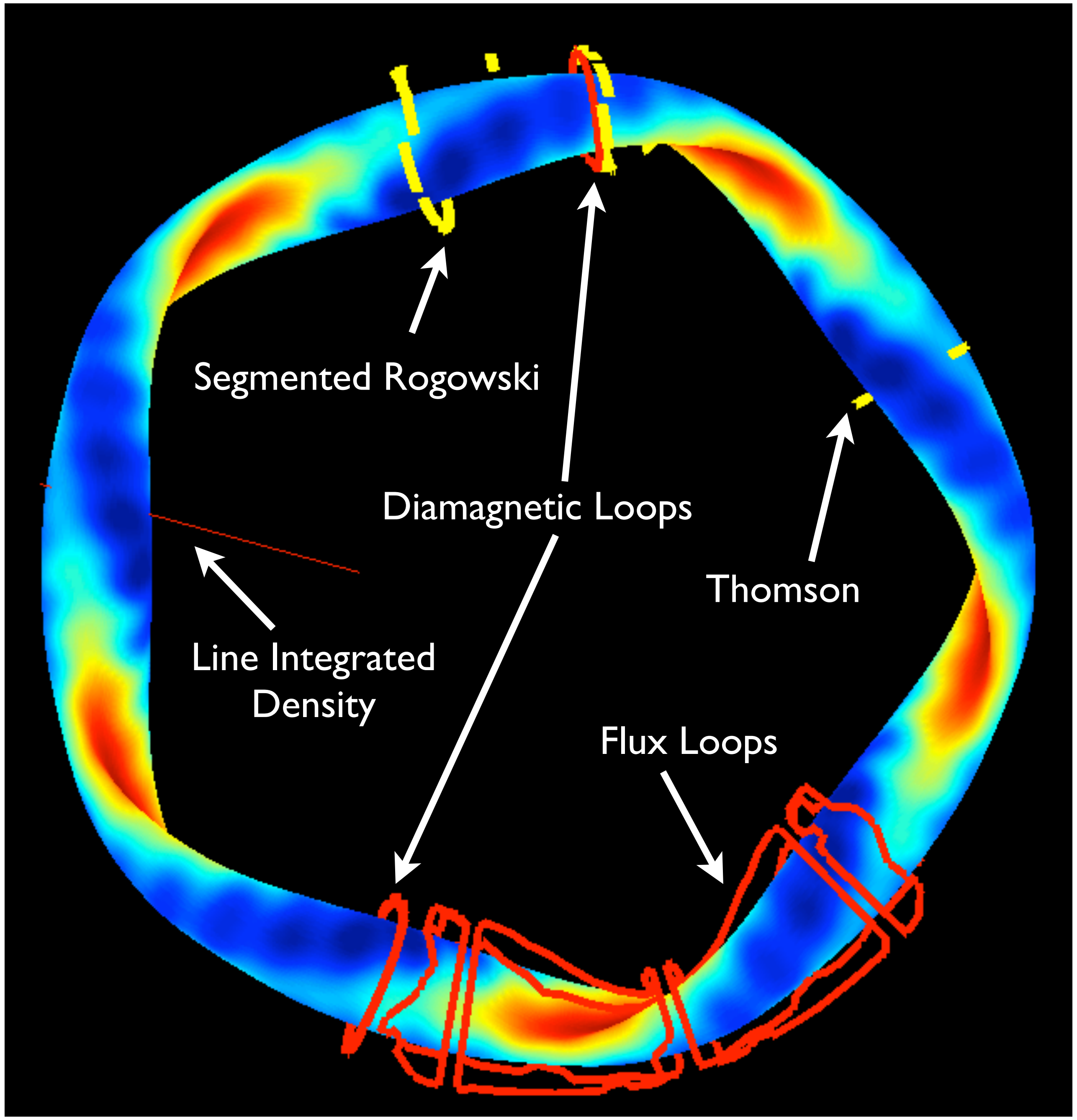
- Provides accurate 3D model for more advanced codes (SPEC, etc)
- Stellarator symmetry constraint in STELLOPT has now been relaxed
- Focus is now shifting back towards stellarators and W7-X

STELLOPT modeling of W7-X has begun

- The W7-X device utilizes an island diverter which requires current profile control
- Forward modeling is being utilized to asses role of magnetic diagnostics in diverter control
- Simulated data being utilized to test equilibrium reconstruction capability

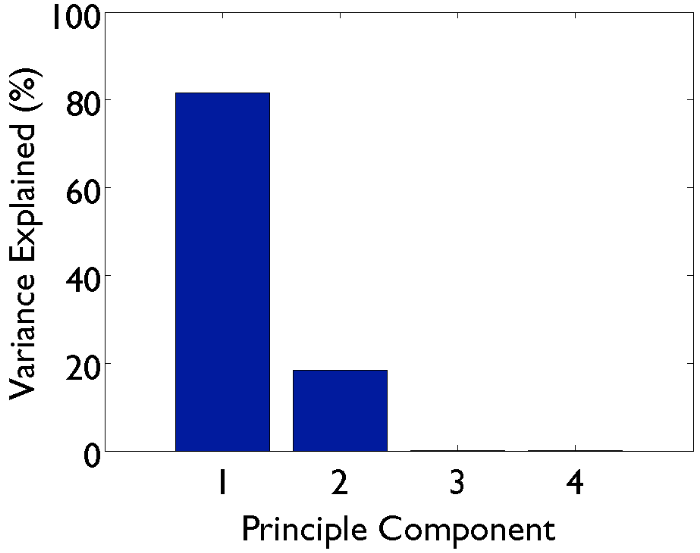
W7-X diagnostics constrain equilibrium

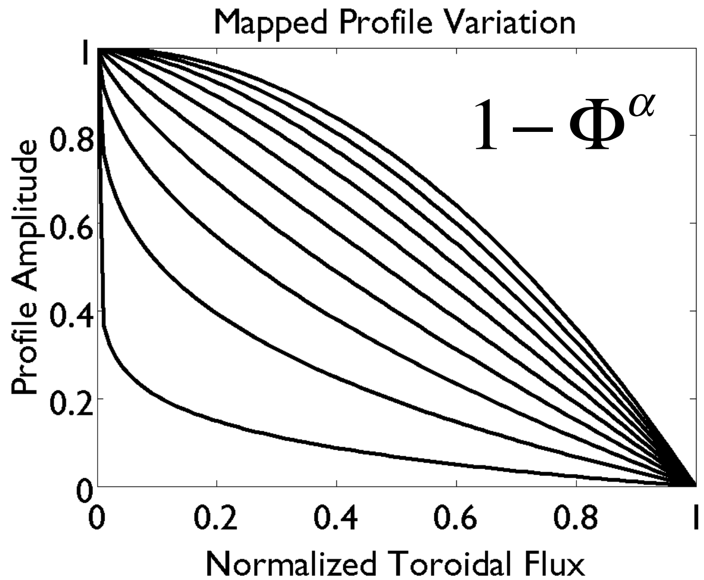
- Flux Loops (8)
- Diamagnetic Loops (2)
- Seg.Rogowski Coils (38)
- Interferrometry (1)
- Thomson Scattering (~100)
- XICS (under development)

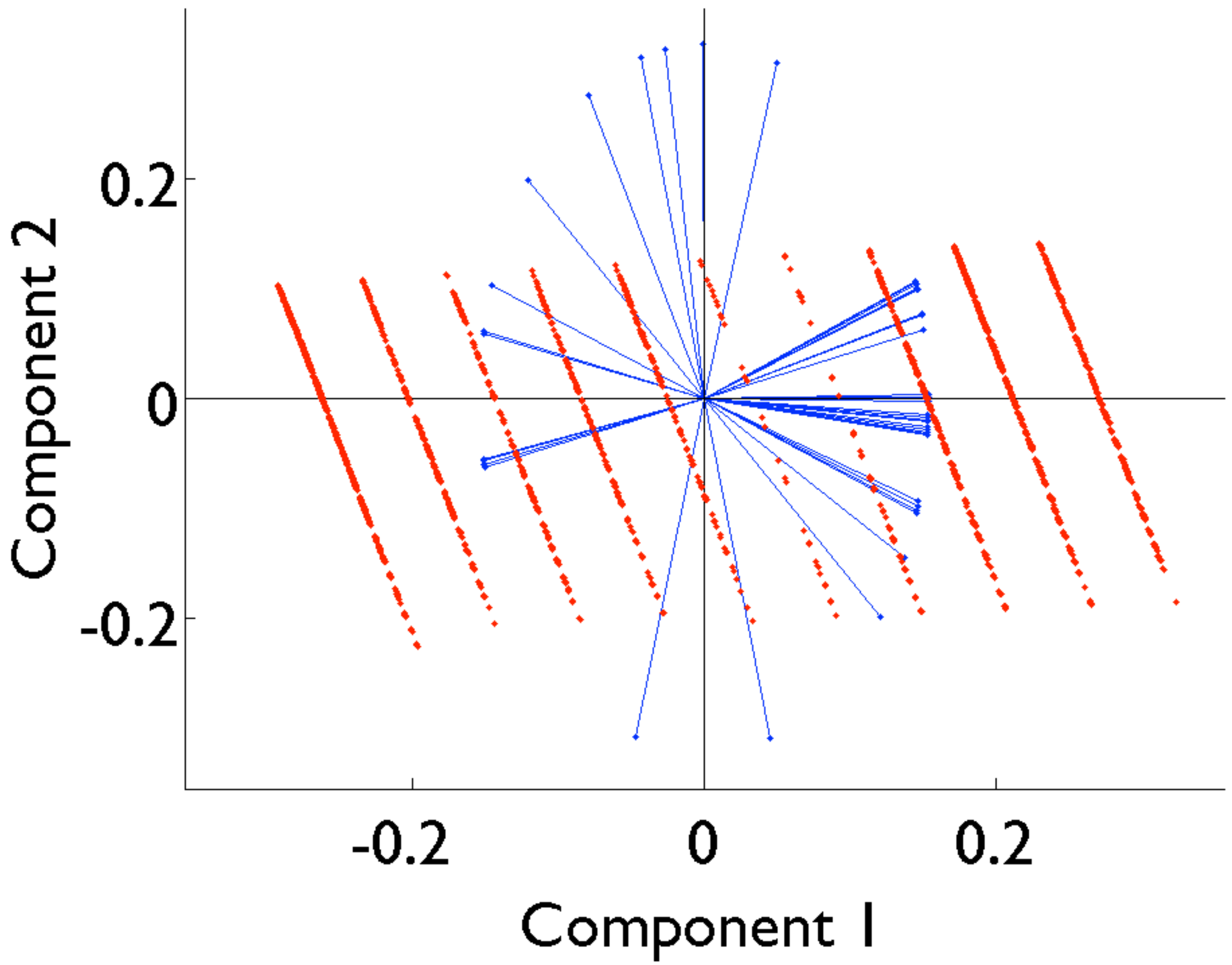


Current magnetic diagnostics on W7-X may not be adequate

- Mapping of toroidal current, pressure scaling, pressure profile, current profile
- Segmented Rogowski's now included







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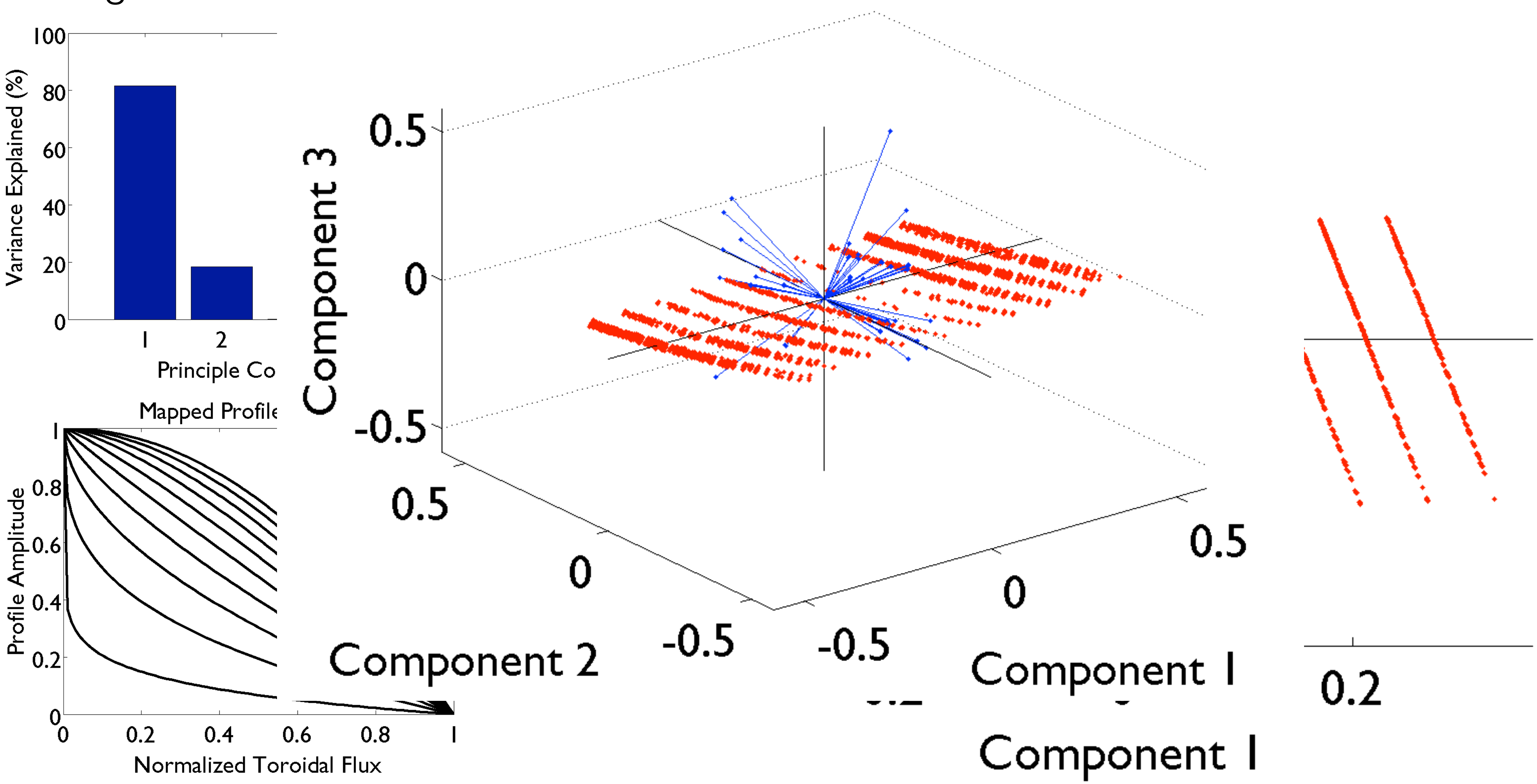
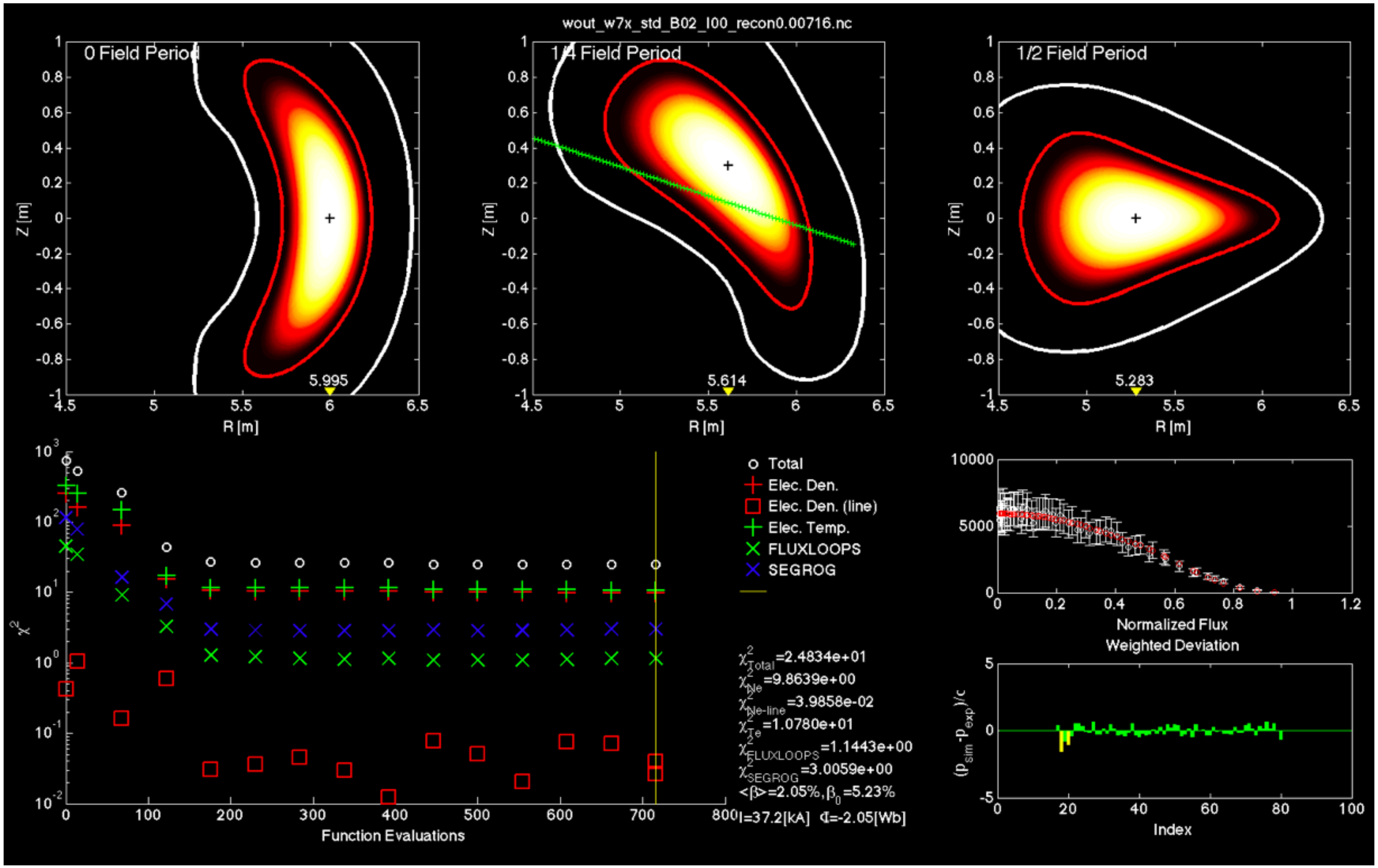


Inclusion of Thomson data allows for accurate reconstructions



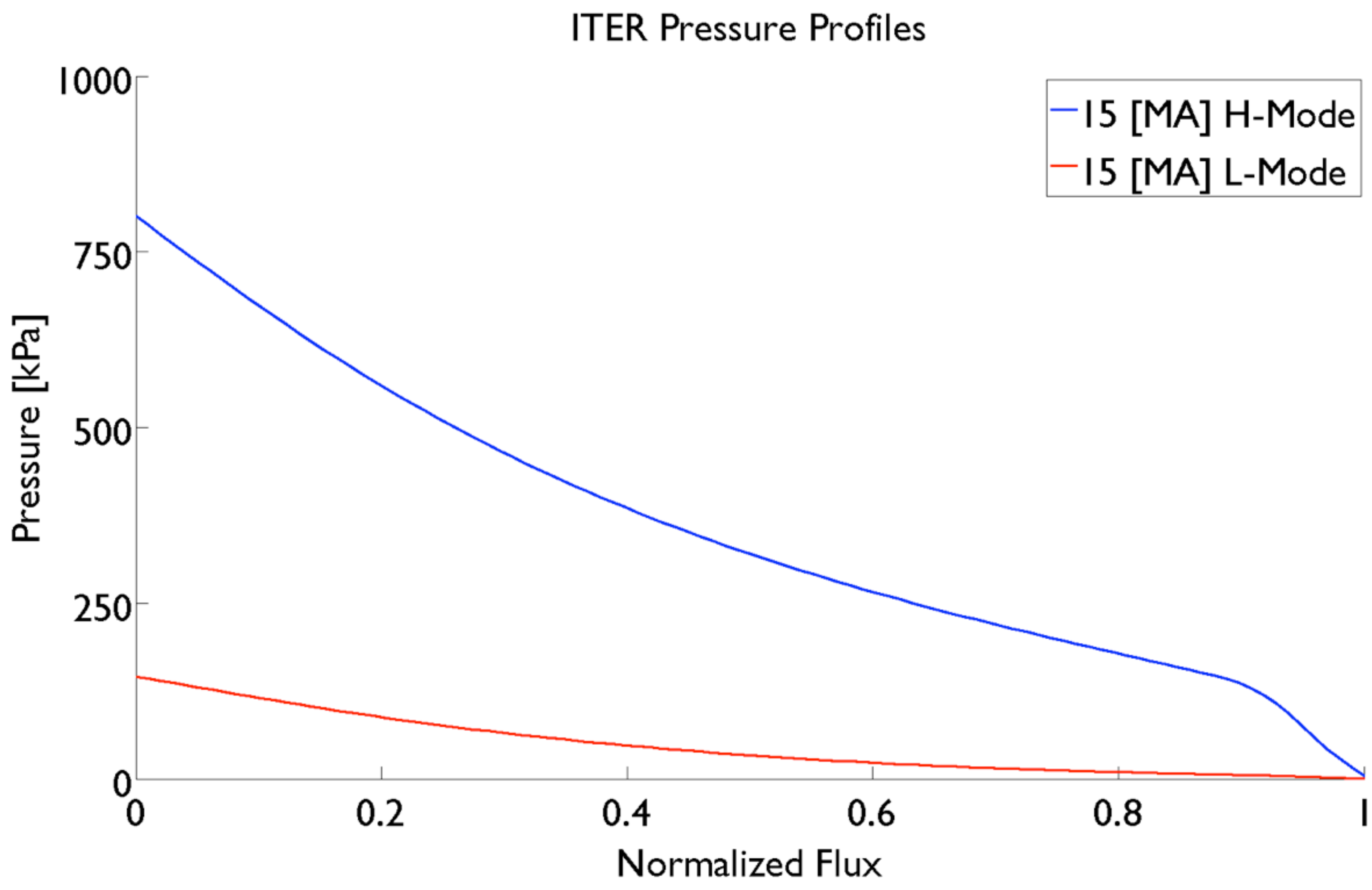
Orig.	Recon.
Beta 2%	Beta 2.05%
I [kA] 38	I [kA] 37.2
Flux [Wb] -1.98	Flux [Wb] -2.05

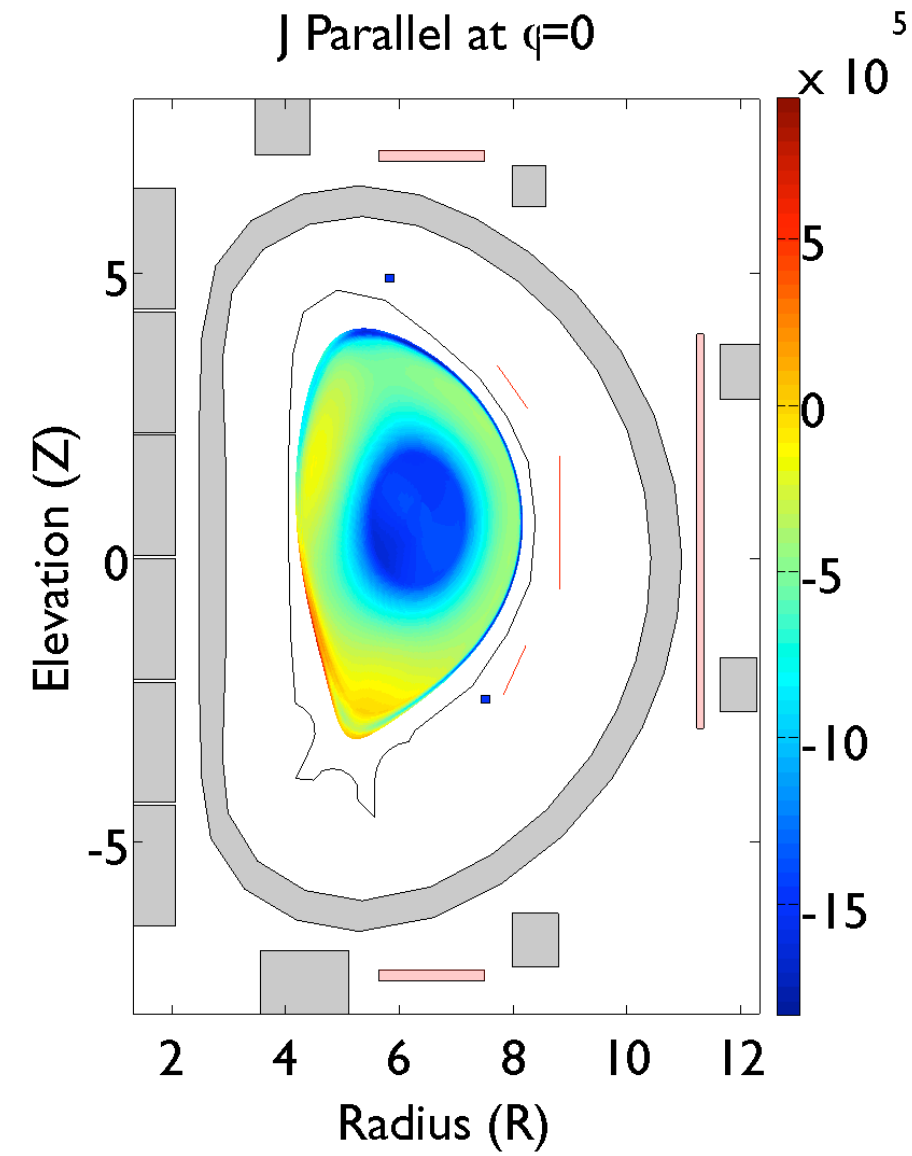
Work is ongoing for W7-X

- Full equilibrium reconstruction will take advantage of all measurements
- Magnetics alone do constrain stored energy and toroidal current
- New magnetics are being explored for profile sensitivity
- SPEC modeling of W7-X is underway
- Forward modeling of ITER now possible

Forward modeling of ITER is now underway

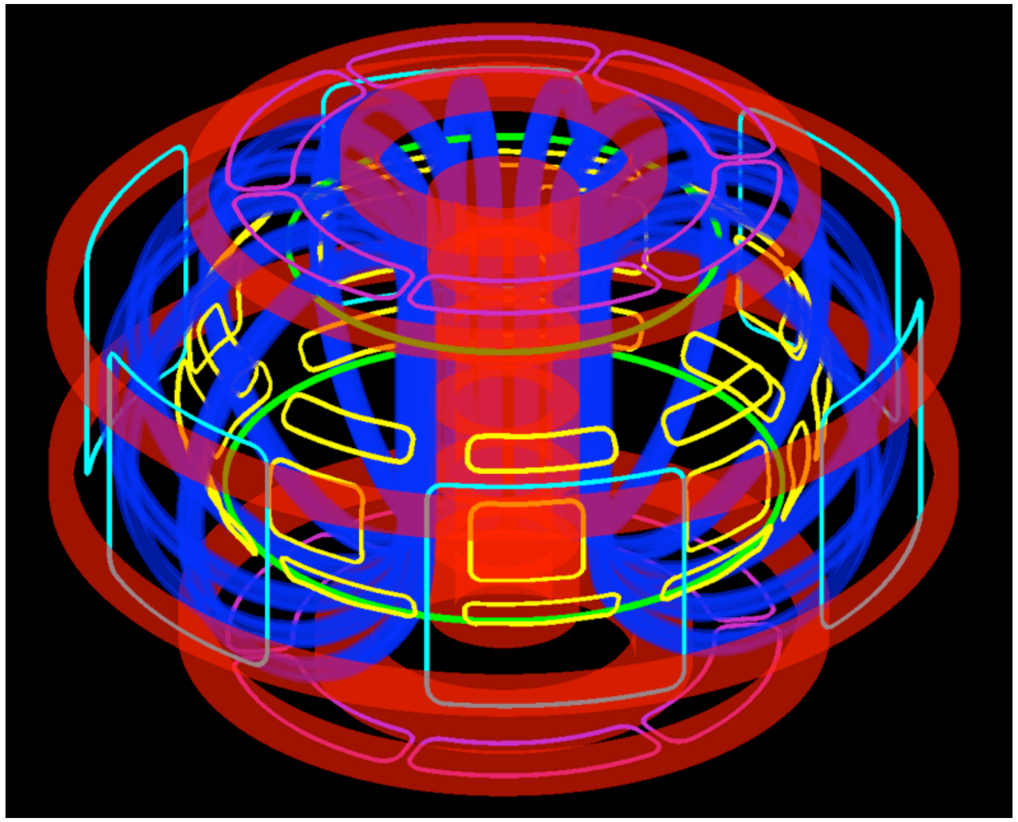
- CORSICA modeling provides profile data
- VMEC is utilized to calculate 2D and 3D free boundary equilibria
- DIAGNO2 then used to forward model the magnetic diagnostic response

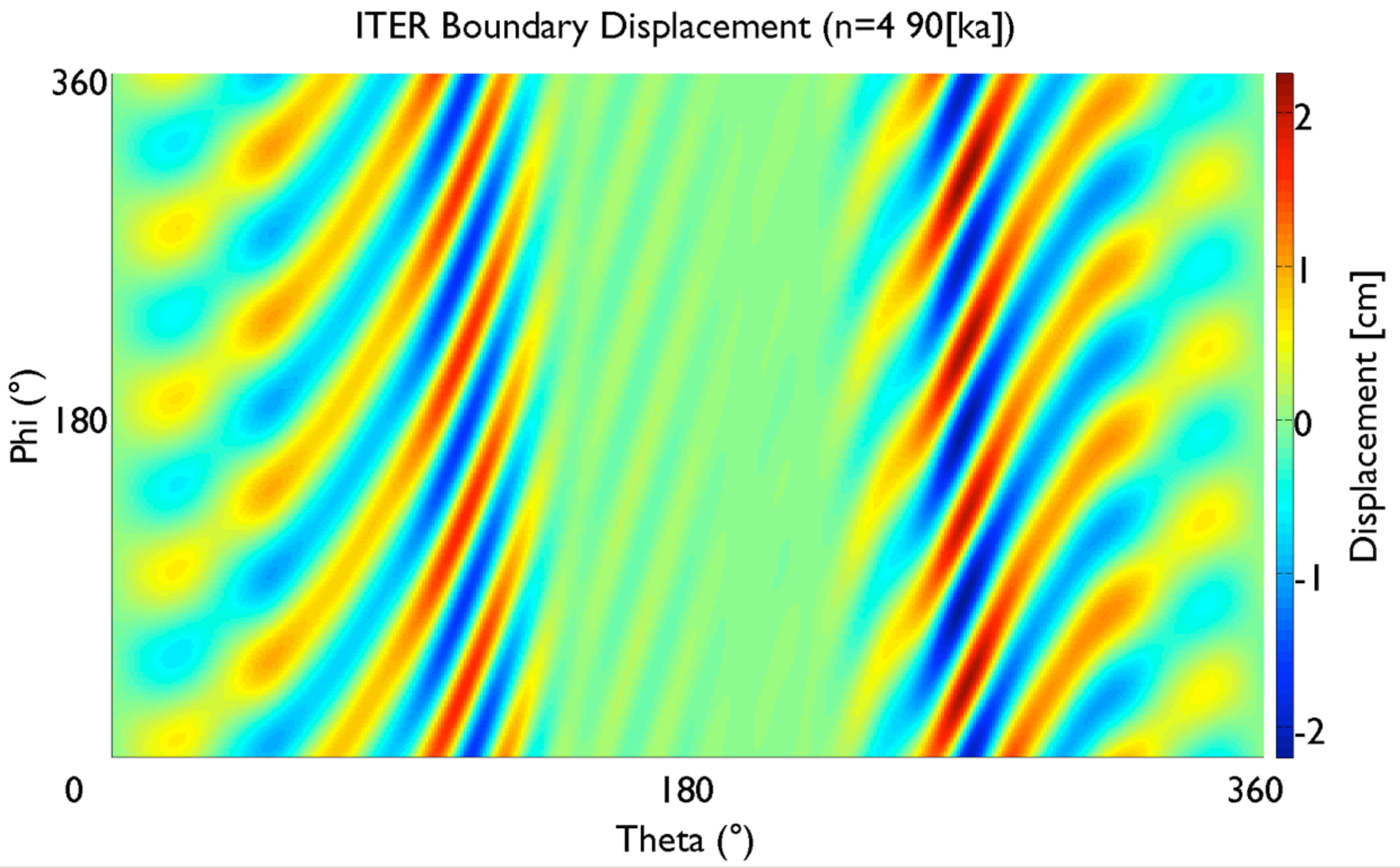




Boundary Displacements with ELM Coils (H-Mode)

In-Vessel Coil Scenario	Maximum Displacement
n=3 45 [kA-t]	0.84 [cm]
n=3 90 [kA-t]	1.36 [cm]
n=4 30 [kA-t]	0.36 [cm]
n=4 90 [kA-t]	1.66 [cm]

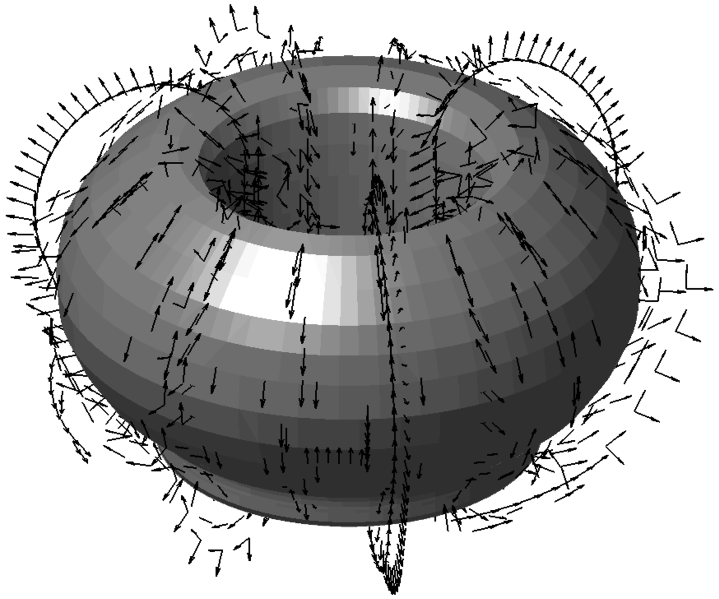


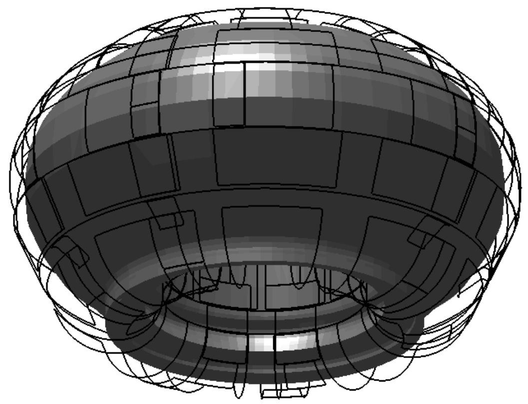


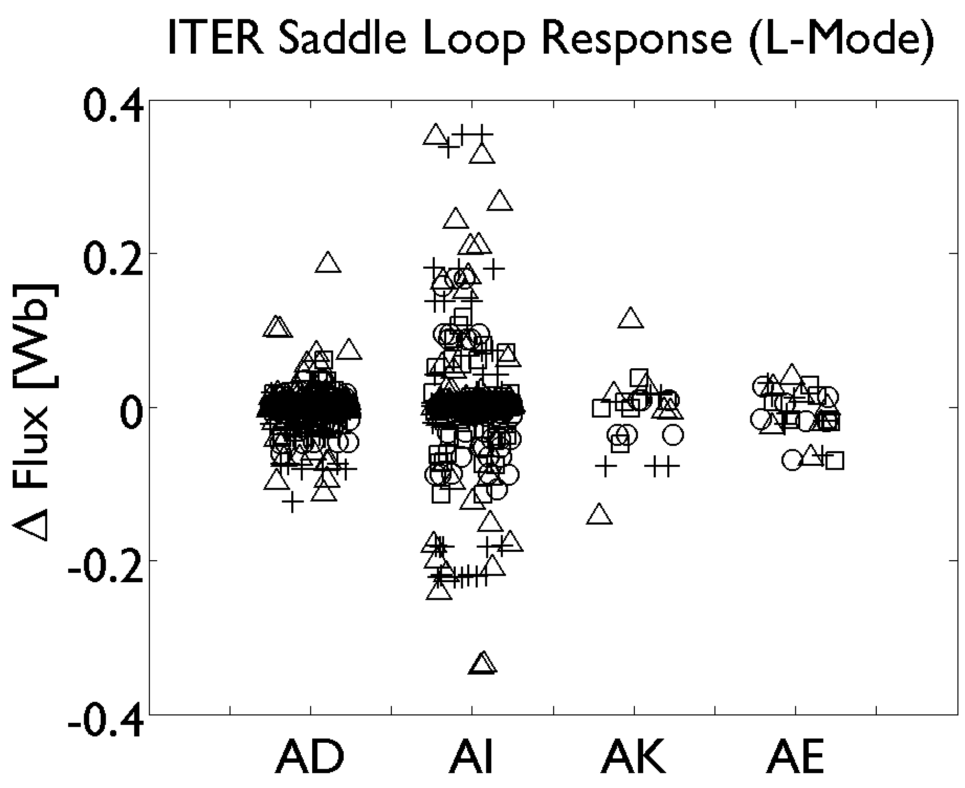
VMEC Resolution s=99,m=24,n=4

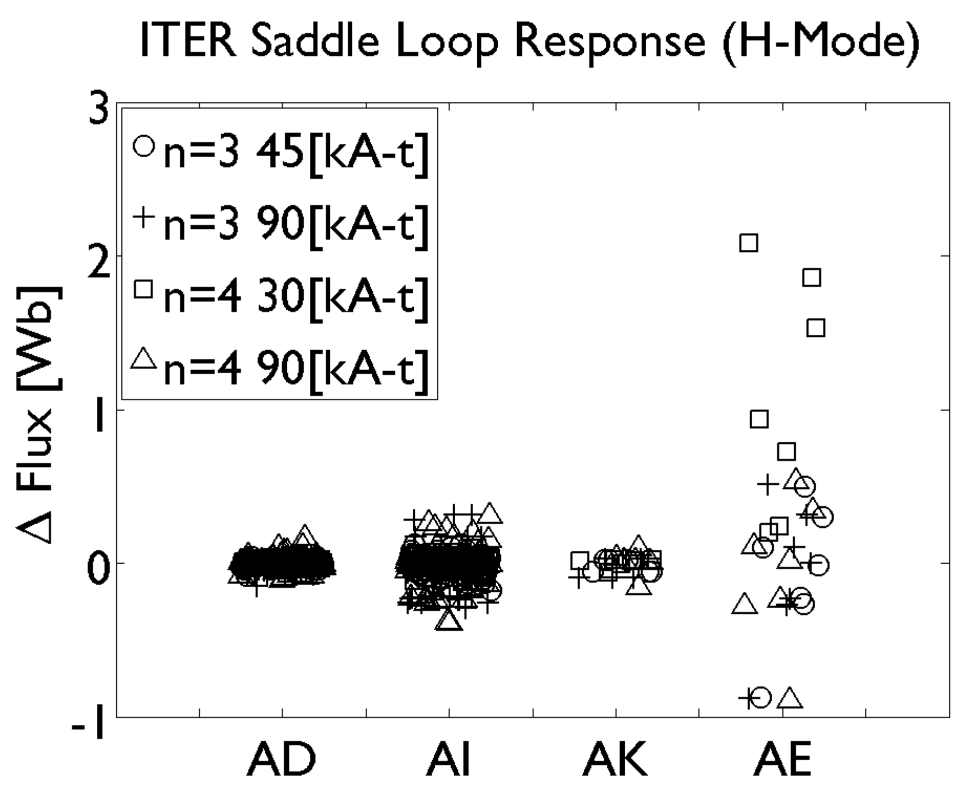
Work is underway to address the diagnostic response

- Large number of probes
- Clear response in axisymmetric flux loops when RMP's are applied to H-Mode plasmas
- Additional modeling necessary









ITER Work is underway

- Analysis of diagnostic response for trial equilibrium has been conducted
- More sophisticated transport analysis is underway
- As-designed magnetics are being assessed for 3D fields
- SPEC modeling of ITER is underway

Future work

- NSTX EP H-Mode boundary discrepancy under investigation
- Additional diagnostics for W7-X under investigation
- IPECOPT under development for NSTX/U 3D coil design

Thank you for listening!