Cosmic Bubble Collisions

Outline

• Background
  – Expanding Universe: Einstein’s Eqn with FRW metric
  – Inflationary Cosmology: model with scalar field
  – QFT → Bubble nucleation → Bubble collisions

• Bubble Collisions in Single Field Theory
  – Results: Classical Tunneling via “Free Passage”

• Concerns:
  – dissipation via interactions with other fields
    • Include interaction with additional scalar field
    • Account for gravity
  – Revisit validity of FP itself for single field case

• Methods
  – Purely classical
    • Analytic
    • Numerical
  – Semiclassical
Expanding Universe

- **Special relativity**
  - correct definition of distance in flat spacetime is \( ds^2 = dt^2 - dx^2 - dy^2 - dz^2 \)
  - index notation: \( ds^2 = g_{\mu\nu} dx^\mu dx^\nu \) (repeated indices summed over)
  - here the metric, \( g_{\mu\nu} \), is the 4x4 matrix: \( diag(1, -1, -1, -1) \)

- **Universe is homogeneous, isotropic, and expanding** on large scales \( \rightarrow \) use FRW metric: \( g_{\mu\nu} = diag(1, -a^2(t), -a^2(t), -a^2(t)) \)

Increasing \( a(t) \) means expansion. How does it evolve?

- **GR:** Einstein’s equation relates curvature of spacetime to mass/energy content of spacetime

\[
R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R + g_{\mu\nu} \Lambda = 8\pi G T_{\mu\nu}
\]

- **For FRW:**

\[
\left( \frac{\dot{a}}{a} \right)^2 = \frac{8\pi G \rho}{3} + \frac{\Lambda}{3} - \frac{k}{a^2}
\]

\( H = \frac{\dot{a}}{a} \)

- Energy density
- spatial curvature=0
- Cosmological constant
- Stress energy tensor
- Combinations of components and deriv of components of metric
- Note: $\Lambda$ equivalent to a constant energy density (does not dilute as universe expands)
- Universe with only $\Lambda$ has $a \propto e^{Ht}$, other types?
  - ordinary (non-rel) matter: $\rho \propto a^{-3} \rightarrow a \propto t^{2/3}$
  - radiation: $\rho \propto a^{-4} \rightarrow a \propto t^{1/2}$
  - Only since recently has (today's) observed $\Lambda$ been dominant form of energy density (~70%) in universe. $\Lambda$ is nearly zero, but dominates since matter and radiation have become so diluted.

![Diagram showing expansion of the universe with scale factor $a$ and cosmic time $t$](image)

- Radiation domination $a \propto t^{1/2}$
- Matter domination $a \propto t^{2/3}$
- Today
Horizon Problem/Inflation

- If we assume only radiation domination followed by matter domination we find that CMB photons originated from region not causally connected
- CMB photons arriving from every direction fit same BB curve, so have same temp to one part in $10^5$

Solution: INFLATION

**Short** period of very rapid exponential expansion, i.e. large, positive before radiation domination. As inflation ends $\Lambda \to \Lambda_{\text{today}}=\text{small}$

- occurred at $\sim 10^{-32}$-$10^{-33}$ s after big bang, lasted $\sim 10^{-36}$ s
- $a(t)$ increased by $\sim 10^{27}$, (vol increased by factor of $10^{78}$)

**How should we model inflation?**

- A scalar field, $\Phi(x)$, has energy density: $\rho = \frac{1}{2} \dot{\phi}^2 + \frac{1}{2} |\nabla \phi|^2 + V(\phi)$
- Model inflation with homogeneous scalar field so $\nabla \phi$ is zero. We call this field the inflaton field. If $d\Phi/dt$ can be ignored rel to $V(\Phi)$, then inflaton field’s $\rho$ enters Friedmann eqn as $V(\Phi)$.

**Slow-roll inflation:**

1. $d\Phi/dt$ starts off small, $V$ has small negative slope here, phi rolls down flat part of potential slowly ($d\Phi/dt$ stays small) until here so $\rho\approx V(\Phi)\approx V_{\text{infl}}$.
2. As $\Phi$ rolls down steeper part of potential $d\Phi/dt$ becomes significant, $V(\Phi)$ is decreasing from $V_{\text{infl}}$ (inflation is ending). Interactions between $\Phi$ and other fields yield particle production in other fields as rolls into potential well and settles into $\Phi^*$ (there is damping due to this particle production and hubble damping term in $\Phi$ field equation
3. $V(\Phi^*)\approx 0$ corresponds to the small $\Lambda$ we observe today.
Quantum Effects

• The inflaton field is a quantum field
• Tunneling:
  – Regular Quantum Mechanics:

    \[ V(x) \]

    \[ E \]

    \[ x \]

  • For IC corresponding to particle coming in from left with energy \( E \), not all of the wavefunction will be reflected off the barrier. Nonzero transmission coeff (nonzero probability of finding particle to right of barrier).

  – Similarly a quantum field can develop regions that fluctuate into configurations outside of the basin of attraction field begins in

Consider a potential \( V(\Phi) \) that looks like:
Bubble Nucleation

- Scalar field Lagrangian: \( \mathcal{L}(\phi) = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi - V(\phi) \)
- In Minkowski space variation of action yields this eqn of motion:
  \[
  g^{\mu\nu} \partial_\mu \partial_\nu \phi = - \frac{\partial V(\phi)}{\partial \phi}
  \]

- Homogeneous classical field initially in \( \Phi_A \) (with \( d\Phi/dt = 0 \)) stays in \( \Phi_A \)
  - For FRW metric with \( da/dt \neq 0 \) the field eqn has a damping term nonzero, so \( \Phi_A \) is stable attractor
- Quantum field initially homogeneous and equal to \( \Phi_A \) develops small regions, or **bubbles**, in which field configuration fluctuates away from \( \Phi_A \)
- Some fluctuations will be in basin of attraction of \( \Phi_B \), such as \( \Phi_B^* \)

\( \Phi_A \) \hspace{1cm} \Phi_A \hspace{1cm} \Phi_A \hspace{1cm} \Phi_B \hspace{1cm} \Phi_B^* \hspace{1cm} \Phi_A \)

- \( \Phi_B^* \) is far enough inside basin of \( \Phi_B \), the field inside bubble quickly evolves into \( \Phi_B \) and this bubble “expands” (in sense that bubble walls move out, encompassing more and more space originally in \( \Phi_A \), now in \( \Phi_B \). This is because \( V(\Phi_B) < V(\Phi_A) \), so outward pressure gradient
- Note: fluctuations into field configurations \( \Phi' \) with \( V(\Phi') > V(\Phi_A) \) collapse so we don’t care about them
Bubble collisions

• Note: region separating inside of bubble (in $\Phi_B$) and surrounding “sea” (in $\Phi_A$) is finite. Bubble walls accelerate as move out, thin walls move near speed of light.

  – Field configuration in a thin wall can be solved for using “relaxation”. schematically (in 1 spatial dim) looks like:

  \[ \Phi(x) \]

  \[ \Phi_B \quad x \rightarrow \quad \Phi_A \]

• Presumably these bubbles are nucleated all over.. So bubble collisions inevitably occur. What then?
  
  – Numerical results for single scalar field theory found **classical tunneling** could occur. Consider a potential with 3 local minima:

  \[ V(\Phi) \]

  initial conditions corresponding to two expanding phiB bubbles in surrounding phiA are evolved numerically according to field eqn:

  \[ g^{\mu\nu} \partial_\mu \partial_\nu \phi = - \frac{\partial V(\phi)}{\partial \phi} \]
Bubble collisions

Explanation: Free passage approximation
Solution to nonhomogeneous wave equation is well approximated by solution to homogeneous wave eqn (mere superposition of wall profiles) up until and shortly after walls reach each other. Hence they “pass through” each other.

After collision interior (overlap region) is in:
\[ \Phi^* = 2\Phi_B - \Phi_A \]

If \( \Phi^* \) is sufficiently far in basin of attraction of \( \Phi_C \) then field inside collision region evolves into \( \Phi_C \) and expands.

And so we have a **classical mechanism for bubble nucleation.**
Concerns

- Most realistic avenues for energy dissipation are closed off to the model because it is a single field theory without gravity.
- Is free passage realistic? Or could it be an artifact of the simplicity of the model?
- Dissipation:
  1. Interactions
     - We know inflaton field interacts with other fields (fermion fields, gauge fields, etc)
     - Violent changes in a field coupled to other fields typically results in bursts of particle production (associated w/ add’l fields)
       - Ex: as inflation ends (violent change in inflaton field) energy is leaked into other fields in form of particle production

Bubble collisions are violent events. Expect particle production if we include interactions. Does opening this avenue for energy dissipation yield a different result?

2. Gravitational Effects
   - The inflaton field (and any additional fields built into the model) source the stress energy tensor, hence affect the metric (which appears in field eqns themselves).
   - By not accounting correctly for this backreaction we ignore gravitational wave production which is another mechanism of energy dissipation AND is one that hypothetically could be observed
Interacting Field theory

- Simplest thing to do: introduce an additional scalar field, a stand in for quarks, etc
- New Lagrangian: \[ \mathcal{L}(\phi, \psi) = \frac{1}{2} g^{\mu\nu} \partial_\mu \phi \partial_\nu \phi + \frac{1}{2} g^{\mu\nu} \partial_\mu \psi \partial_\nu \psi - V(\phi, \psi) \]

Eqns of motion in Minkowski space:

- Subtleties: not all interactions will provide a legitimate test of classical tunneling via free passage. Both fields would participate in FP since FP in multifield case involves superpositions of two component (scalar) vectors in field space.
  - Single field case: \( \Phi^* = \Phi_A + 2\Delta\Phi = 2\Phi_B - \Phi_A \) had to be in basin of attraction of \( \Phi_C \) to get classical tunneling
  - Analog in multifield: \( 2(\Phi, \Psi)_B - (\Phi, \Psi)_A \) has to be in basin of attraction of \( (\Phi, \Psi)_C \)
  \( \rightarrow \) local minima in \( \Phi-\Psi \) plane have to lie roughly on a line

- so looking at 2 interactions in particular: \( V_{\text{int}} \sim \Phi^2\Psi^2, \ V_{\text{int}} \sim \Phi^3\Psi \)
Summary of Methods

\[ V(\Phi, \Psi) = V_1(\Phi) + V_{\text{int}}(\Phi, \Psi) + m^2 \Psi^2/2: \]

- **Purely Classical Treatment:**
  - Analytic: for \( g\Phi^3\Psi \)
    Assume FP for \( \Phi \), solution to \( \Psi \) eqn given in terms of retarded Green’s function. Consistent with \( E \) conservation?
  - Numerical:
    Don’t assume \( \Phi_{\text{FP}} \), approx solution to the (2) coupled nonlinear wave equations for relevant ICs

  **New:** Revisit Free passage argument for single field case

- **Semi-classical Treatment:** for \( \Phi^2\Psi^2 \) interaction
  Treat \( \Phi \) field classically and as given by FP, \( \Psi \) field as quantized
  - Background Field Method: expand action in path integral about \( \Psi = \Psi_{\text{classical}} \) (which is \( \Phi \) dep, so nonhomogeneous since \( \Phi_{\text{FP}} \) is nonhomogeneous). Consistent does \( \Phi_{\text{FP}} \) correspond to tremendous \( \Psi \) particle production that would violate \( E \) conservation?
  - Bogoliubov Transformations, assume \( \Phi_{\text{FP}} \), compute map between early time, and late time \( (\Psi) \) creation/annihilation operators \( \rightarrow \) compute late time particle content of early time vacuum. Difficulty here is that \( \Phi_{\text{FP}} \) is nonhomogeneous. Consistent?
Gravitational Effects

- Field(s) backreact via affect on metric
  \[ S = S_{\text{Einstein-Hilbert}} + \int \sqrt{-g} \, d^4 x \, \mathcal{L}(\phi, \psi) \]
- Vary action:
- Eqns of motion:
  \[
  g^\mu{}_{\nu} \partial_\mu \partial_\nu \phi + \frac{\partial^\mu \phi \partial_\mu \sqrt{-g}}{\sqrt{-g}} = -\frac{\partial V(\phi, \psi)}{\partial \phi} \\
  g^\mu{}_{\nu} \partial_\mu \partial_\nu \psi + \frac{\partial^\mu \psi \partial_\mu \sqrt{-g}}{\sqrt{-g}} = -\frac{\partial V(\phi, \psi)}{\partial \psi} \\
  R_{\mu\nu} - \frac{1}{2} g_{\mu\nu} R = 8\pi G T_{\mu\nu}(\phi, \psi)
  \]
- Highly nonlinear… \(\rightarrow\) solve numerically
- Introduce new variables, spatial metric and extrinsic curvature, so that we can formulate Einstein’s eqn as a Cauchy problem (ADM eqns)
- Does this result in significant gravitational wave production? Observable?