PLASMA IN CONTACT WITH METAL: RF ANTENNA NEAR FIELD BEHAVIORS IN TOKAMAKS (AND INDUSTRIAL PLASMAS)

Presenter: David N. Smithe
Co-author: Tom Jenkins
Tech-X Corporation

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If You Don’t Already Know Me

• ’87 – Ph.D. (Univ. Mich.) PostDoc (PPPL)
  – ICRF Core Absorption, 1D Parallel “All Orders” (METS)
  – Early FEM 2.5D Core Solvers (SHOOT, SPRUCE)

• ‘89 Hired by Mission Research Corp.
  – Defense contractor, FDTD PIC codes and radar (MAGIC)
  – ’95, PPPL (Phillips) 1D perp “All Orders” for HHFW (METS95)

• ’05 – MRC bought by ATK, I went to Tech X
  – Back to Fusion (and some accelerator work) (Vorpal)
  – Trying to see what 3D FDTD can do in Fusion … Antenna

• Unusual Talent
  – Good with 3D geometry
The Original Problem

- We want to model this (~1 meter) antenna:

- Which has this (~10 microns) sheath
Because …

- RF antennas create impurities
  - which reduce the $Z_{\text{effective}}$ of the plasma
  - Which reduces the fusion yield
  - Which is bad, especially in ITER

- Impurities are thought to arise from sputtering, when ions fall through the rectified RF sheath, hitting wall with
  - ~ 100 volts, a bit higher than $T_{e,\text{edge}}$.

- Work by D’Ippolito and Myra at Lodestar
  - Local electric fields $\sim 10^4$ to $10^5$ V/m
  - RF Rectified Sheaths get you $\sim$ 100 Volts
What is an RF Rectified Sheath?

- Additional plasma DC bias in the sheath, due to RF, so plasma potential never goes wrong way.
  - (Show Movie 1)
  - 1D Parallel Plate
  - Start with Debye sheath (blue)
  - Oscillating potential (black)

- Still very small (10 microns – 100 microns)
  - Antenna is a meter, grid size is 1 mm
How to Defeat the Scale Problem

- **Analogy**: in EM-FDTD, we don’t resolve the metallic skin depth, $\delta=(\mu\sigma\omega)^{-1/2}$, we just set $E_\parallel=0$, as a boundary condition.

- If we need ohmic losses in the skin depth, we use the local $H$ field to compute current and power, as in $R_s|H|^2$.
  - E.g., the metallic skin depth is a “sub-grid model”
  - We model its effect, $E_\parallel=0$, not its full physics

- We “want” to do the same with the sheath.
  - Can we!? Not as simple as $E_\parallel=0$.
  - But lots of sheath work to draw from …
D’Ippolito and Myra Boundary Condition

Model sheath as capacitive gap.

Simple parameter, $\Delta$, characterizes capacitor.

Follow the action:
- Capacitor has oscillating $\phi_{\text{sheath}}$ (RF sheath Voltage)
- Nonlinear Capacitor spacing, $\Delta(|\phi_{\text{sheath}}|)$, Child-Langmuir
- Rectified potential, $\phi_{\text{DC,\,rectified}} \sim |\phi_{\text{sheath}}|$ is computed after the fact, usable for ion sputtering analysis

\[
C_s = \frac{\varepsilon_0 A}{\lambda_{De} \left[ 1 + \left( \frac{0.6q_i \Delta V_x}{T_e} \right)^2 \right]^{3/8}}
\]
Any EE’s in the Audience?

- Looks like this.
Do We Believe This?

- We’ve done 1D PIC (particle-in-cell) benchmarking. From first principles.
- For industrial plasmas (and maybe fusion), we add a thin layer of dielectric to the metal wall.
- The Dielectric layer DOES behave like a capacitor, with well known $C/Area=(d\varepsilon)^{-1}$.
- (Show Movie 2)
- Dielectric (blue)
Yes, We Believe

- Voltage is split, from capacitances in series, we know one, we suspect we know the other. Sheath capacitance based on $\Delta(|\phi_{\text{sheath}}|)$ gives correct split.
Also Need a Plasma

• Sheath potentials will depend on local electric field.

• That is only correct if the plasma in front of the antenna is correct
  \[ \varepsilon_{\text{plasma}} \sim 1000 \ldots \pm! \]

• Time domain cold plasma:
  
  “Finite-difference time-domain simulation of fusion plasmas at radiofrequency time scales”, Smithe, Physics of Plasmas 14, 1 2007

• (This is actually where the story begins.)
2007 PoP Paper

- **Critical Realization:** Cold Plasma Dielectric is zero-D – can make it point-wise implicit in time domain, to get all cold-plasma waves, *stable*, regardless of cutoff / resonances.

- **Benchmarking:** E.g., ICW mode conversion

![Dispersion Relation](image)
Time Domain
Cold Plasma Equations

• Convert frequency-domain linear plasma dielectric tensor to “auxiliary differential equation” (ADE) time domain method.

\[
\varepsilon = 1 - \sum_{\text{species}} \begin{bmatrix}
\frac{\omega_p^2}{\omega^2 - \Omega^2} & \left(\frac{\Omega}{-i\omega}\right) \frac{\omega_p^2}{\omega^2 - \Omega^2} & 0 \\
\left(\frac{\Omega}{i\omega}\right) \frac{\omega_p^2}{\omega^2 - \Omega^2} & \frac{\omega_p^2}{\omega^2 - \Omega^2} & 0 \\
0 & 0 & \frac{\omega_p^2}{\omega^2}
\end{bmatrix}
\]

• “Auxiliary Fields” are the linear plasma currents, one per species.

\[
\{\partial_t + v_s\}J_s = \varepsilon_0 \omega_{ps}^2 E - \Omega_s \times J_s
\]
“Super-Boris” Time Domain Cold Plasma Implicit Algorithm

• $3x'(N_{\text{species}}+1)$ matrix is analytically invertible.
• But messy!

\[
\lambda_s = \frac{1 - \frac{1}{2} \nu_s \delta t}{1 + \frac{1}{2} \nu_s \delta t}, \quad \omega_s = \frac{\Omega_s \delta t}{1 + \frac{1}{2} \nu_s \delta t}, \quad \omega_{p0}^2 = \sum_s \frac{\omega_{ps}^2 \delta t^2}{1 + \frac{1}{2} \nu_s \delta t}
\]

\[
\gamma^2 = \sum_s \left( \frac{\omega_{ps}^2 \Omega_s \delta t^4}{(1 + \frac{1}{2} \nu_s \delta t)^3} \right), \quad \delta = \sum_s \left( \frac{\omega_{ps}^2 \Omega_s \delta t^3}{(1 + \frac{1}{2} \nu_s \delta t)^2} \right), \quad \Lambda^2 = \frac{\delta^2}{1 + \frac{1}{4} \omega_{p0}^2}
\]

\[
F^{n+1} = \frac{1 - \frac{1}{4} \omega_{p0}^2 - \frac{1}{64} \Lambda^2}{1 + \frac{1}{4} \omega_{p0}^2 + \frac{1}{64} \Lambda^2} F^n + \frac{1}{1 + \frac{1}{4} \omega_{p0}^2 + \frac{1}{64} \Lambda^2} K
\]

\[
\gamma^2 = \frac{1}{64} \Lambda^2 - \frac{1}{16} \gamma^2 \left( \frac{1 + \frac{1}{4} \omega_{p0}^2 + \frac{1}{64} \Lambda^2}{1 + \frac{1}{4} \omega_{p0}^2 + \frac{1}{16} \gamma^2} \right) b b \cdot \left[ 2 F^n + K \right]
\]

\[
K = L - \frac{\delta t}{2} \sum_s \left( 1 + \lambda_s \right) \left[ \frac{1}{1 + \frac{1}{4} \omega_s^2} J_s^n + \frac{1}{4} \omega_s^2 b b \cdot J_s^n - \frac{1}{2} \omega_s b \times J_s^n \right]
\]

\[
L = \frac{\omega_s^2}{1 + \frac{1}{4} \omega_s^2} J_s^n + \left( \frac{1}{\delta t} \right) \frac{\omega_{p0}^2 \delta t^2}{1 + \frac{1}{2} \nu_s \delta t} \frac{1}{2} \left( F^{n+1} + F^n \right) + \frac{1}{4} \omega_s^2 b b \cdot \left[ (\lambda_s + 1) J_s^n + \frac{1}{2} \left( F^{n+1} + F^n \right) \right]
\]

\[
- \frac{1}{2} \omega_s b \times \left[ (\lambda_s + 1) J_s^n + \frac{1}{2} \left( F^{n+1} + F^n \right) \right]
\]

• Steps over $\omega_{pe}$, $\Omega_{ce}$ times.
In the Boundary Cells …

- Key Realization is that Plasma Currents from Time Domain Plasma is Capacitor Current.
  - E.g., the sheath IS the boundary condition for $J_{\text{plasma}}$
- One More Equation, for sheath potential.
  - Ugh, redo all that math, make sure energy conserving

The following equations shows the energy balance.

\[
\begin{align*}
\frac{1}{\delta x} \phi_{\text{sheath}} & \quad \{ \quad (\varepsilon_0/\Delta) \partial_t \phi_{\text{sheath}} = n \cdot J - (\varepsilon_0 \omega v/\Delta) \phi_{\text{sheath}} \quad \} \\
H & \quad \{ \quad \mu_0 \partial_t H = -\nabla \times E \quad \} \\
E & \quad \{ \quad \varepsilon_0 \partial_t E = \nabla \times H - J \quad \} \\
J & \quad \{ \quad (1/\varepsilon_0 \omega_p^2) \partial_t J = [E + n(1/\delta x)\phi_{\text{sheath}}] \quad \}
\end{align*}
\]

The sum of all of these gives

\[
\partial_t \left\{ \frac{1}{2} \varepsilon_0 (1-\Delta/\delta x)\|E\|^2 + \frac{1}{2} \varepsilon_0 (\Delta/\delta x)\|\phi/\Delta\|^2 + \frac{1}{2} \mu_0 \|H\|^2 + \frac{1}{2} (1/\varepsilon_0 \omega_p^2) \|J\|^2 \right\} = -\nu \omega \varepsilon_0 (\Delta/\delta x) \|\phi/\Delta\|^2
\]
The Advantages of Time Domain

• Explicit FDTD EM code is very well established and fast. (e.g., we already had the code!)
• And handles complex metallic boundary conditions.

• Of course, multiple simultaneous frequencies.
• Which allows for inclusion of nonlinearity, without a priori guess at modes and frequencies.
An Aside: Example of Nonlinearity: Parametric Decay Instability

• Use linear plasma electrons, and kinetic ions, (needed for IBW wave in PDI)
  – Sometimes as expected (comparison to J. Rost Thesis, MIT)
  – Sometimes not

The Disadvantage of Time Domain

- Yee Cell! \((B_x \text{ is at different location of } B_y, B_z)\)
- Complicates the \(\mathbf{J} \times \mathbf{B}\) in plasma force equation.

Also, no warm plasma effects, since no longer point physics, \(\nabla\)-operations.

So no IBW.
ITER Antenna Simulations

- (Show Movie 3 and 4)
- Simulations of ¼ ITER antenna, with surrounding box shows
  - slow-wave enhanced sheaths on the surrounding antenna box,
  - an interesting circulation pattern around the box that vanishes when field aligned
New Work on ITER Antenna

• Simulations pre-date energy conserving form
  – So sheath voltages are not necessarily correct

• Simulations pre-date Titan usage
  – Can do all 24 straps now

• (Hopefully) more material for APS ITER session talk later this fall.

• And then there is the question of the slow wave…
The Slow Wave

- I was given a $10^3$ range in edge densities to look at for ITER. (!)
- Some of those densities show a short wavelength mode in front of the antenna
- (Show Movie 5).
“Layer” moves with Density

Layer e17 at Density 0.2

Layer e18 at Density 0.3

Layer e19 at Density 0.4

0.5 Lowest Density

Highest Density
Does This Have Anything to do with NSTX Anomalous Losses?

• This question is presently under study.
• Concern is that it might be there for ITER as well.

• I’ve been trying to get a better instinctive idea of what is going on with these slow waves.
• Cold Plasma, they have to essentially be lower hybrid resonances.
• Resonance in the edge is $\omega \sim \omega_{LH} \sim \omega_{p,ion}$
• So there is a critical density, but $\omega_{pi}$, not $\omega_{pe}$. 
We are looking at the CMod Field Aligned Antenna as well
We can see slow waves there too, sometimes at high magnitude.
Slow Wave Dispersion

• The slow waves are NOT a thin layer trapped between two cutoffs, they are a thin layer trapped between a cutoff and a resonance.

• The resonance is what allows for energy to be in the layer, otherwise would be too thin to excite, since need a $k_\perp$. 
\[ \omega_{LH} \text{ in the Edge} \]

- From Stix: 
  \[ \omega_{LH}^{-2} = (\omega_{pi}^2 + \Omega_{ci}^2)^{-1} + (\Omega_{ci} \Omega_{ce})^{-1} \]

- At edge density, last term neglects away.

- At edge, 
  \[ (\omega_{RF}/\Omega_{ci})^2 \sim (n_{Harm}R_{outer}/R_0)^2 \gg 1 \]

- So: 
  \[ \omega_{LH}^2/\omega_{RF}^2 \approx \omega_{pi}^2/\omega_{RF}^2 + \text{SmallNumber} \]
  
  \[ (0.2 - 0.02) \]
Critical Density

• Since $\omega_{LH} \sim \omega_{pi}$, insensitive to B field, only density
  – Assume D plasma.

• For 30 MHz: $1.7 \times 10^{17} / m^3$

• For 80 MHz: $2.7 \times 10^{17} / m^3$
What are the Reported Densities?

- NSTX: Expect $5-10 \times 10^{17} m^3$, at least 3x over critical density.
  - Is there some other explanation why density might be lower than reported, in front of antenna. Ponderamotive pressure?
  - The game is on …

- ITER: Density is unknown. Hopefully high enough! The frequency is higher … so happens at higher density.
An Aside: Industrial Applications

- Many industrial plasmas are maintained by similar DC + RF fields, with energetic particles generated in the sheath as the ionization source.

- (Show Movie 6.)
Phase II SBIR Work

- Still have the scale problem, so still want to use sub-grid sheath.
- Want EED, use “test” particles.
- Have created special particle boundaries that see the sheath “kick” before impact, or after secondary emission, in an attempt to reproduce the “beam” electrons without resolving the sheath.
- Much interest in estimating steady-state of such plasmas a priori.
• Ultimate goal is estimation of impurity generating sputtering in ITER, and other tokamaks.
• Resolved scale problem with RF sheath sub-grid model, e.g., single $\phi_{\text{sheath}}$ value represents sheath.
• Coupled with pre-existing time-domain cold plasma algorithm, and pre-existing 3D CAD-importing EM software.
• Slow wave is a very interesting tangent.
• SBIR work will also include particles and industrial concerns.
THANK YOU!

- Questions, Comments?