PARTICLE-IN-CELL MODELING OF SPECIES SEPARATION IN TWO-SPECIES PLASMAS

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PARTICLE-IN-CELL MODELING OF SPECIES SEPARATION IN TWO-SPECIES PLASMAS

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OUTLINE

• Brief overview of pulsed power: from toasters to terawatts

• Frozen-in flux and magnetic penetration

• Simulations of magnetic penetration and species separation
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PULSED POWER

FROM TOASTERS TO TERAWATTS

LOAD

100 ns

POWER CONDITIONING

μs

CAPACITOR BANK (MARX)

100 s

HIGH VOLTAGE POWER SUPPLY

~100 V, ~10 A

MV, MA
LOAD

POWER CONDITIONING

CAPACITOR BANK (MARX)

HIGH VOLTAGE POWER SUPPLY

100 ns

μs

100 s

~100 V, ~10 A

MV, MA
Radiograph for Stockpile Stewardship
Detection of fissile material
• Nuclear Weapon Effects Simulation
  • Advanced bremsstrahlung diode sources for warm and hot x-rays
  • Plasma radiation sources for cold x-rays
  • Ion beam simulation of cold x-ray effects

• Other Pulsed-Power Applications
  • Detection of SNM – (IPAD: Intense Pulse Active Detection)
  • Electromagnetic Launchers
  • High-power, pulsed radiography
  • Advanced Energetics
  • Inertial confinement fusion
GAMBLE II: WATER LINE GENERATOR, 1.5 TW, ±1.5 MV, 1 MA, 60 NS (1978)

- Oil-insulated Marx
  0.5 MJ, 5 MV

- Water-filled coax (capacitive power conditioning)

- Vacuum load region
  ± 1.5 MV, 1 MA, 50 ns
**MERCURY**: INDUCTIVE VOLTAGE ADDER GENERATOR, +5 TO -8 MV, 360 TO 200 KA, 50 NS (2004)
MERCURY: INDUCTIVE VOLTAGE ADDER GENERATOR, +5 TO -8 MV, 360 TO 200 KA, 50 NS (2004)
HAWK: INDUCTIVE STORAGE GENERATOR, 0.6 MV, 800 KA, 1200 NS (1990)
A plasma opening switch can be used to conduct current from a pulsed power generator, then open on a faster time scale. Plasma is injected into the region between the inner and outer conductors. Current initially flows through the plasma as magnetic energy accumulates. At some point, the current in the plasma decreases rapidly (the switch opens) and generator current flows to downstream load. The physics of the opening process is not fully understood, but some experimental evidence points to fast magnetic penetration and species separation playing a role.
OUTLINE

• Brief overview of pulsed power: from toasters to terawatts

• Frozen-in flux and magnetic penetration
  • Ideal MHD and Frozen-in Flux
  • Hall-driven Magnetic Penetration
  • Collisionless Magnetic Piston

• Simulations of magnetic penetration and species separation
The ideal MHD Ohm’s law implies that the plasma is “frozen” to magnetic field lines

\[
\Phi = \int_{\Delta S} \mathbf{B} \cdot d\mathbf{A}
\]

\[
\dot{\Phi} = \int_{\Delta S} \dot{\mathbf{B}} \cdot d\mathbf{A} + \int_{\Delta S} \mathbf{B} \cdot d\mathbf{A}
\]

\[
\dot{\Phi} = \int_{\Delta S} \left( \nabla \times (\mathbf{v} \times \mathbf{B}) \right) \cdot d\mathbf{A} + \oint \mathbf{B} \cdot (\mathbf{u} \times d\mathbf{l})
\]

\[
\dot{\Phi} = \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} + \oint (\mathbf{B} \times \mathbf{u}) \cdot d\mathbf{l}
\]

\[
\therefore \dot{\Phi} = 0 \quad \text{if} \quad \mathbf{u} = \mathbf{v}
\]
RELAXATION OF FROZEN-IN-FLUX

• Keeping additional terms in Ohm’s Law leads to cases where frozen-flux law no longer holds

• Generalized Ohm’s Law:

\[
\frac{m_e m_i}{\rho_M e^2} \frac{\partial \mathbf{J}}{\partial t} = \frac{m_i \nabla P}{2\rho_M e} + \mathbf{E} + \frac{\mathbf{v} \times \mathbf{B}}{c} - \frac{m_i}{\rho_M e c} \mathbf{J} \times \mathbf{B} - \frac{\mathbf{J}}{\sigma}
\]

Hall Term
HALL-DRIVEN MAGNETIC PENETRATION

• Rather than considering MHD with the generalized Ohm’s law, consider the fixed-ions limit (EMHD), where electrons carry the current,

\[ \mathbf{J} = -n_e \mathbf{V}_e \]

• As described in [1], this analysis can be performed as a two-fluid model (and we take the ion fluid as fixed). Then the equation for the magnetic field becomes

\[
\frac{\partial \mathbf{B}}{\partial t} - \frac{c \mathbf{B}}{4 \pi e} \left( \nabla n_e^{-1} \cdot (\nabla \times \mathbf{B}) \right) = 0
\]

• In cartesian geometry, with the density gradient parallel to the current, this becomes a Burgers’ Equation, which has magnetic shock solutions which propagate into the plasma with speed

\[
\nu_s = \frac{1}{2} \nu_{\text{Hall}} = \frac{cB_0}{8\pi en_e L_n}
\]

where \( L_n \) is the density gradient length scale

A second phenomenon which can play an important role in the regimes of interest is the “collisionless magnetic piston” [2].

In this model, a magnetic field drives a boundary current, which pushes on the plasma in a piston-like way.

Reference [2] gives a self-consistent solution for the fields and currents, and shows that the speed of the piston is half the Alfvén speed.

Consideration of pressure balance also allows one to derive the piston speed.

Magnetic front moving with speed $v_b$ imparts momentum to ions:

\[ \Delta p = M(v_{\text{final}} - v_{\text{initial}}) = \Delta x \Delta z \rho v_{\text{final}} \]

\[ \Delta t = \Delta z/v_b \]

\[ F = \Delta p/\Delta t = \Delta x \rho v_{\text{final}} v_b \]

\[ P = F/\Delta x = \rho v_{\text{final}} v_b \]

Balancing total pressure with magnetic pressure gives

\[ P = B^2/8\pi = \rho v_{\text{final}} v_b \]

For the case of specular reflection as in ref. [2],

\[ |v_{\text{initial}}| = |v_{\text{final}}| \] in the moving frame, so we have $v_{\text{final}} = 2 v_b$

or

\[ v_b = \sqrt{(B^2/16 \pi \rho)} = \frac{v_{\text{Alfvén}}}{2} \]
COMBINING HALL PENETRATION AND MAGNETIC PISTON

- The two phenomena previously described can combine together to give a “leaky piston” model

- Specifically, in a multi-ion-species plasma with density gradients, the magnetic field can penetrate one ion species via the Hall process, while simultaneously pushing another via a collisionless piston process

- For the right choice of density gradients and ion masses, this combination can lead to separation of ion species
LEAKY PISTON IN MULTI-SPECIES PLASMA

• Consider a magnetic front moving with speed $v_b$, which imparts momentum to ions:
  \[ \Delta p = M(v_{\text{final},i} - v_{\text{initial},i}) = \Delta x \Delta z m_i n_i v_{\text{final},i} \]
  \[ \Delta t = \Delta z/v_b \]
  \[ F = \Delta p/\Delta t = \Delta x m_i n_i v_{\text{final},i} v_b \]
  \[ P_i = F/\Delta x = m_i n_i v_{\text{final},i} v_b \]

• This partial pressure can be written in terms of the total pressure and total number density: $P_i = P n_i/n$

• Balancing total pressure with magnetic pressure gives
  \[ P/n = B^2/8\pi n = m_i v_{\text{final},i} v_b \]

• or
  \[ v_{\text{final},i} = B^2/(8 \pi n m_i v_b) \]
SPECIES SEPARATION IN THE LEAKY PISTON

• For a fixed piston speed $v_b$, the final ion speed is inversely proportional to mass.

• For massive ions, $v_{\text{final}}$ is very small.

• As $m_i$ is decreased in a series of simulations, $v_{\text{final}}$ will increase, and there should be some transition (when $v_{\text{final}} \sim v_b$) to specular reflection of ions.

• For a multi-component plasma, the light-ion component can be reflected, while the heavy-ion component is penetrated by the magnetic field.

• Transition should occur when $v_{\text{final},i} = v_b = B^2/(8\pi nm_i v_b)$.

• The corresponding mass threshold is $m^* = B^2/8\pi n v_b^2$. 
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• Brief overview of pulsed power: from toasters to terawatts

• Frozen-in flux and magnetic penetration

• Simulations of magnetic penetration and species separation
  • Hall-driven magnetic penetration
  • Leaky piston behavior in multi-component plasmas
  • Ion species separation
SOME PREDICTIONS

• Things to look for in simulations include:

• Hall-driven magnetic penetration in single species plasma

• Leaky piston behavior in multi-component plasma, specifically:
  • Quantitative prediction: final ion velocity \( v_{\text{final},i} = B^2 / (8\pi n m_i v_b) \)
  • Quantitative prediction: mass threshold \( m^* = B^2 / 8\pi n v_b^2 \) for transition from magnetic penetration of light ions, to specular reflection of light ions
  • Qualitative prediction: species separation in multi-component plasma
SIMULATIONS

• To simulate these effects, we start with simulating Hall penetration into a plasma composed of infinitely massive ions. This makes the Hall speed faster than any ion time-scale.

• This lets us estimate the proportionality factor $\alpha$, where $v_b = \alpha v_{\text{Hall}}$. Recall that for the cartesian case which gives rise to the Burgers’ Equation, $\alpha = 1/2$.

• We then do simulations of two-component plasmas, where one component is infinitely massive.

• The mass of the light ions is varied in a series of simulations, in order to observe the transition from magnetic penetration to specular reflection and species separation.
DESCRIPTION OF SIMULATION SETUP

• 2D cartesian geometry, 5 cm AK gap

• Magnetic field from pulse rises to 25 kG in 3 ns

• Plasma with 3 cm axial extent bridges the “AK gap”

• Higher density near the anode gives rise to Hall penetration

• Simulations with mobile ions will include 0.5 cm layer of immobile ions near the cathode, to mitigate edge effects

• Form of density profile chosen so that transition mass \( m^* \) is the same across the plasma \( [m^* = \text{const} \Rightarrow n_L n^2 = \text{const}] \)

\[
n_e(x) = \frac{1}{(a (x-x_0)/2 + n_0^{-1/2})^2} \\
= -1.7625 \times 10^{-8} \text{ cm}^{1/2}
\]
Field line-outs taken midway between anode and cathode, at $x = 2.5 \text{ cm}$

Plasma located at $z = 0$ to $z = 3 \text{ cm}$
This simulation shows that $v_b \approx 0.65 \, v_{\text{Hall}} = 76.32 \, \text{cm/µs}$

\[
v_H = \frac{cB}{4\pi neL_n}
\]

\[
v_b = \alpha v_H
\]
MAGNETIC FIELD IN 2D, AT 15 NS

FIXED ION SIMULATION

predicted location of front moving at 100% Hall Speed ($\alpha=1$)

predicted location of front moving at 65% Hall Speed ($\alpha=0.65$)

\[
v_H = \frac{cB}{4\pi neL_n}
\]

\[
v_b = \alpha v_H
\]
FINAL ION VELOCITY: THEORY

\[ v_H = \frac{cB}{4\pi neL_n} \]

\[ v_b = \alpha v_H \]

\[ v_{\text{final}} = \frac{B^2}{8\pi nm_i v_b} \]
80% FIXED IONS
20% KR IONS
FIELD PENETRATION AT X=2.5 CM
20% KR IONS SIMULATION

7.20 \times 10^{-2} \text{ cm/ns}

3 ns, 6 ns, 9 ns, 12 ns, 15 ns

Position in \( z \) [cm]

Time [ns]

Hall Front, 65% Speed
Magnetic Front (@5 kG) From Simulation
20% Krypton Ions Simulation: Movie

B Field

Light Ion Density
ION VELOCITY, X = 2.4 TO 2.6 CM, 15 NS

20% KR IONS SIMULATION

Ion Density at x = 2.5 cm

Phase Space Particle Plot
\[ v_H = \frac{cB}{4\pi neL_n} \]
\[ v_b = \alpha v_H \]
\[ v_{\text{final}} = \frac{B^2}{8\pi nm_i v_b} \]
SIMULATION 3

80% FIXED IONS
20% B IONS
FIELD PENETRATION AT X=2.5 CM

20% B IONS SIMULATION

![Graph showing field penetration at X=2.5 cm for 20% ions simulation.](image)

- The graph depicts the magnetic field strength $B_z$ in kG as a function of position $z$ in cm for different times: 3 ns, 6 ns, 9 ns, 12 ns, and 15 ns.
- The magnetic field strength decreases with time as the ions penetrate through the field.
- The rate of decrease is given as $8.33 \times 10^{-2} \text{ cm/ns}$.

![Graph showing position in z as a function of time.](image)

- The graph shows the position in z (cm) on the y-axis and time (ns) on the x-axis.
- The magnetic field front is denoted by a dashed purple line, labeled "Hall Front, 65% Speed".
- The magnetic front at 5 kG is shown by a solid black line, labeled "Magnetic Front (at 5 kG) From Simulation".

Time [ns]: 0, 2.5, 5, 7.5, 10, 12.5, 15

Position in z [cm]: 0, 0.2, 0.4, 0.6, 0.8, 1.0, 1.2
20% B IONS SIMULATION: MOVIE

B Field

Light Ion Density

1 cm
t = 0.1 ns

30 kG

4.80E+14/cc

1 cm
t = 0.1 ns

D/cc
ION VELOCITY, X=2.4 TO 2.6 CM, 15 NS

20% B IONS SIMULATION

Ion Density at x = 2.5 cm

Phase Space Particle Plot
\[ v_H = \frac{cB}{4\pi n e L_n} \]

\[ v_b = \alpha v_H \]

\[ v_{\text{final}} = \frac{B^2}{8\pi n m_i v_b} \]
SIMULATION 4

80% FIXED IONS
20% HE IONS
FIELD PENETRATION AT X = 2.5 CM

20% HE IONS SIMULATION

![Graph showing field penetration and simulation results.]

- $8.91 \times 10^{-2}$ cm/ns
- Times: 3 ns, 6 ns, 9 ns, 12 ns, 15 ns
- Hall Front, 65% Speed
- Magnetic Front (@5 kG) From Simulation
20% HE IONS SIMULATION: MOVIE

B Field

Light Ion Density

1 cm

$\tau = 0.1\,\text{ns}$

$30\,\text{kG}$

$4.00 \times 10^{14}/\text{cc}$
ION VELOCITY, X = 2.4 TO 2.6 CM, 15 NS

20% HE IONS SIMULATION

Ion Density at x = 2.5 cm

Phase Space Particle Plot
SIMULATION 5

80% FIXED IONS
20% H IONS
FIELD PENETRATION AT X = 2.5 CM

20% H IONS SIMULATION

\[ 9.05 \times 10^{-2} \text{ cm/ns} \]
20% H IONS SIMULATION: MOVIE

B Field

Light Ion Density

1 cm
t = 0.1 ns

30 kG

4.00E+14/cc

2/cc
B Field

Light Ion Density

1 cm
t = 0.1 ns

30 kG

4.00E+14/cc

0/cc
ION VELOCITY, X = 2.4 TO 2.6 CM, 15 NS

20% HE IONS SIMULATION

Ion Density at x = 2.5 cm

Phase Space Particle Plot

velocity prediction: 556 cm/µs
FINAL ION VELOCITY: SIMULATIONS

\[ v_H = \frac{cB}{4\pi neL_n} \]

\[ v_b = \alpha v_H \]

\[ v_{\text{final}} = \frac{B^2}{8\pi nm_i v_b} \]
SUMMARY

• Hall-driven magnetic penetration observed in single species plasma simulations

• Leaky piston behavior observed in multi-component plasma simulations, specifically:
  
  • Final ion velocity $v_{\text{final},i} = \frac{B^2}{(8\pi \eta m_i v_b)}$ scaling agrees with simulation results, with simulated velocity slightly less than predicted
  
  • Species separation observed in multi-component plasma simulations where $m_{\text{light}} < m^*$

  • Mass threshold $m^* = \frac{B^2}{8\pi n v_b^2}$ agrees with simulation results
QUESTIONS?