PARTICLE-IN-CELL MODELING OF SPECIES SEPARATION IN TWO-SPECIES PLASMAS

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SPONSOR: NRL BASIC AND APPLIED RESEARCH PROGRAM

OUTLINE

- Brief overview of pulsed power: from toasters to terawatts

- Frozen-in flux and magnetic penetration

- Simulations of magnetic penetration and species separation

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- **Brief overview of pulsed power: from toasters to terawatts**
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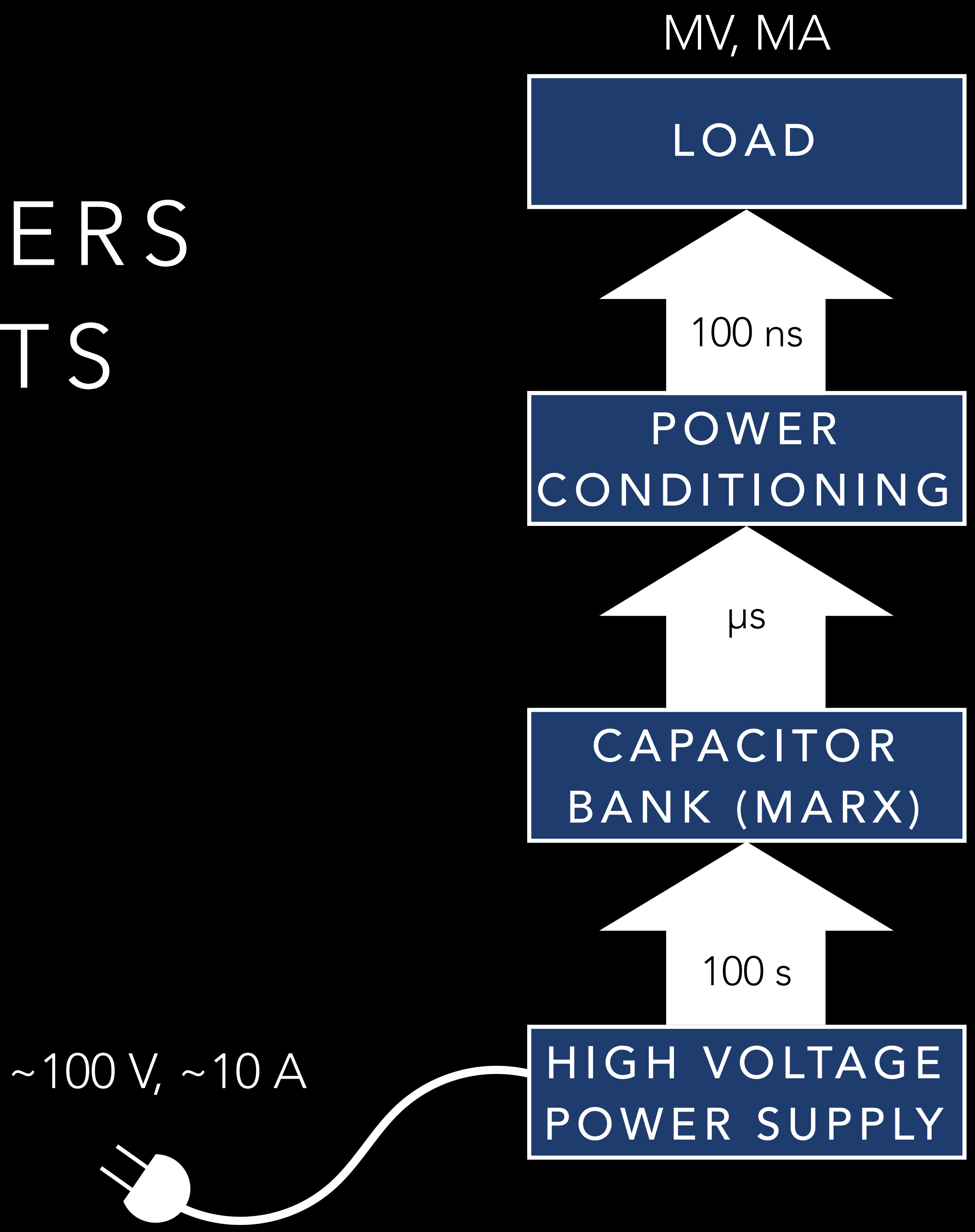
PULSED POWER

FROM TOASTERS TO TERAWATTS



MV, MA

LOAD

100 ns

POWER CONDITIONING

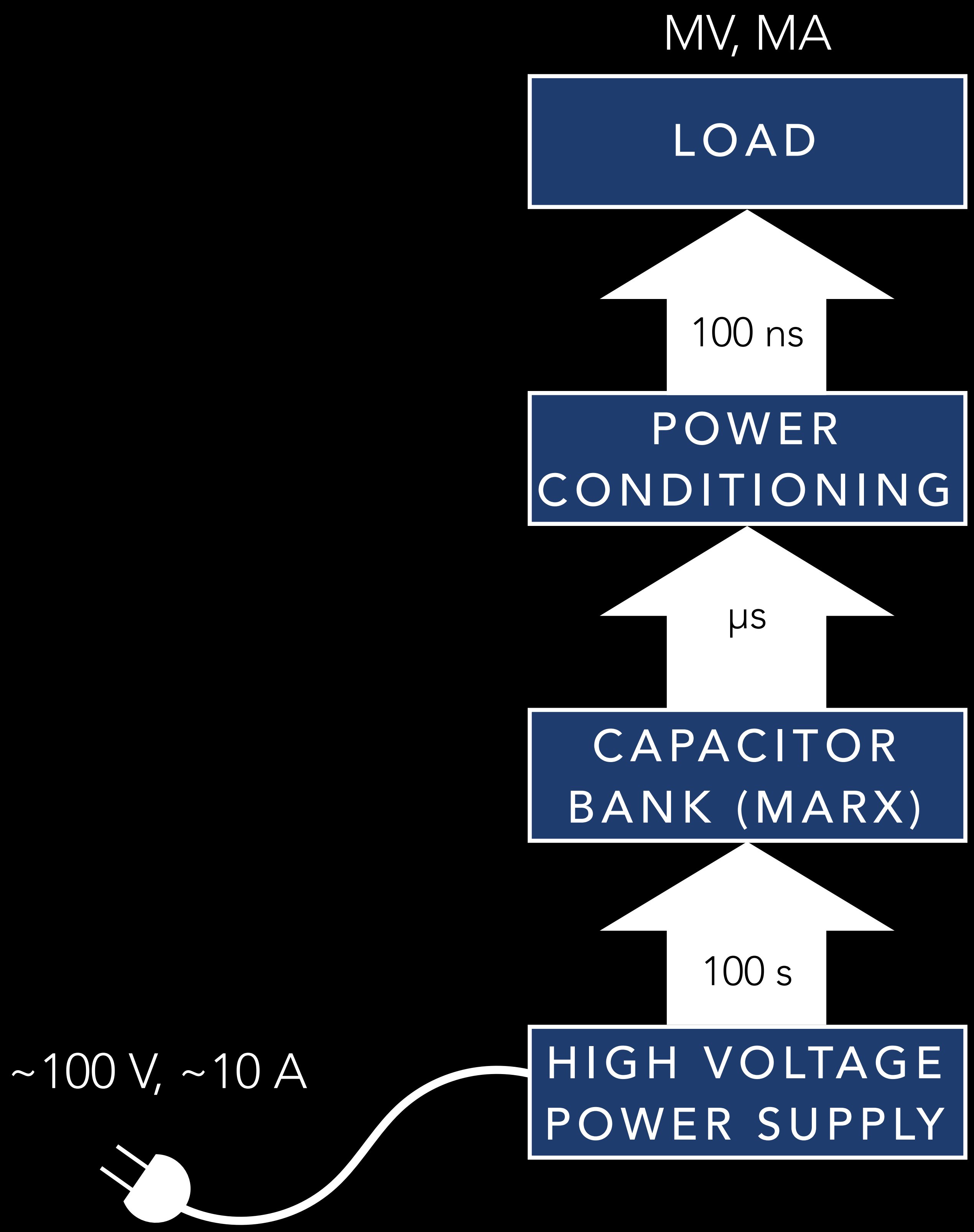
μs

CAPACITOR BANK (MARX)

100 s

HIGH VOLTAGE POWER SUPPLY

~100 V, ~10 A



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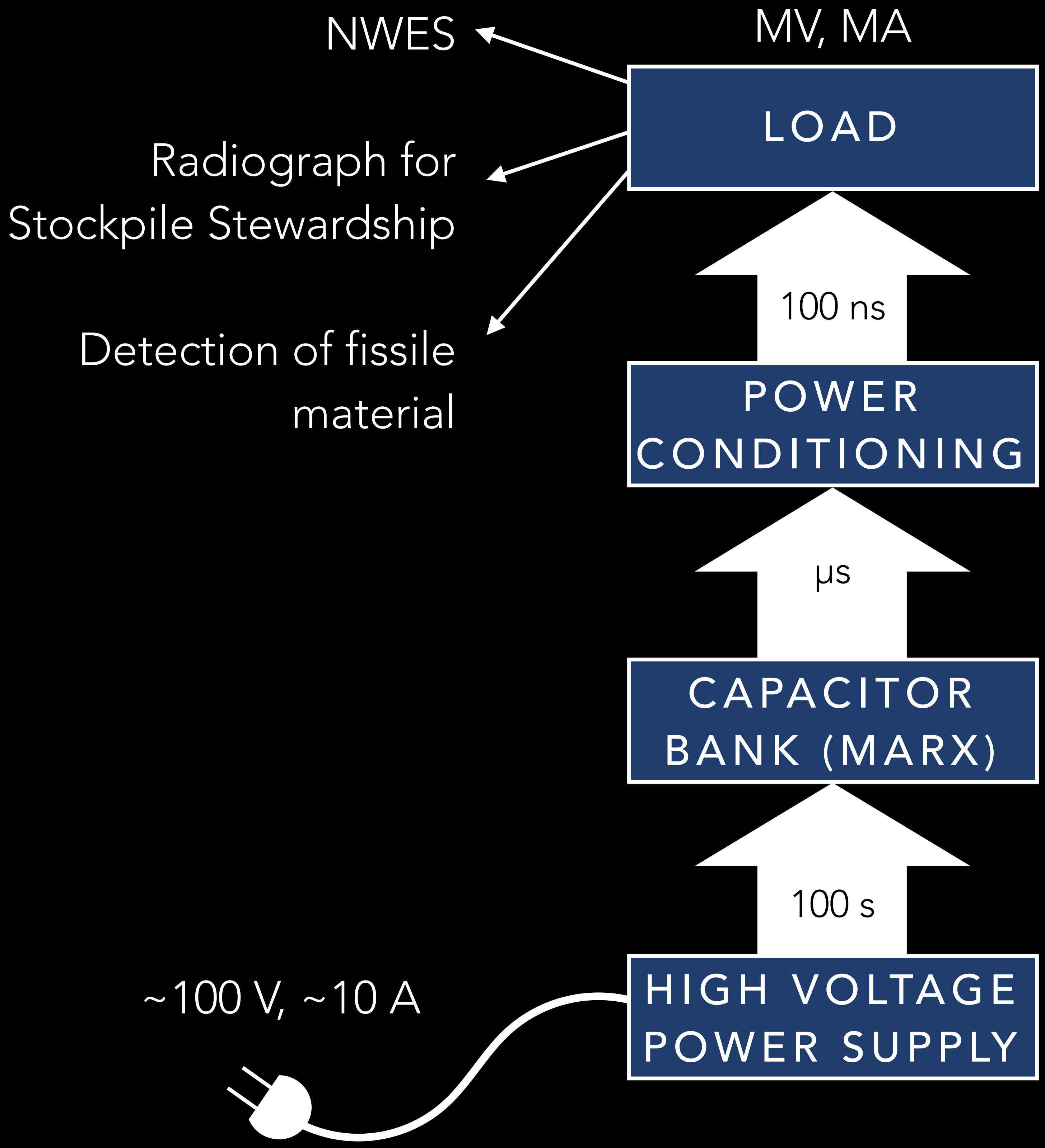


MV, MA

LOAD

NWES

Radiograph for Stockpile Stewardship

Detection of fissile material

100 ns

POWER CONDITIONING

μs

CAPACITOR BANK (MARX)

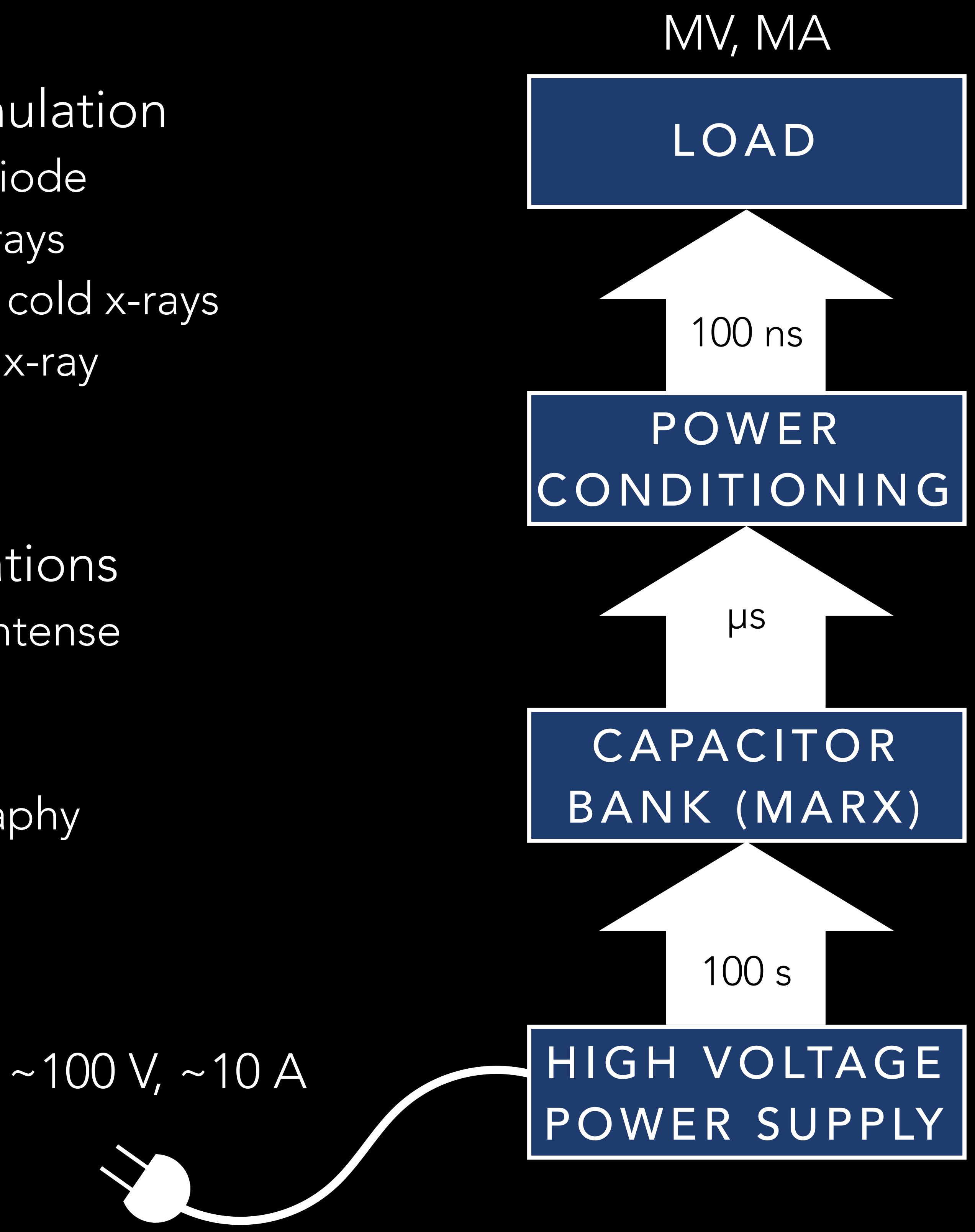
100 s

HIGH VOLTAGE POWER SUPPLY

~100 V, ~10 A

- Nuclear Weapon Effects Simulation
  - Advanced bremsstrahlung diode sources for warm and hot x-rays
  - Plasma radiation sources for cold x-rays
  - Ion beam simulation of cold x-ray effects

- Other Pulsed-Power Applications
  - Detection of SNM – (IPAD: Intense Pulse Active Detection)
  - Electromagnetic Launchers
  - High-power, pulsed radiography
  - Advanced Energetics
  - Inertial confinement fusion



MV, MA

LOAD

100 ns

POWER CONDITIONING

µs

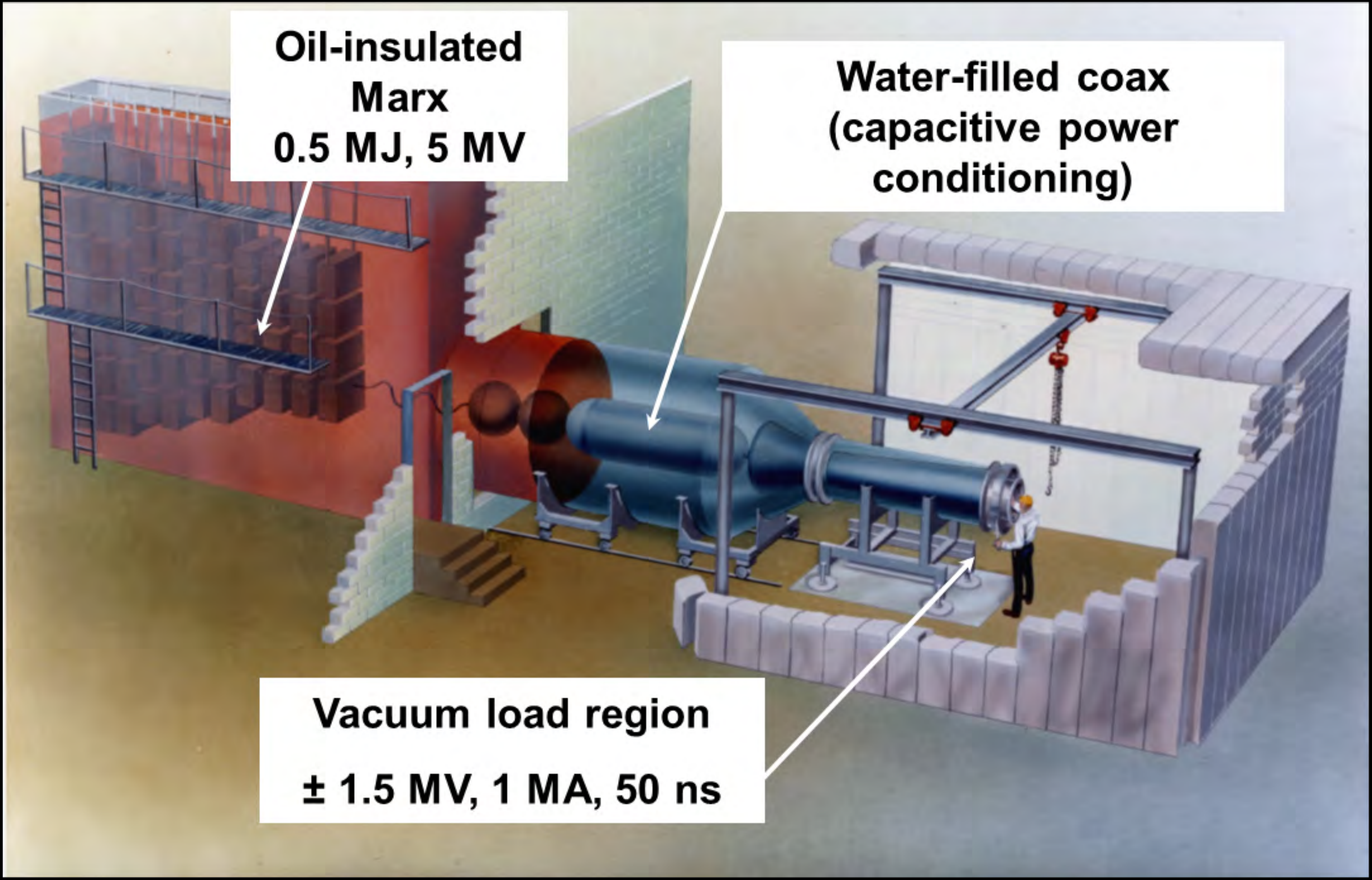
CAPACITOR BANK (MARX)

100 s

HIGH VOLTAGE POWER SUPPLY

~100 V, ~10 A

GAMBLE II: WATER LINE GENERATOR, 1.5 TW, ±1.5 MV, 1 MA, 60 NS (1978)



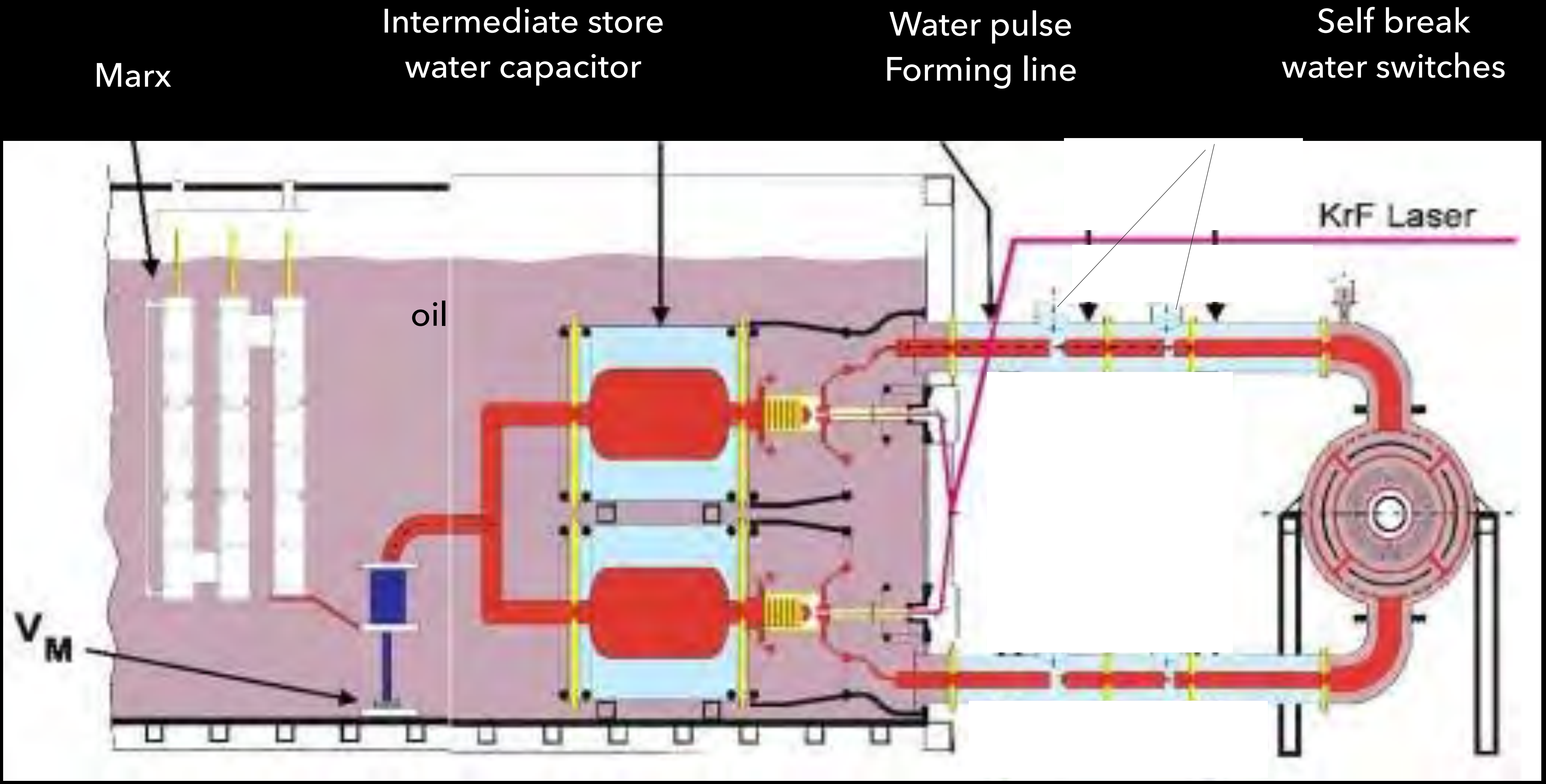
Oil-insulated Marx 0.5 MJ, 5 MV

Water-filled coax (capacitive power conditioning)

Vacuum load region

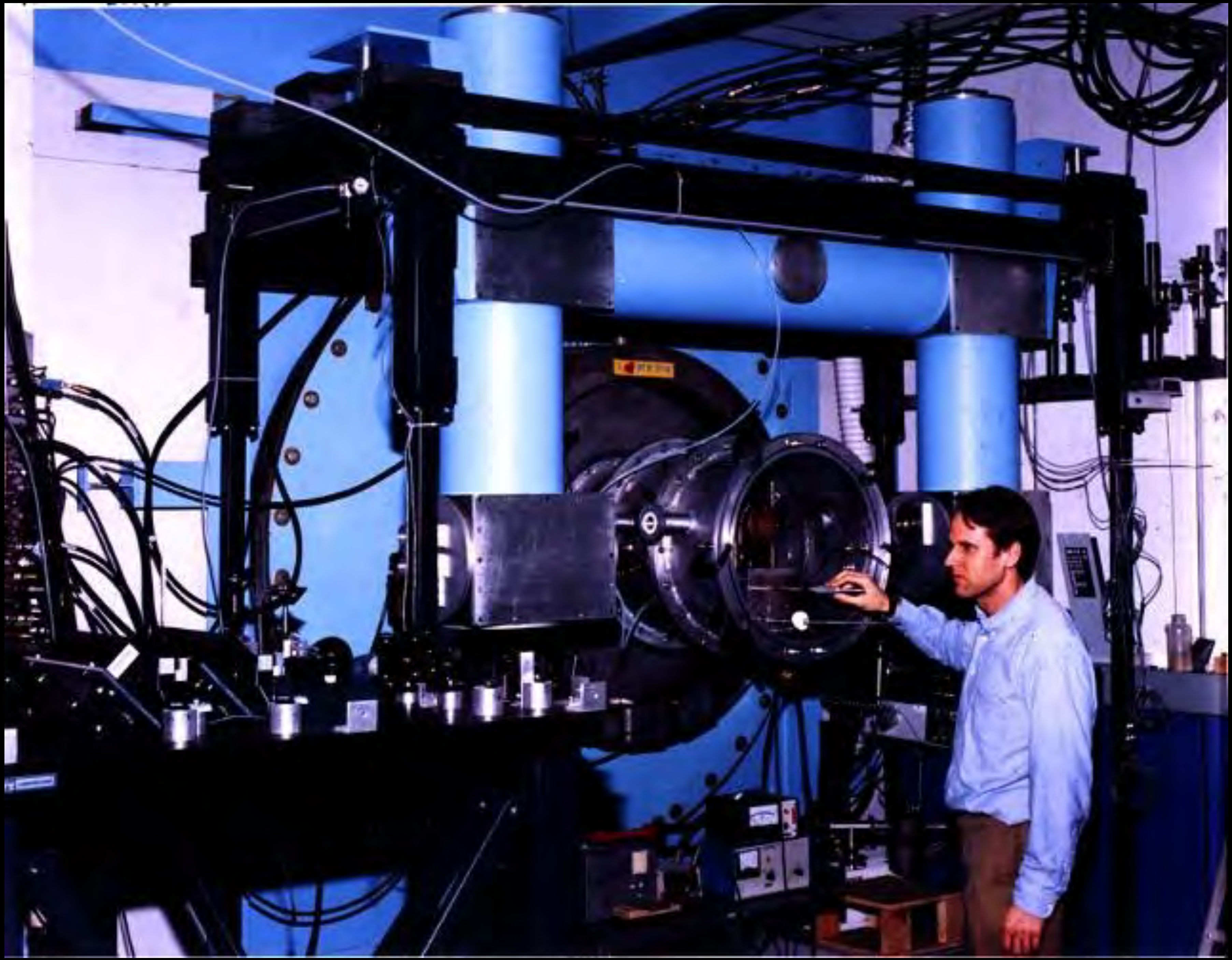
± 1.5 MV, 1 MA, 50 ns

MERCURY: INDUCTIVE VOLTAGE ADDER GENERATOR, +5 TO -8 MV, 360 TO 200 KA, 50 NS (2004)



MERCURY: INDUCTIVE VOLTAGE ADDER GENERATOR, +5 TO -8 MV, 360 TO 200 KA, 50 NS (2004)

HAWK: INDUCTIVE STORAGE GENERATOR, 0.6 MV, 800 KA, 1200 NS (1990)



THE PLASMA OPENING SWITCH, OR WHY AM I TALKING ABOUT MAGNETIC PENETRATION INTO PLASMA?

- A plasma opening switch can be used to conduct current from a pulsed power generator, then open on a faster time scale
- Plasma is injected into the region between the inner and outer conductors
- Current initially flows through the plasma as magnetic energy accumulates
- At some point, the current in the plasma decreases rapidly (the switch opens) and generator current flows to downstream load
- The physics of the opening process is not fully understood, but some experimental evidence points to fast magnetic penetration and species separation playing a role

OUTLINE

- Brief overview of pulsed power: from toasters to terawatts
- **Frozen-in flux and magnetic penetration**
  - Ideal MHD and Frozen-in Flux
  - Hall-driven Magnetic Penetration
  - Collisionless Magnetic Piston
- Simulations of magnetic penetration and species separation

FROZEN-IN-FLUX

- The ideal MHD Ohm’s law implies that the plasma is “frozen” to magnetic field lines

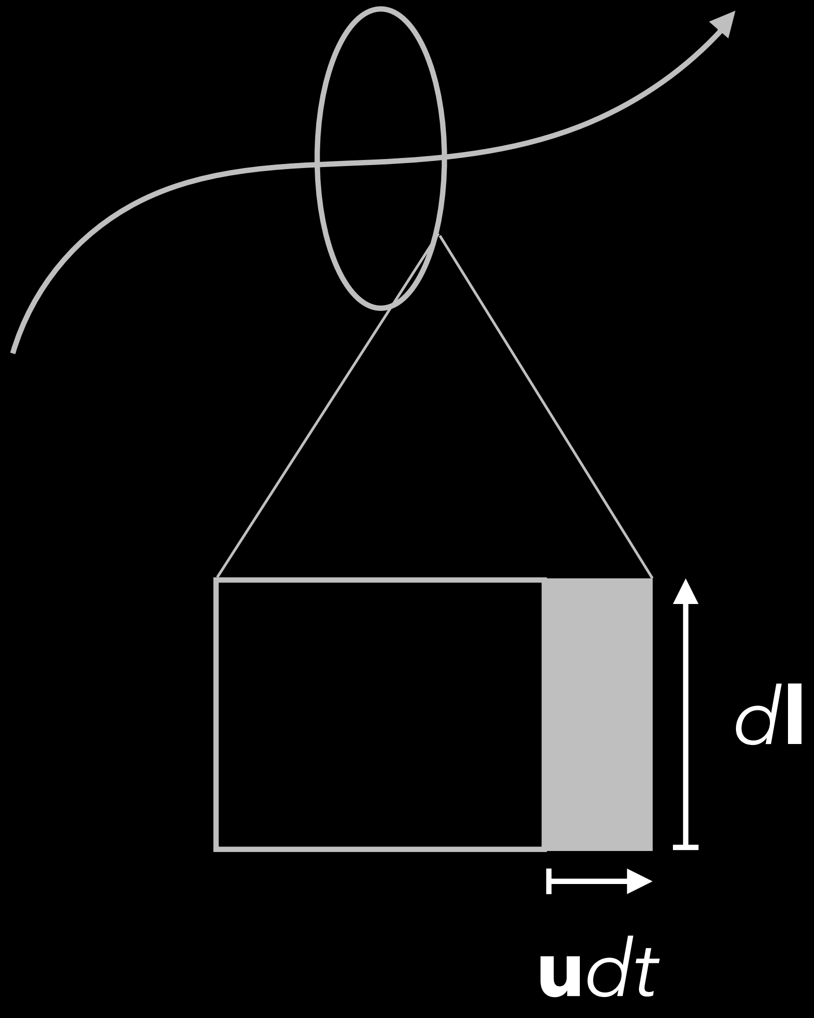
$$\Phi = \int_{\Delta S} \mathbf{B} \cdot d\mathbf{A}$$

$$\dot{\Phi} = \int_{\Delta S} \dot{\mathbf{B}} \cdot d\mathbf{A} + \int_{\Delta S} \mathbf{B} \cdot d\dot{\mathbf{A}}$$

$$\dot{\Phi} = \int_{\Delta S} (\nabla \times (\mathbf{v} \times \mathbf{B})) \cdot d\mathbf{A} + \oint \mathbf{B} \cdot (\mathbf{u} \times d\mathbf{l})$$

$$\dot{\Phi} = \oint (\mathbf{v} \times \mathbf{B}) \cdot d\mathbf{l} + \oint (\mathbf{B} \times \mathbf{u}) \cdot d\mathbf{l}$$

$$\therefore \dot{\Phi} = 0 \quad \text{if} \quad \mathbf{u} = \mathbf{v}$$



RELAXATION OF FROZEN-IN-FLUX

- Keeping additional terms in Ohm's Law leads to cases where frozen-flux law no longer holds
  - Generalized Ohm's Law:

$$\frac{m_e m_i}{\rho_M e^2}\frac{\partial \mathbf{J}}{\partial t} = \frac{m_i \nabla P}{2\rho_M e} + \mathbf{E} + \frac{\mathbf{v}\times\mathbf{B}}{c} - \boxed{\frac{m_i}{\rho_M e c}\mathbf{J}\times\mathbf{B}} - \frac{\mathbf{J}}{\sigma}$$

Hall Term

HALL-DRIVEN MAGNETIC PENETRATION

- Rather than considering MHD with the generalized Ohm's law, consider the fixed-ions limit (EMHD), where electrons carry the current,

$$\mathbf{J} = -n_e \mathbf{V}_e$$

- As described in [1], this analysis can be performed as a two-fluid model (and we take the ion fluid as fixed). Then the equation for the magnetic field becomes

$$\frac{\partial \mathbf{B}}{\partial t} - \frac{c\mathbf{B}}{4\pi e}\left(\nabla n_e^{-1} \cdot (\nabla \times \mathbf{B})\right) = 0$$

- In cartesian geometry, with the density gradient parallel to the current, this becomes a Burgers' Equation, which has magnetic shock solutions which propagate into the plasma with speed

$$v_s = \tfrac{1}{2} v_{\mathrm{Hall}} = \frac{cB_0}{8\pi e n_e L_n}$$

where L_n is the density gradient length scale

[1] A. S. Kingsep, Y. V. Mokhov, and K. V. Chukbar, "Nonlinear skin effect in plasmas," Sov. J. Plamsa Phys., vol. 10, no. 4, pp. 495–498, 1984.

COLLISIONLESS MAGNETIC PISTON

- A second phenomenon which can play an important role in the regimes of interest is the “collisionless magnetic piston” [2]

- In this model, a magnetic field drives a boundary current, which pushes on the plasma in a piston-like way

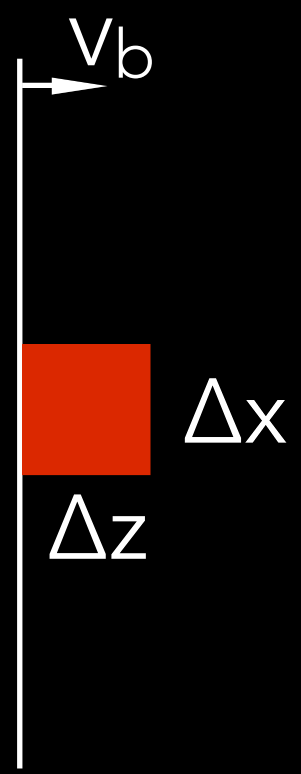
- Reference [2] gives a self-consistent solution for the fields and currents, and shows that the speed of the piston is half the Alfvén speed

- Consideration of pressure balance also allows one to derive the piston speed

[2] M. Rosenbluth, “Infinite conductivity theory of the pinch,” Tech. Rep. LA-1850, Los Alamos National Laboratory, 1954.

COLLISIONLESS PISTON AND PRESSURE BALANCE

- Magnetic front moving with speed v_b imparts momentum to ions:
  $\Delta p = M(v_{final} - v_{initial}) = \Delta x\ \Delta z\ \rho\ v_{final}$
  $\Delta t = \Delta z / v_b$
  $F = \Delta p / \Delta t = \Delta x\ \rho\ v_{final}\ v_b$
  $P = F / \Delta x = \rho\ v_{final}\ v_b$



- Balancing total pressure with magnetic pressure gives
  $P = B^2/8\pi = \rho\ v_{final}\ v_b$

- For the case of specular reflection as in ref. [2],
  $|v_{initial}| = |v_{final}|$ in the moving frame, so we have $v_{final} = 2\ v_b$

- or $\boxed{v_b = \sqrt{(B^2/16\ \pi\ \rho)} = v_{Alfvén}\ /\ 2}$

COMBINING HALL PENETRATION AND MAGNETIC PISTON

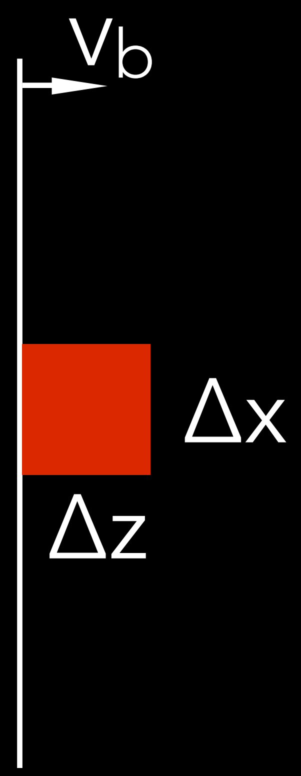
- The two phenomena previously described can combine together to give a “leaky piston” model

- Specifically, in a multi-ion-species plasma with density gradients, the magnetic field can penetrate one ion species via the Hall process, while simultaneously pushing another via a collisionless piston process

- For the right choice of density gradients and ion masses, this combination can lead to separation of ion species

LEAKY PISTON IN MULTI-SPECIES PLASMA

- Consider a magnetic front moving with speed v_b, which imparts momentum to ions:
  $\Delta p = M(v_{final,i} - v_{initial,i}) = \Delta x\, \Delta z\, m_i\, n_i\, v_{final,i}$
  $\Delta t = \Delta z / v_b$
  $F = \Delta p / \Delta t = \Delta x\, m_i\, n_i\, v_{final,i}\, v_b$
  $P_i = F/\Delta x = m_i\, n_i\, v_{final,i}\, v_b$



- This partial pressure can be written in terms of the total pressure and total number density: $P_i = P\, n_i/n$

- Balancing total pressure with magnetic pressure gives
  $P/n = B^2/8\pi n = m_i\, v_{final,i}\, v_b$

- or $\boxed{v_{final,i} = B^2/(8\,\pi\, n\, m_i\, v_b)}$

SPECIES SEPARATION IN THE LEAKY PISTON

- For a fixed piston speed v_b, the final ion speed is inversely proportional to mass
- For massive ions, v_{final} is very small
- As m_i is decreased in a series of simulations, v_{final} will increase, and there should be some transition (when $v_{final} \sim v_b$) to specular reflection of ions
- For a multi-component plasma, the light-ion component can be reflected, while the heavy-ion component is penetrated by the magnetic field
- Transition should occur when $v_{final,i}=v_b=B^2/(8\pi n m_i v_b)$
- The corresponding mass threshold is $m^*=B^2/8\pi n v_b^2$

OUTLINE

- Brief overview of pulsed power: from toasters to terawatts
- Frozen-in flux and magnetic penetration
- **Simulations of magnetic penetration and species separation**
    - Hall-driven magnetic penetration
    - Leaky piston behavior in multi-component plasmas
    - Ion species separation

SOME PREDICTIONS

- Things to look for in simulations include:

- Hall-driven magnetic penetration in single species plasma

- Leaky piston behavior in multi-component plasma, specifically:
    - Quantitative prediction: final ion velocity $v_{\text{final},i} = B^2/(8\pi n m_i v_b)$
    - Quantitative prediction: mass threshold $m^* = B^2/8\pi n v_b^2$ for transition from magnetic penetration of light ions, to specular reflection of light ions
    - Qualitative prediction: species separation in multi-component plasma

SIMULATIONS

- To simulate these effects, we start with simulating Hall penetration into a plasma composed of infinitely massive ions. This makes the Hall speed faster than any ion time-scale.

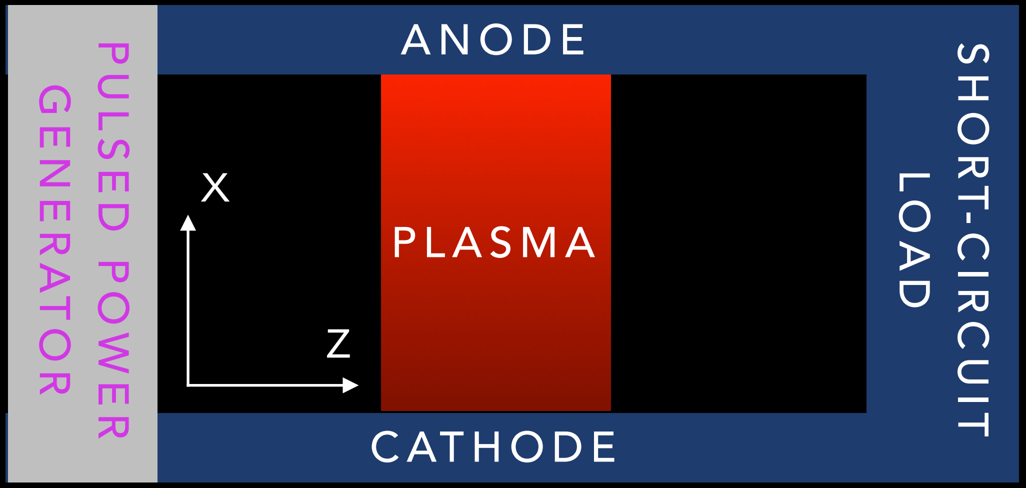
- This lets us estimate the proportionality factor α, where $v_b = \alpha v_{Hall}$. Recall that for the cartesian case which gives rise to the Burgers' Equation, $\alpha = 1/2$.

- We then do simulations of two-component plasmas, where one component is infinitely massive.

- The mass of the light ions is varied in a series of simulations, in order to observe the transition from magnetic penetration to specular reflection and species separation.

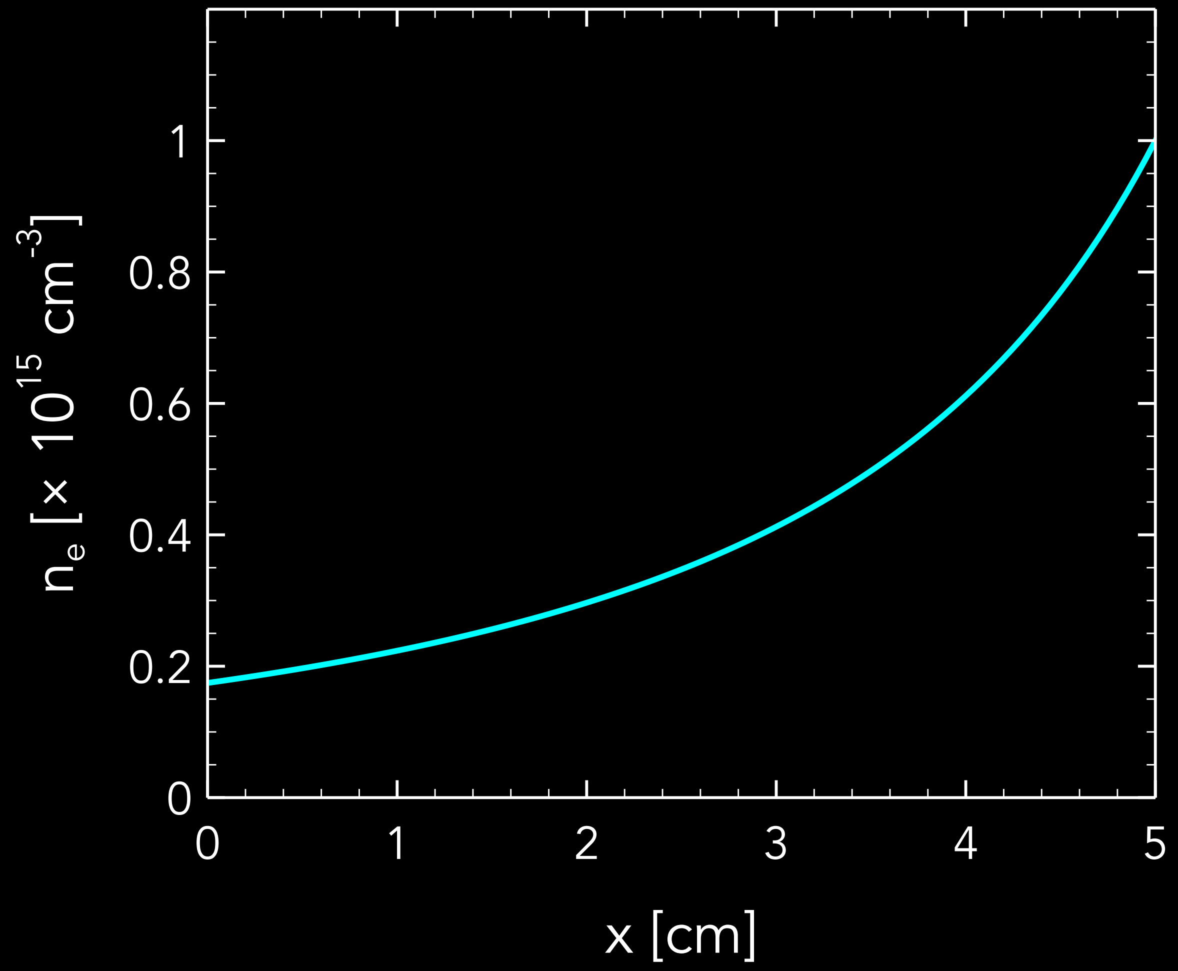
DESCRIPTION OF SIMULATION SETUP

- 2D cartesian geometry, 5 cm AK gap
- Magnetic field from pulse rises to 25 kG in 3 ns
- Plasma with 3 cm axial extent bridges the “AK gap”
- Higher density near the anode gives rise to Hall penetration
- Simulations with mobile ions will include 0.5 cm layer of immobile ions near the cathode, to mitigate edge effects
- Form of density profile chosen so that transition mass m* is the same across the plasma [$m^* = \text{const} \Rightarrow nL_n^2 = \text{const}$]



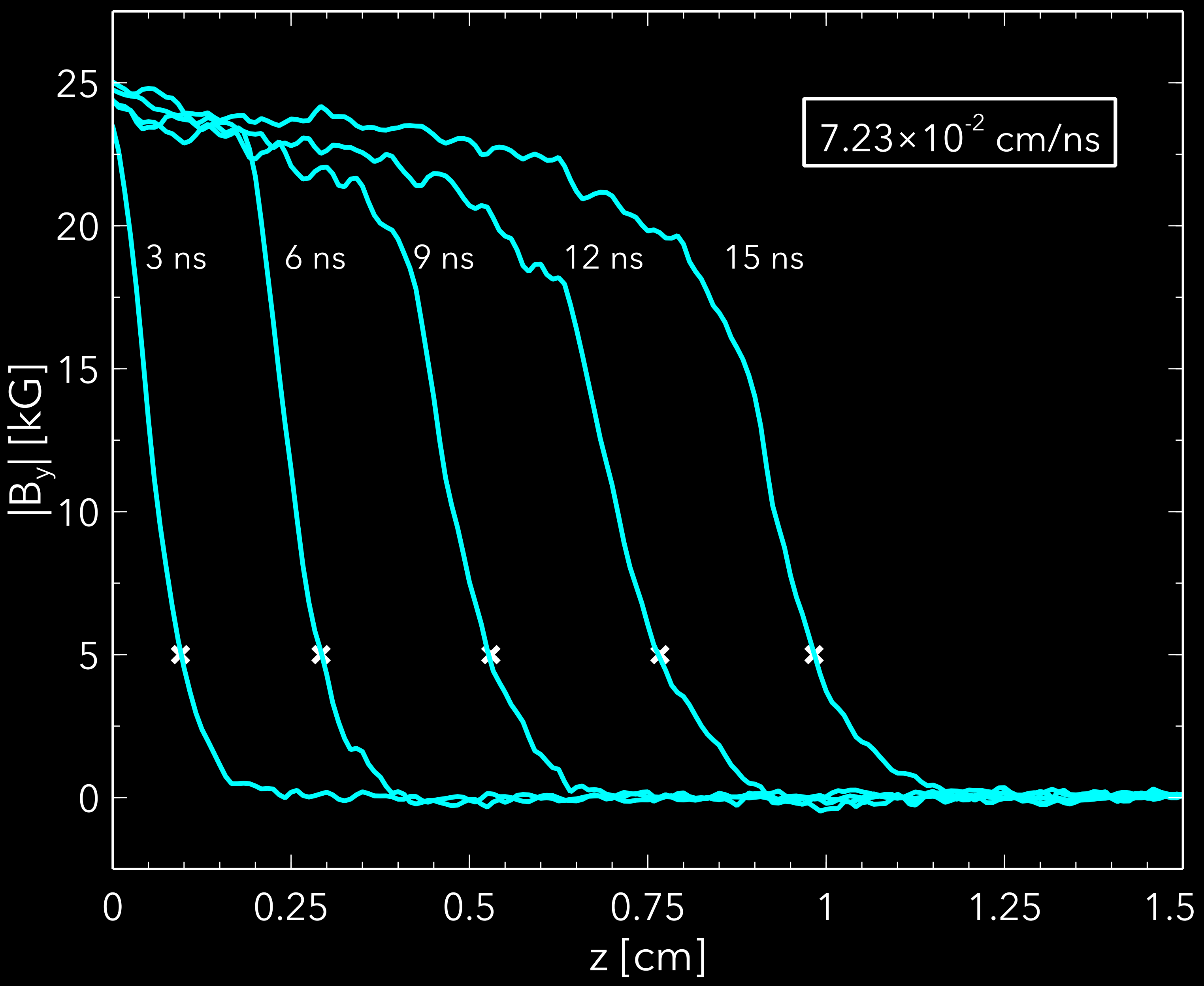
$$n_e(x) = 1/(a\,(x-x_0)/2 + n_0^{-1/2})^2$$

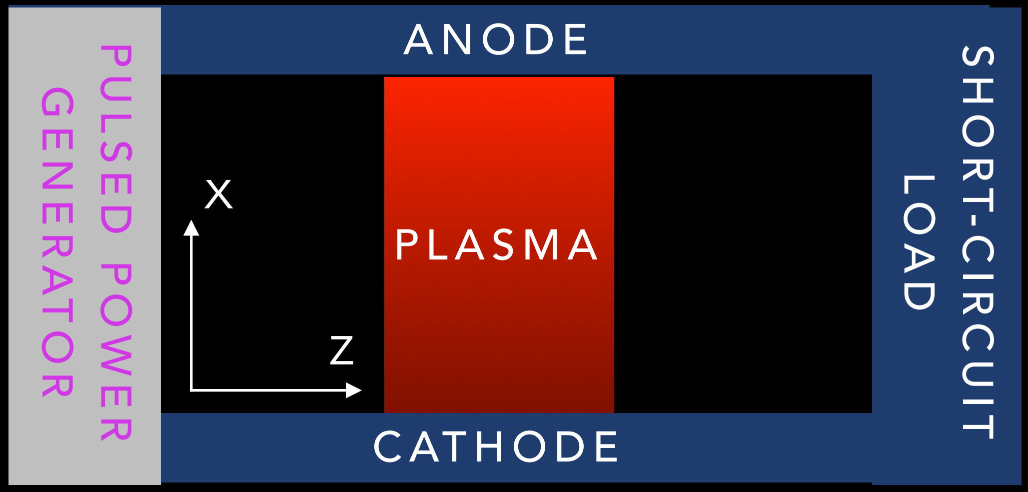
$$a=-1.7625 \times 10^{-8}\ \text{cm}^{1/2}$$



FIXED ION SIMULATION

MAGNETIC FIELD PENETRATION



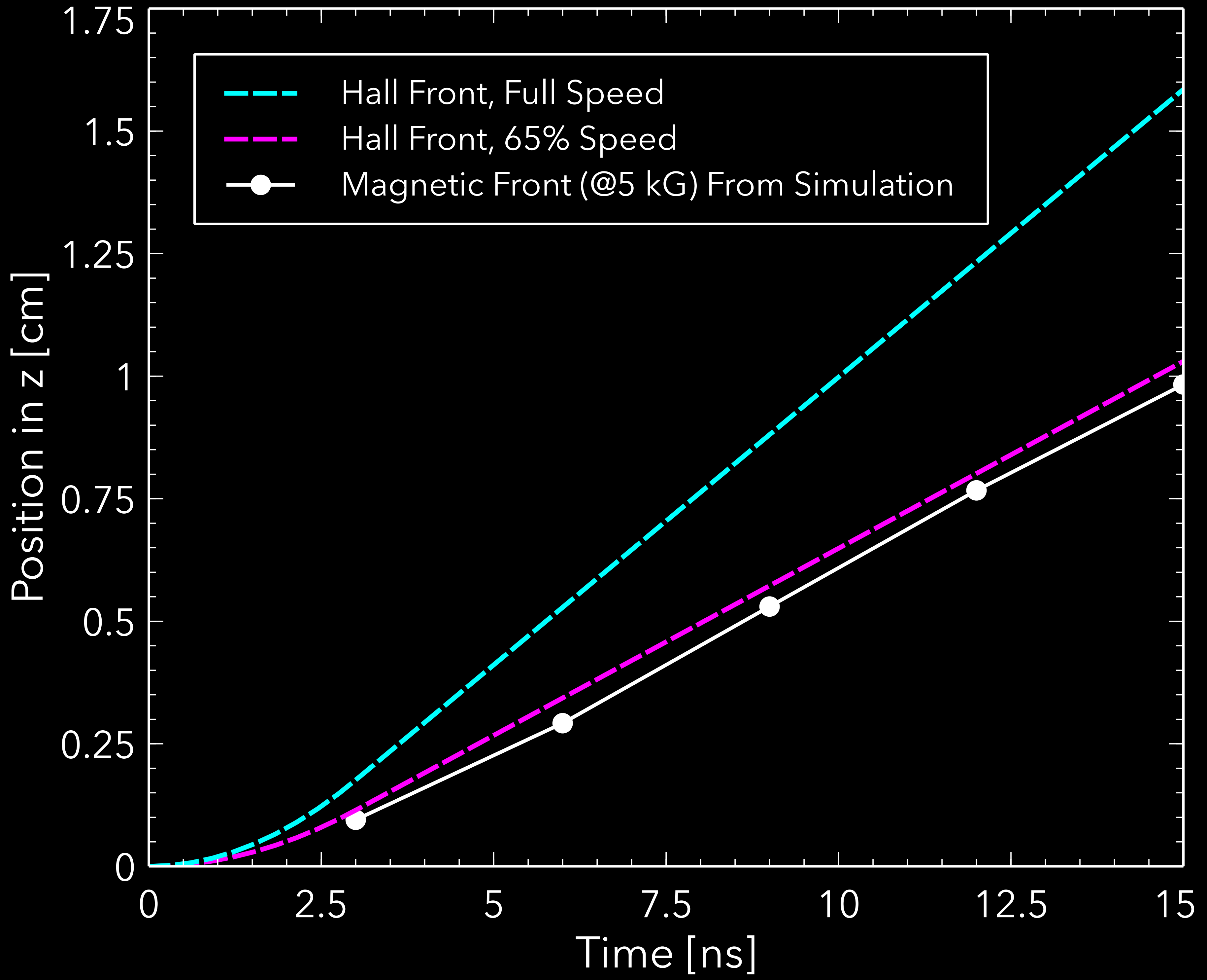


Field line-outs taken midway between anode and cathode, at x = 2.5 cm

Plasma located at z = 0 to z = 3 cm

FIELD PENETRATION AT X=2.5 CM

FIXED ION SIMULATION



This simulation shows that $v_b \approx 0.65\ v_{Hall}$ $= 76.32$ cm/μs

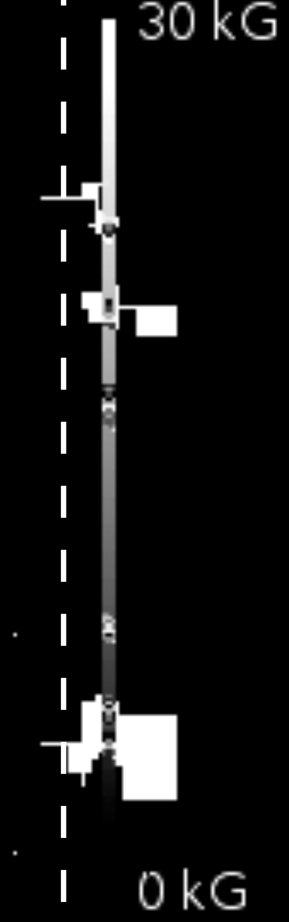
$$v_H = \frac{cB}{4\pi n e L_n}$$

$$v_b = \alpha v_H$$

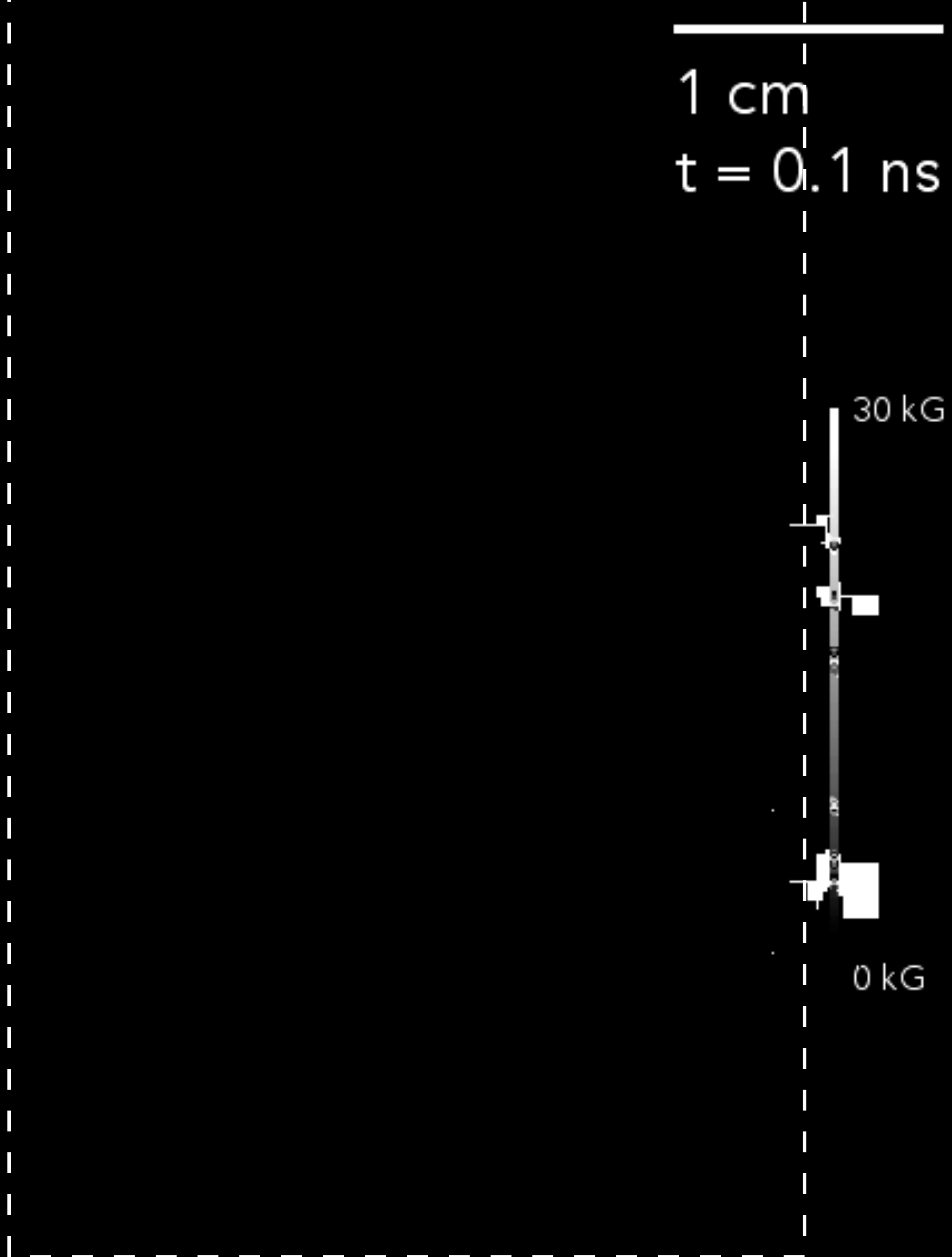
Magnetic field movie

1 cm

t = 0.1 ns

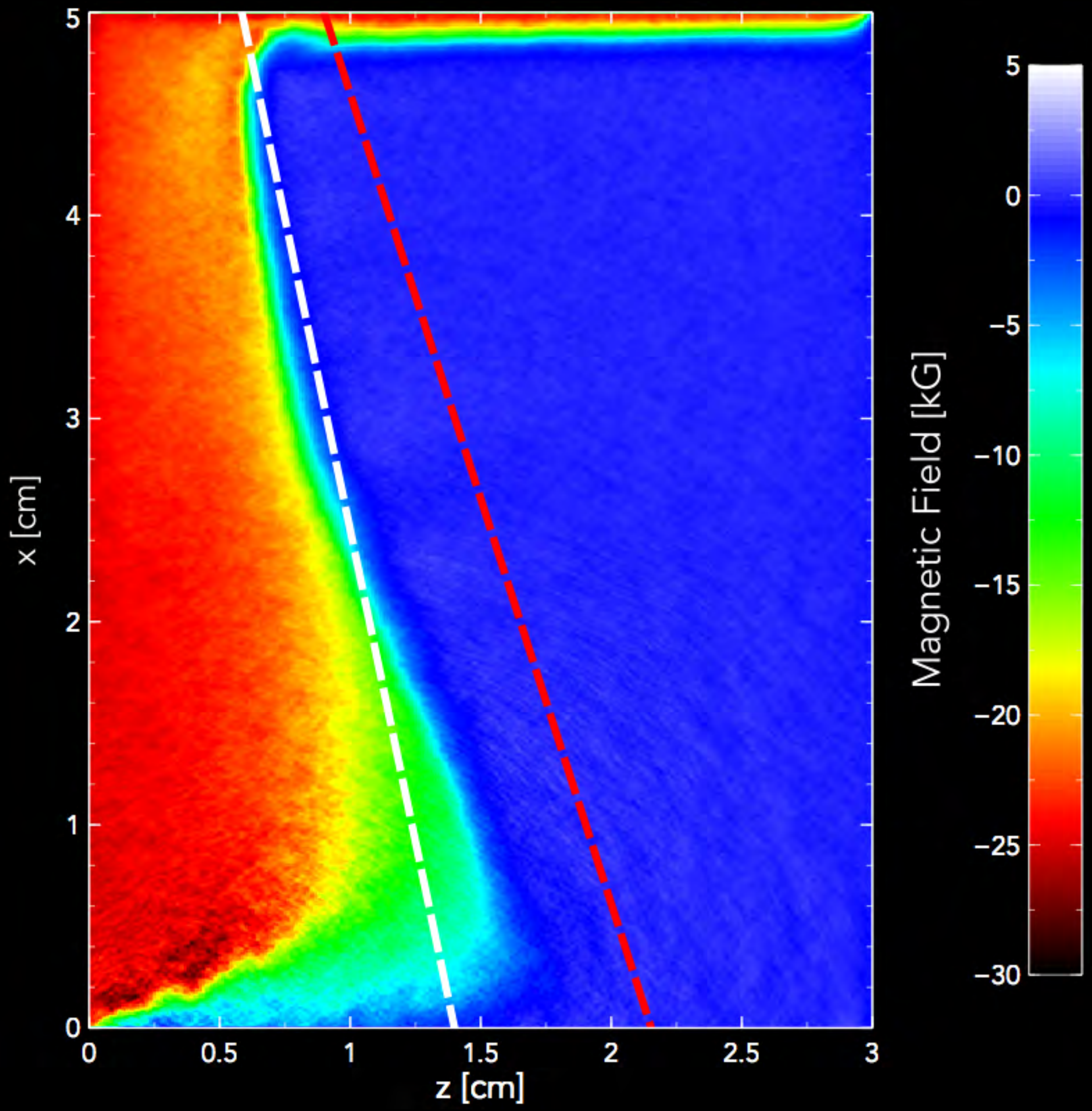


Magnetic field movie



MAGNETIC FIELD IN 2D, AT 15 NS

FIXED ION SIMULATION



predicted location of front moving at 100% Hall Speed (α=1)

predicted location of front moving at 65% Hall Speed (α=0.65)

$$v_H = \frac{cB}{4\pi n e L_n}$$

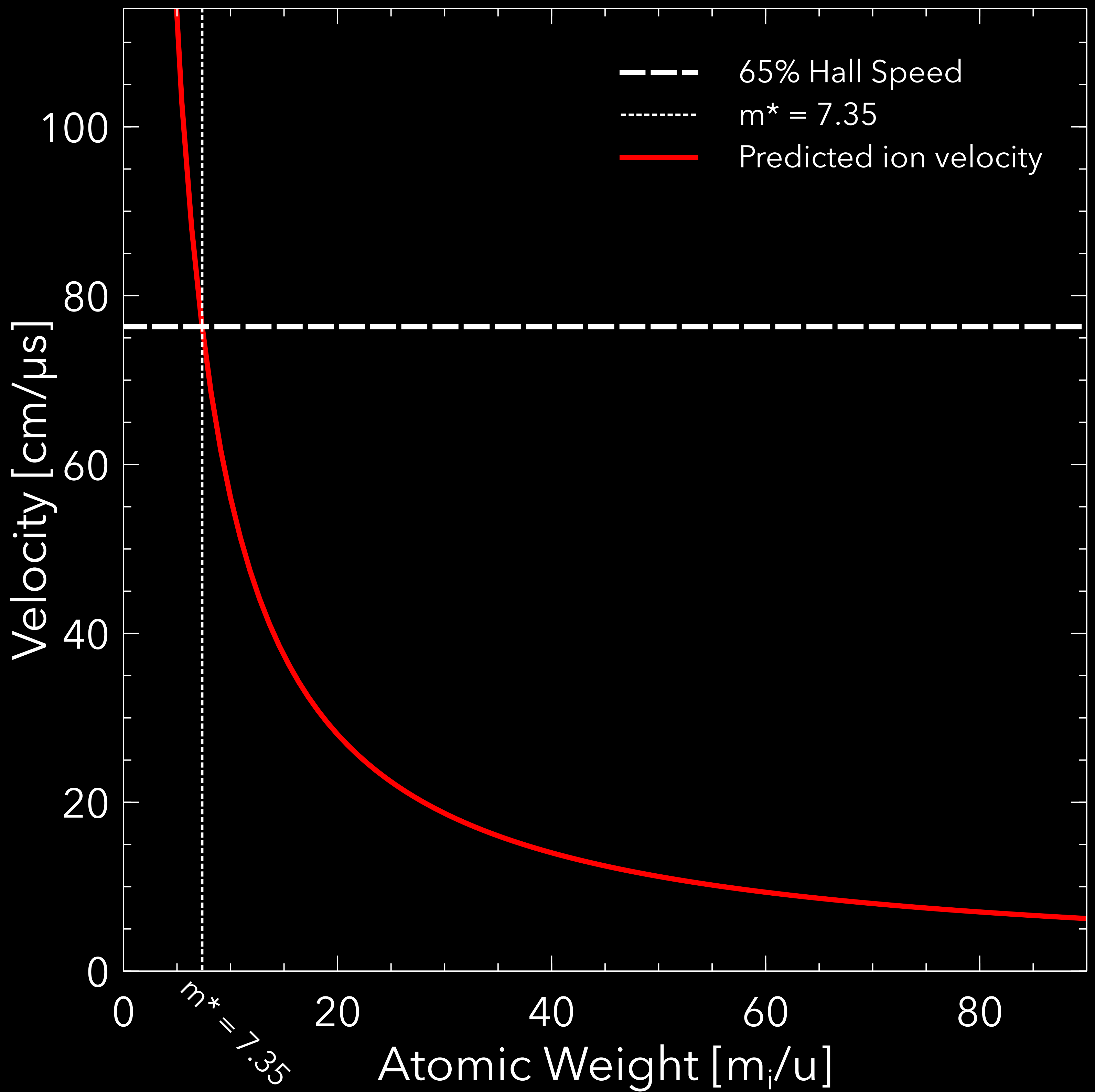
$$v_b = \alpha v_H$$

FINAL ION VELOCITY: THEORY

$$v_H = \frac{cB}{4\pi n e L_n}$$

$$v_b = \alpha v_H$$

$$v_{final} = \frac{B^2}{8\pi n m_i v_b}$$

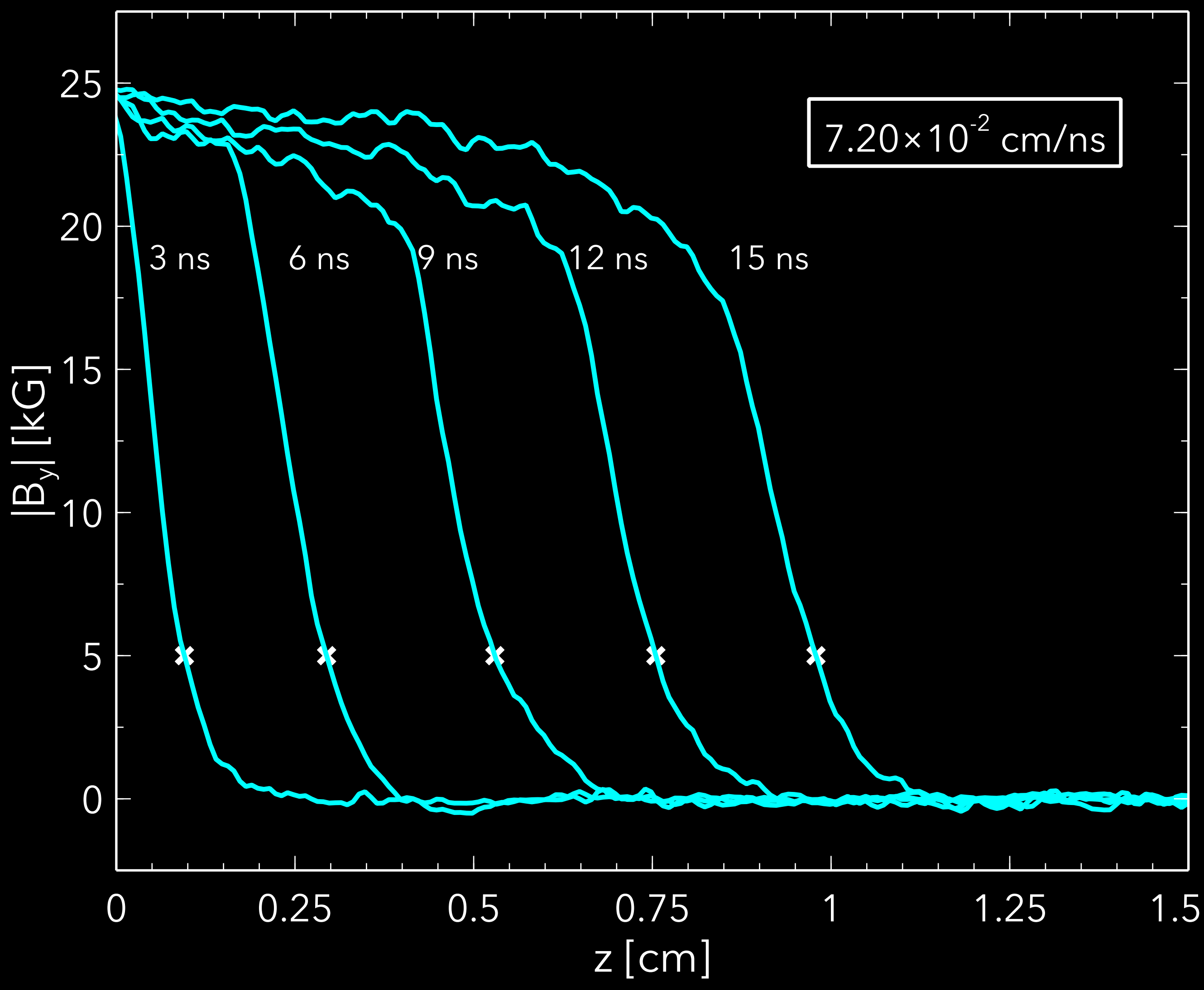


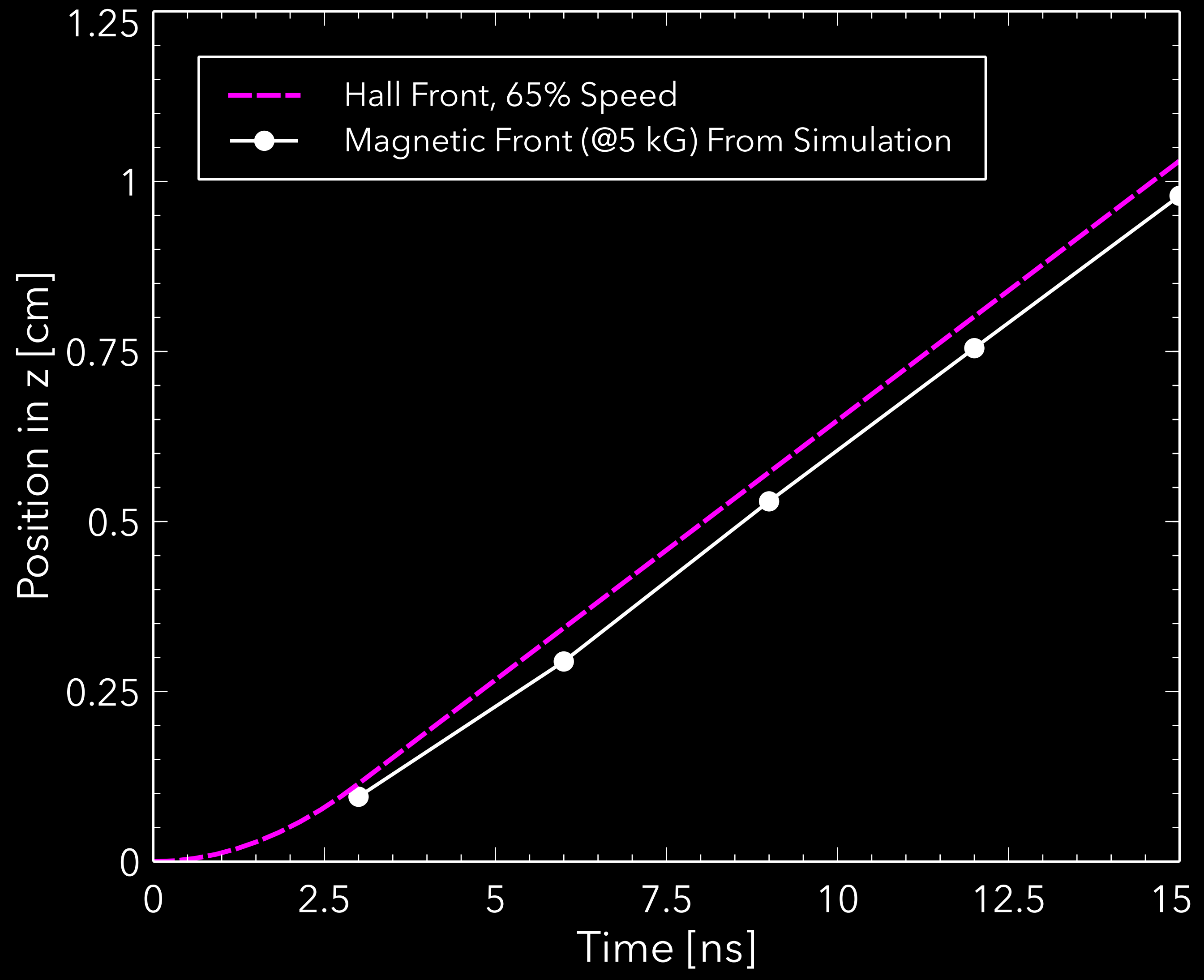
SIMULATION 2

80% FIXED IONS
20% KR IONS

FIELD PENETRATION AT X=2.5 CM

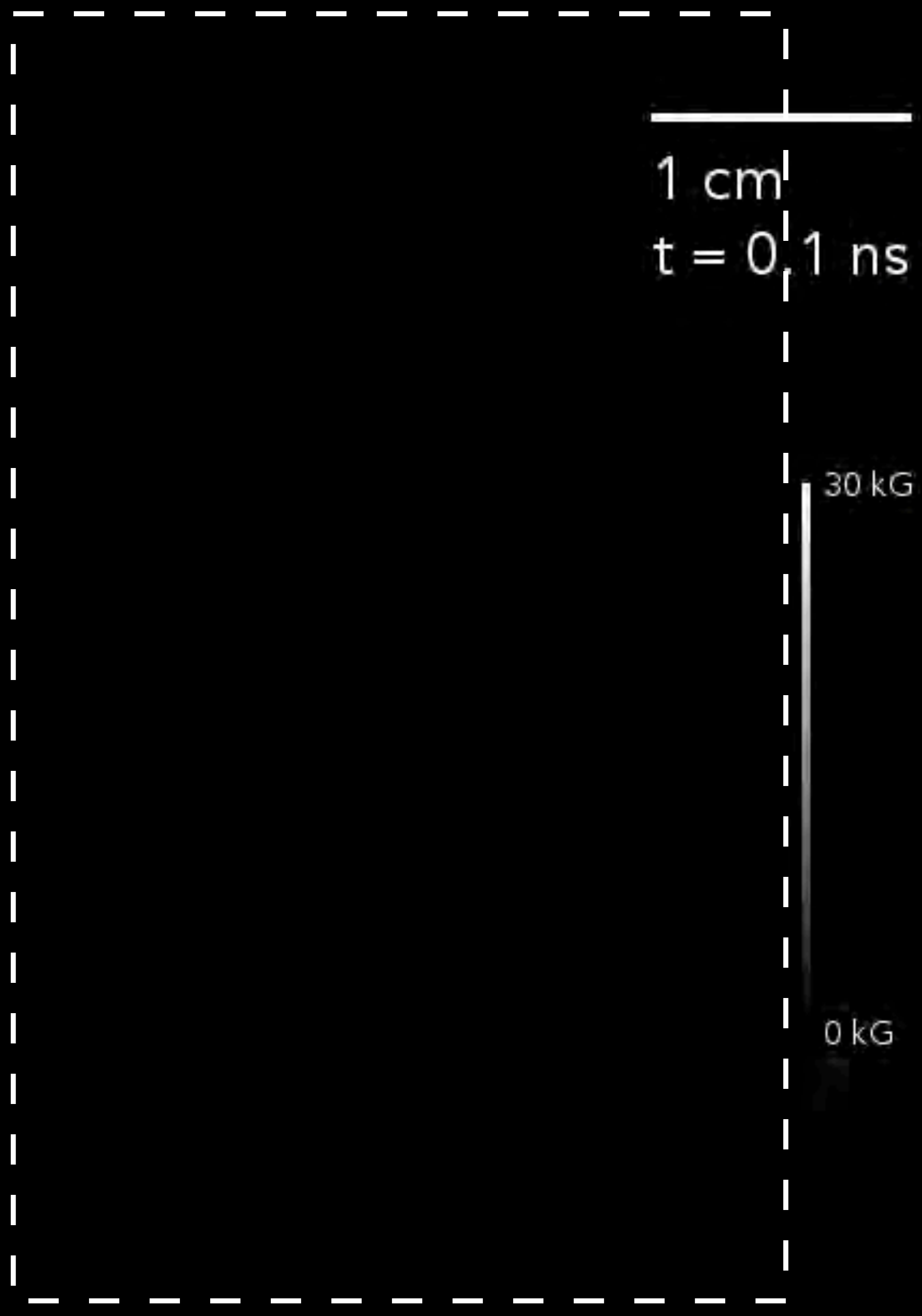
20% KR IONS SIMULATION



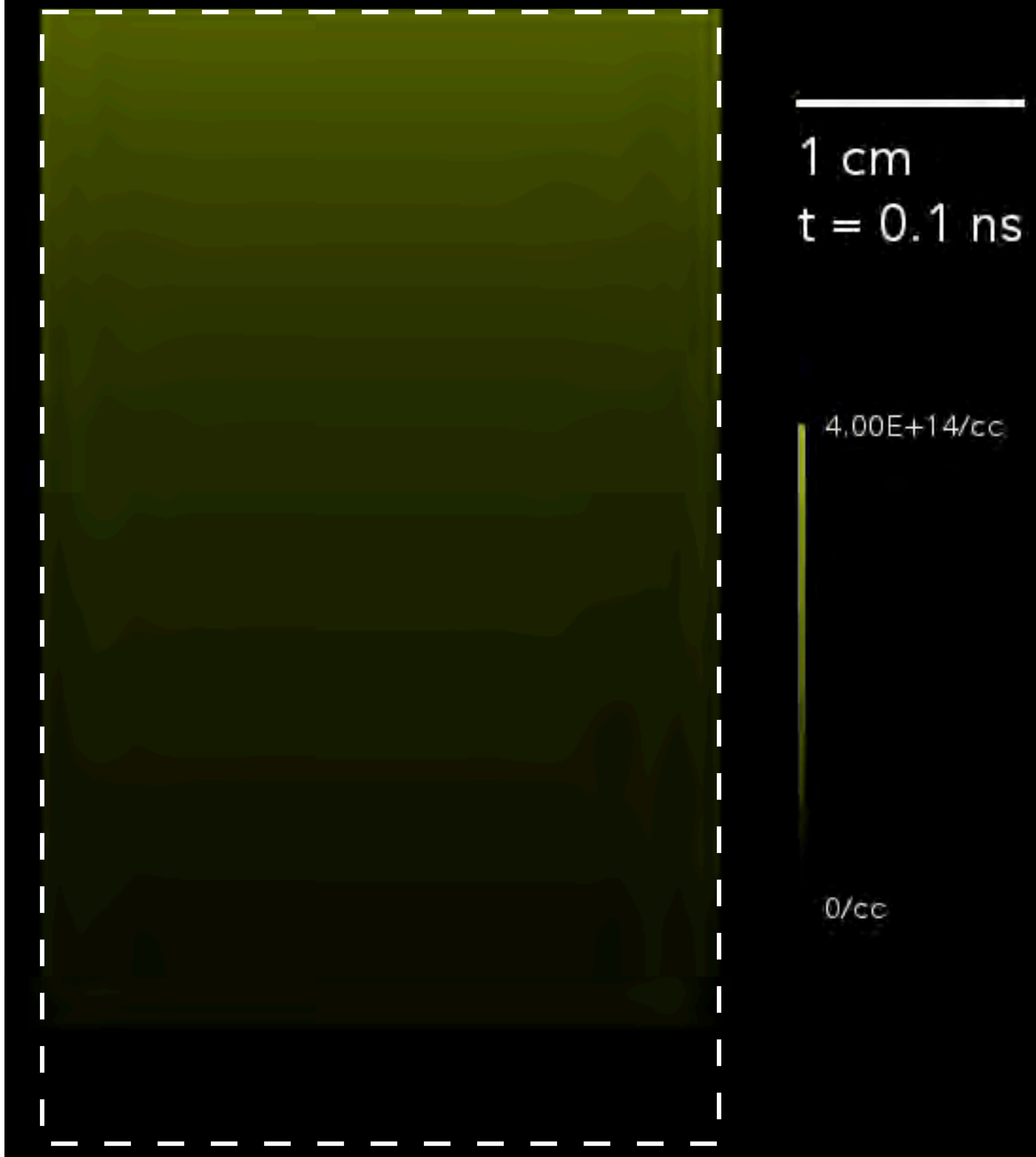


20% KR IONS SIMULATION: MOVIE

B Field

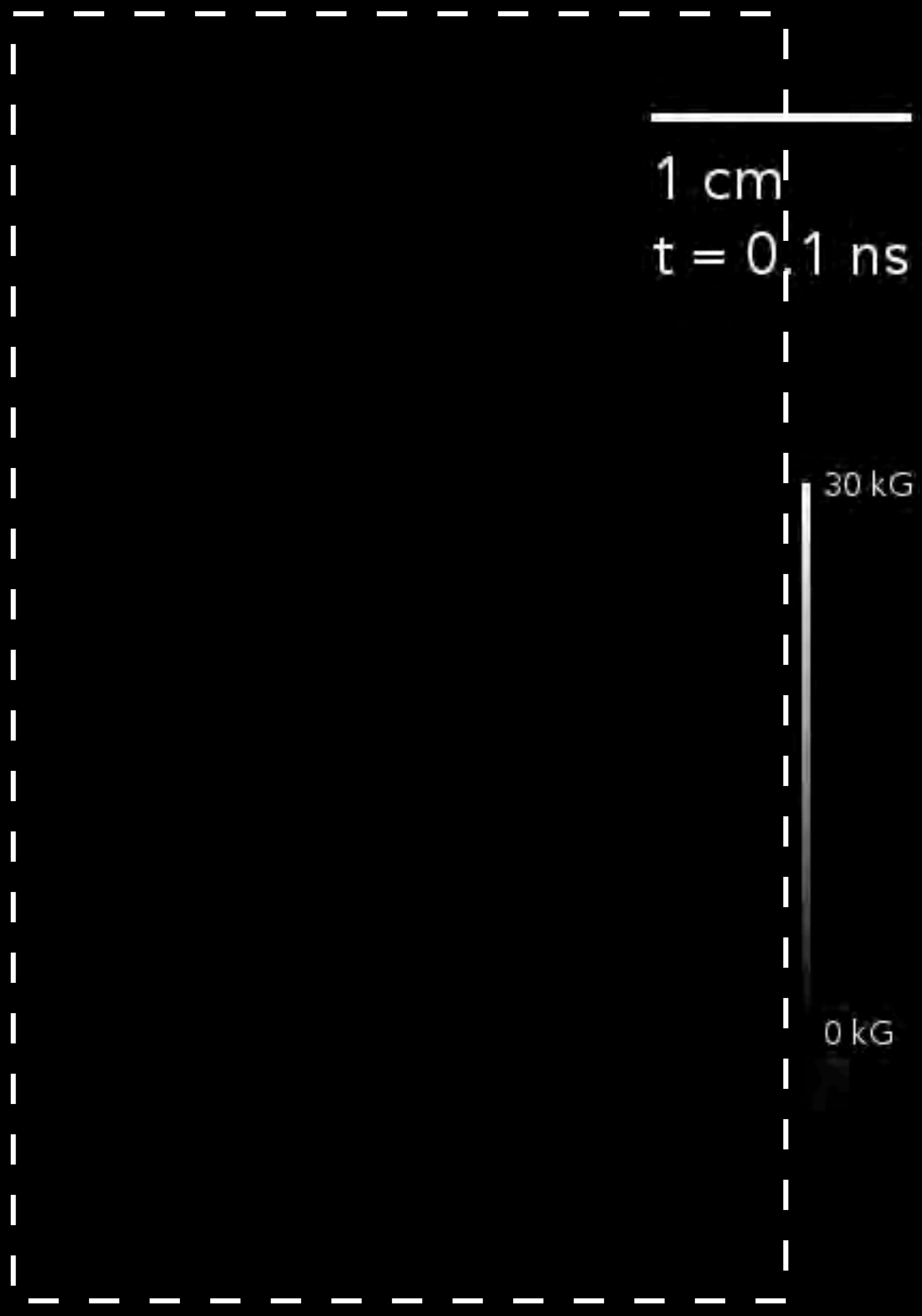


Light Ion Density

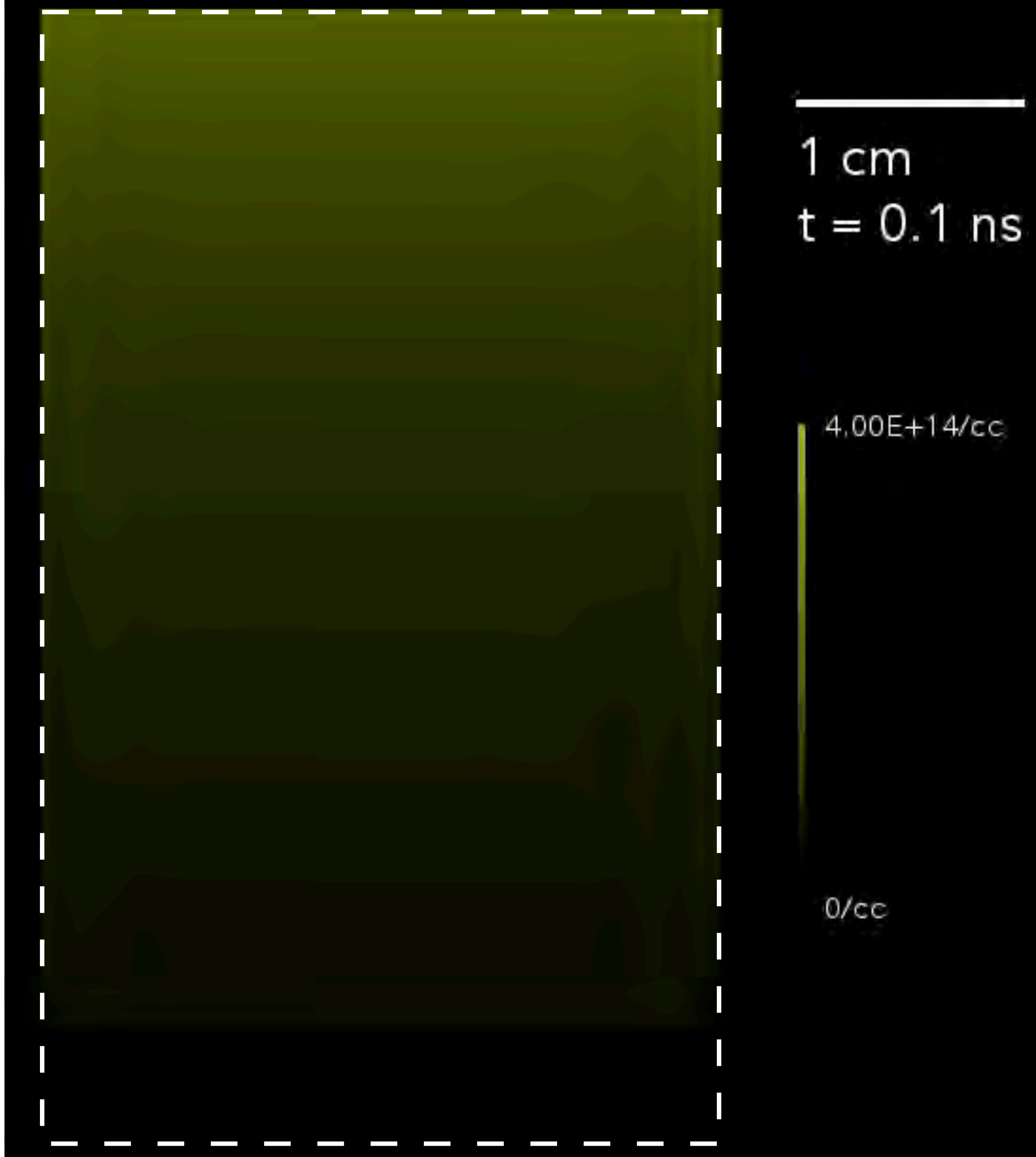


20% KR IONS SIMULATION: MOVIE

B Field



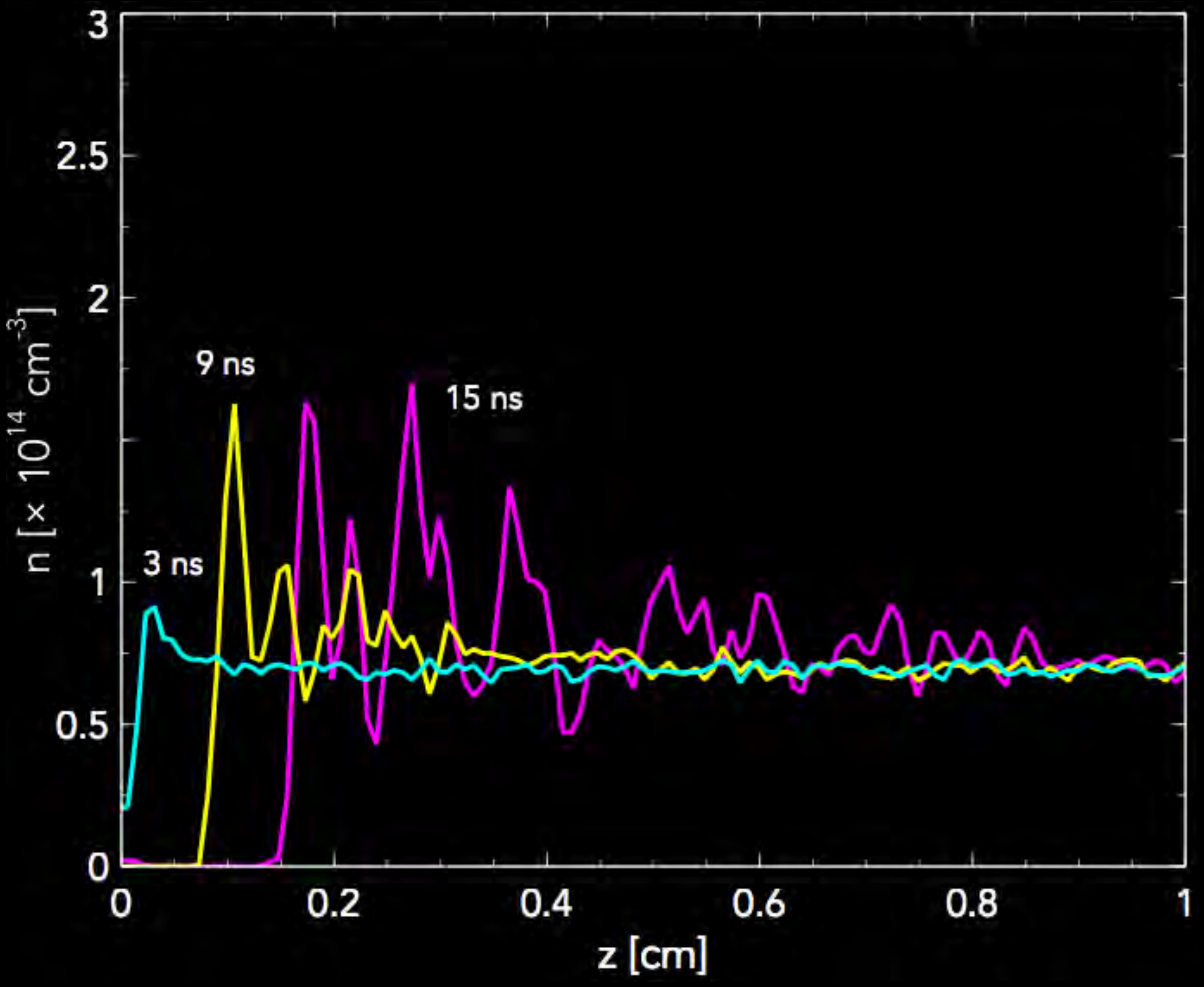
Light Ion Density



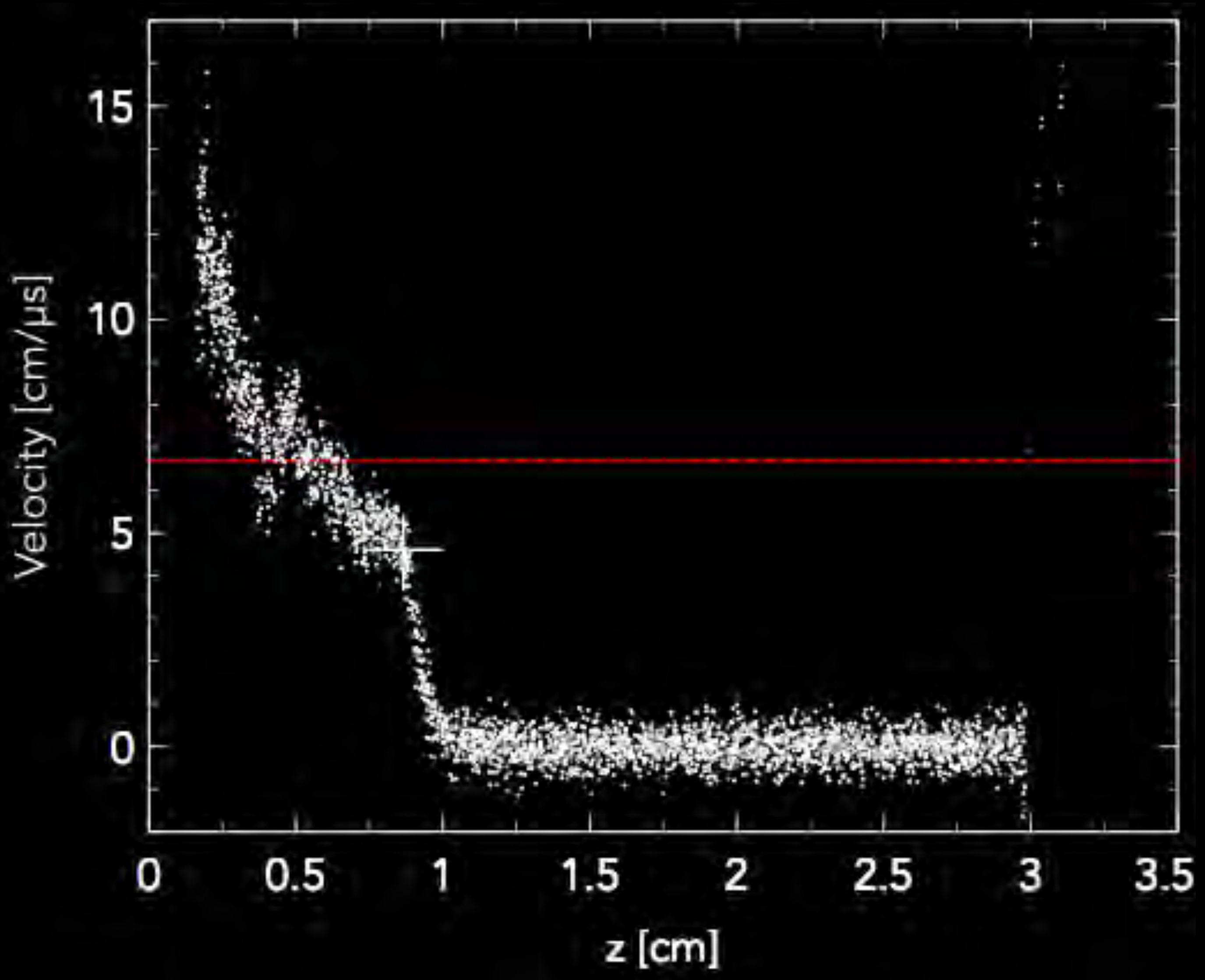
ION VELOCITY, X=2.4 TO 2.6 CM, 15 NS

20% KR IONS SIMULATION

Ion Density at x = 2.5 cm



Phase Space Particle Plot

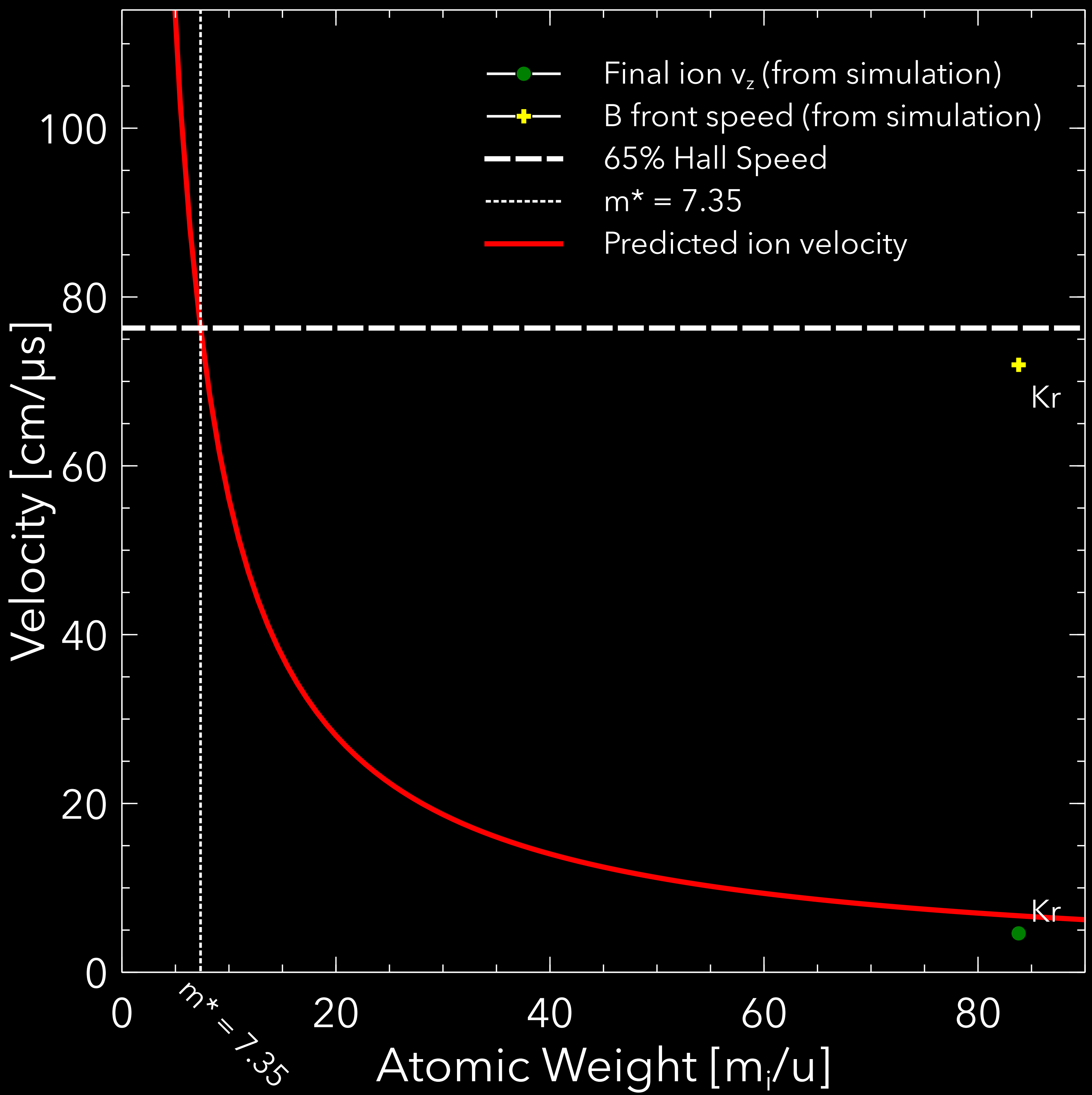


FINAL ION VELOCITY: SIMULATIONS

$$v_H = \frac{cB}{4\pi n e L_n}$$

$$v_b = \alpha v_H$$

$$v_{final} = \frac{B^2}{8\pi n m_i v_b}$$

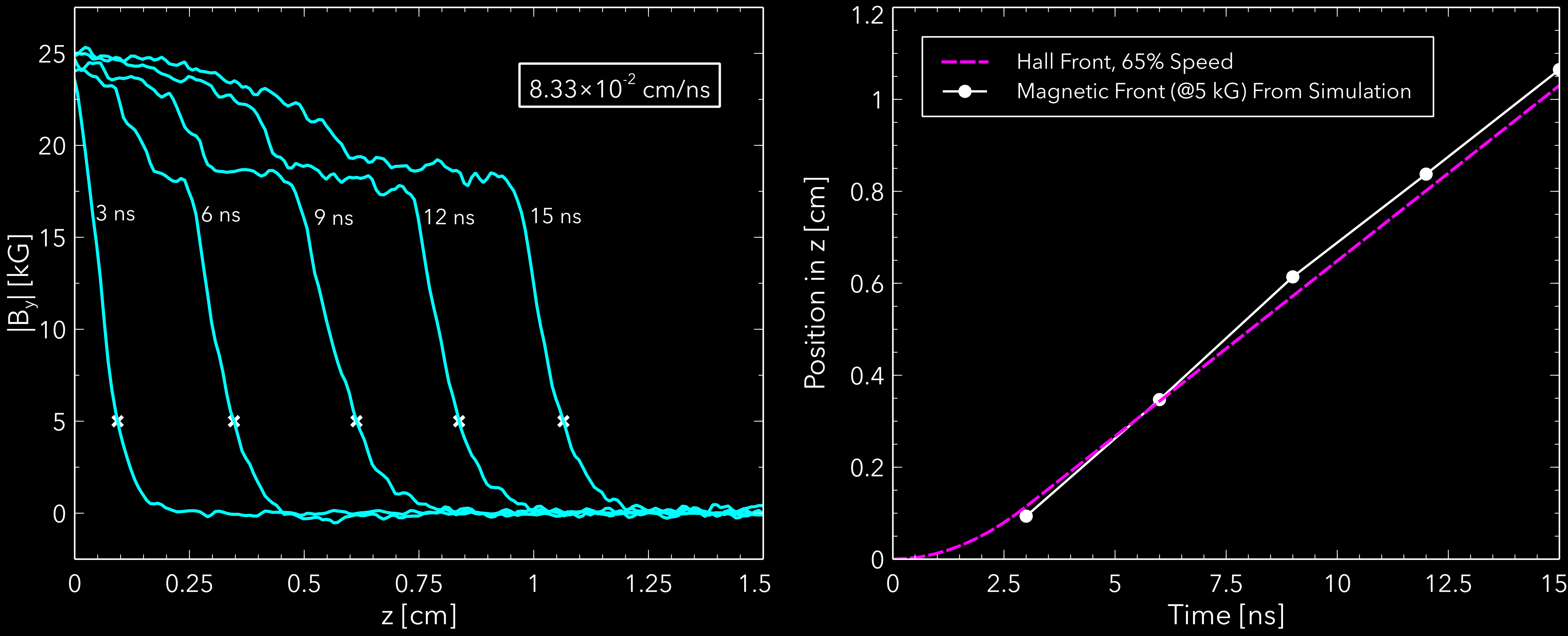


SIMULATION 3

80% FIXED IONS
20% B IONS

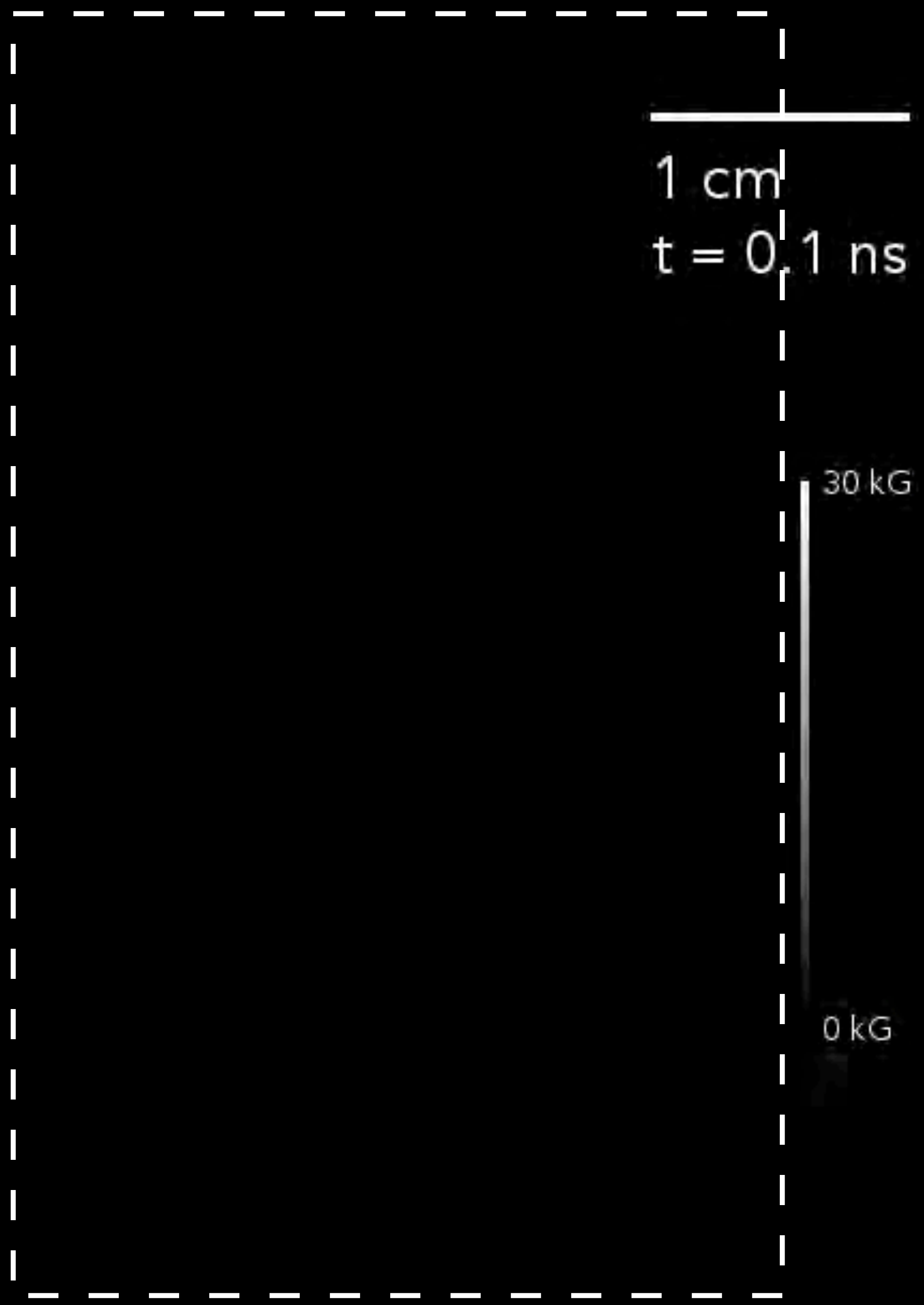
FIELD PENETRATION AT X=2.5 CM

20% B IONS SIMULATION

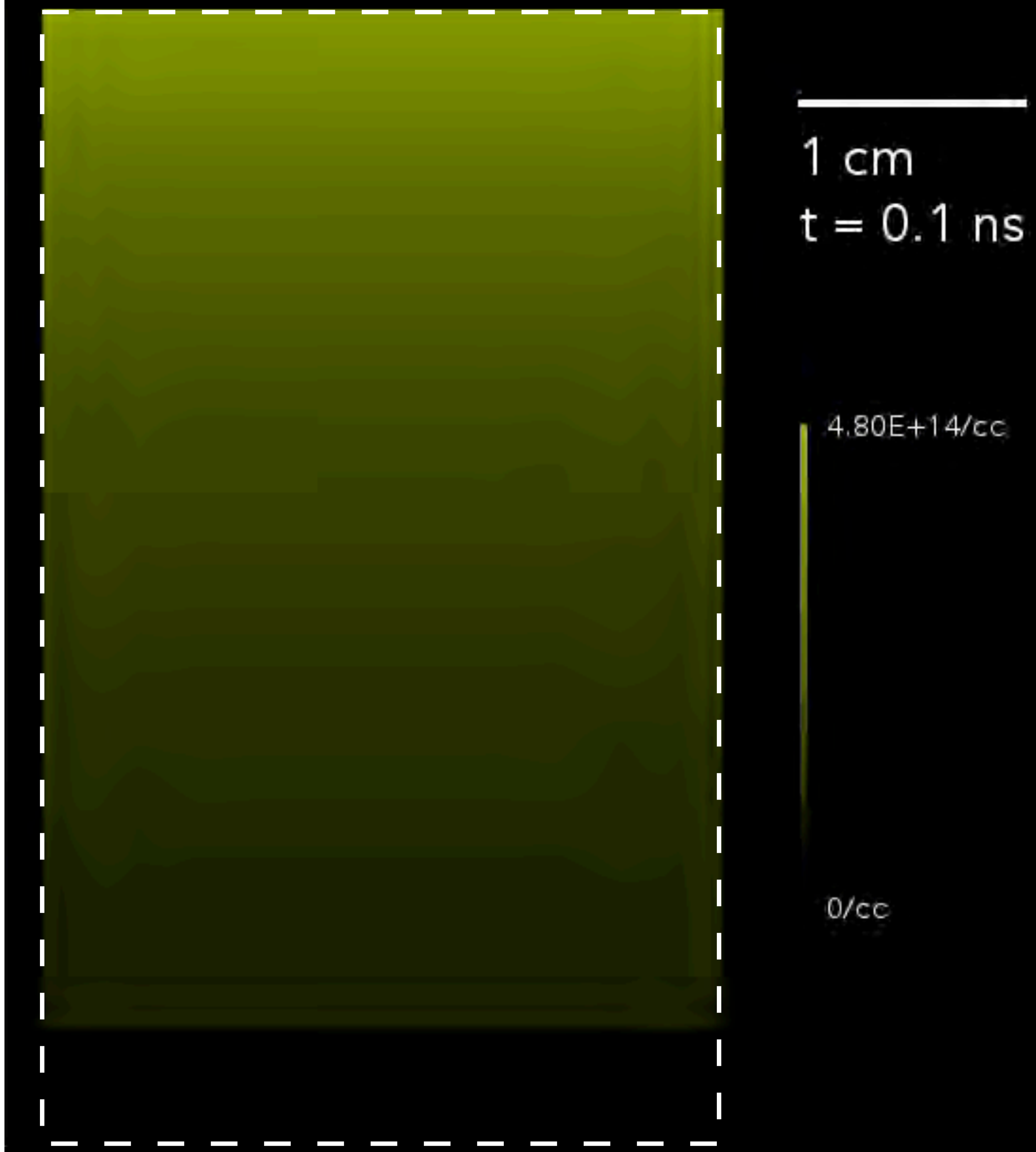


20% B IONS SIMULATION: MOVIE

B Field

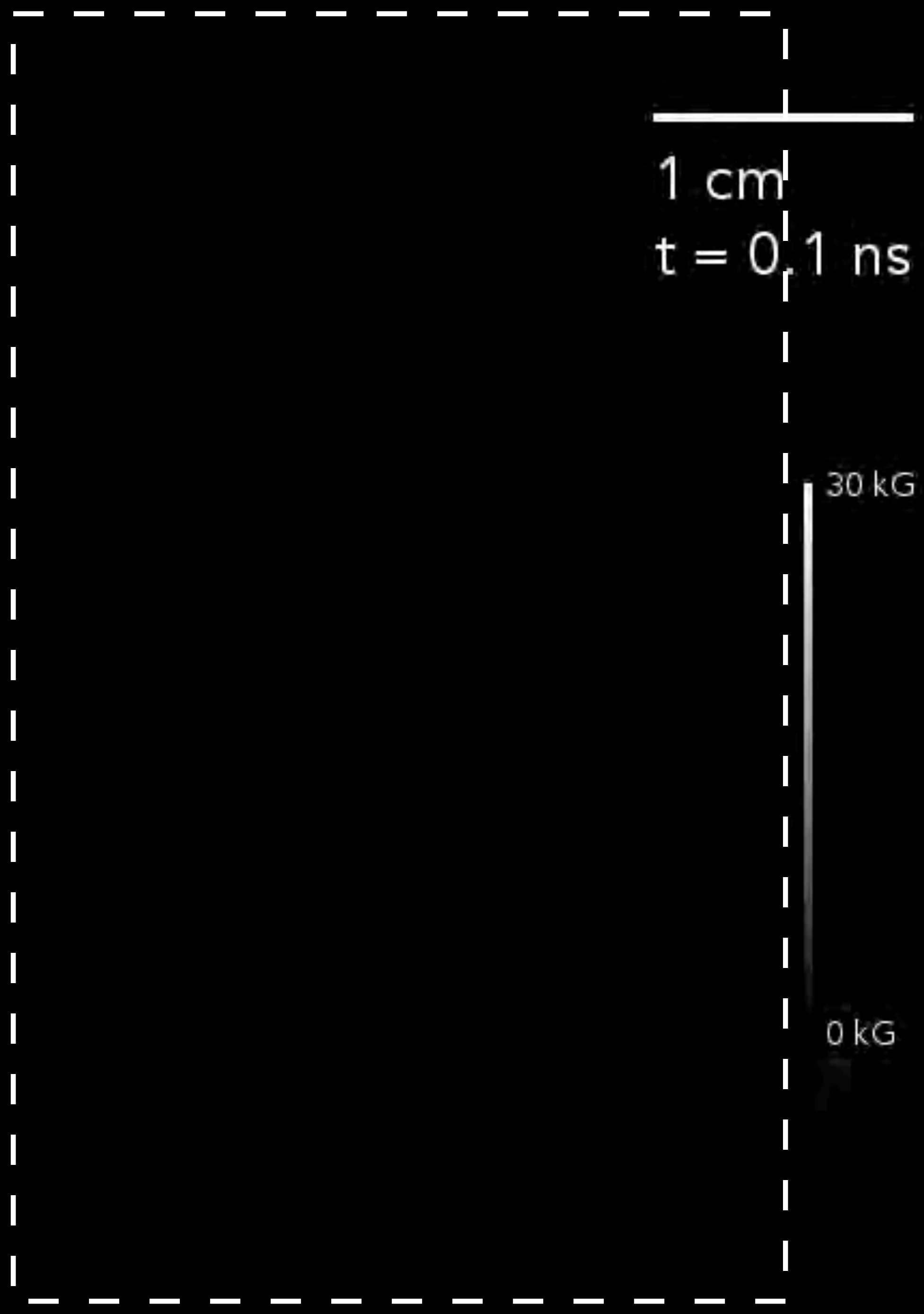


Light Ion Density

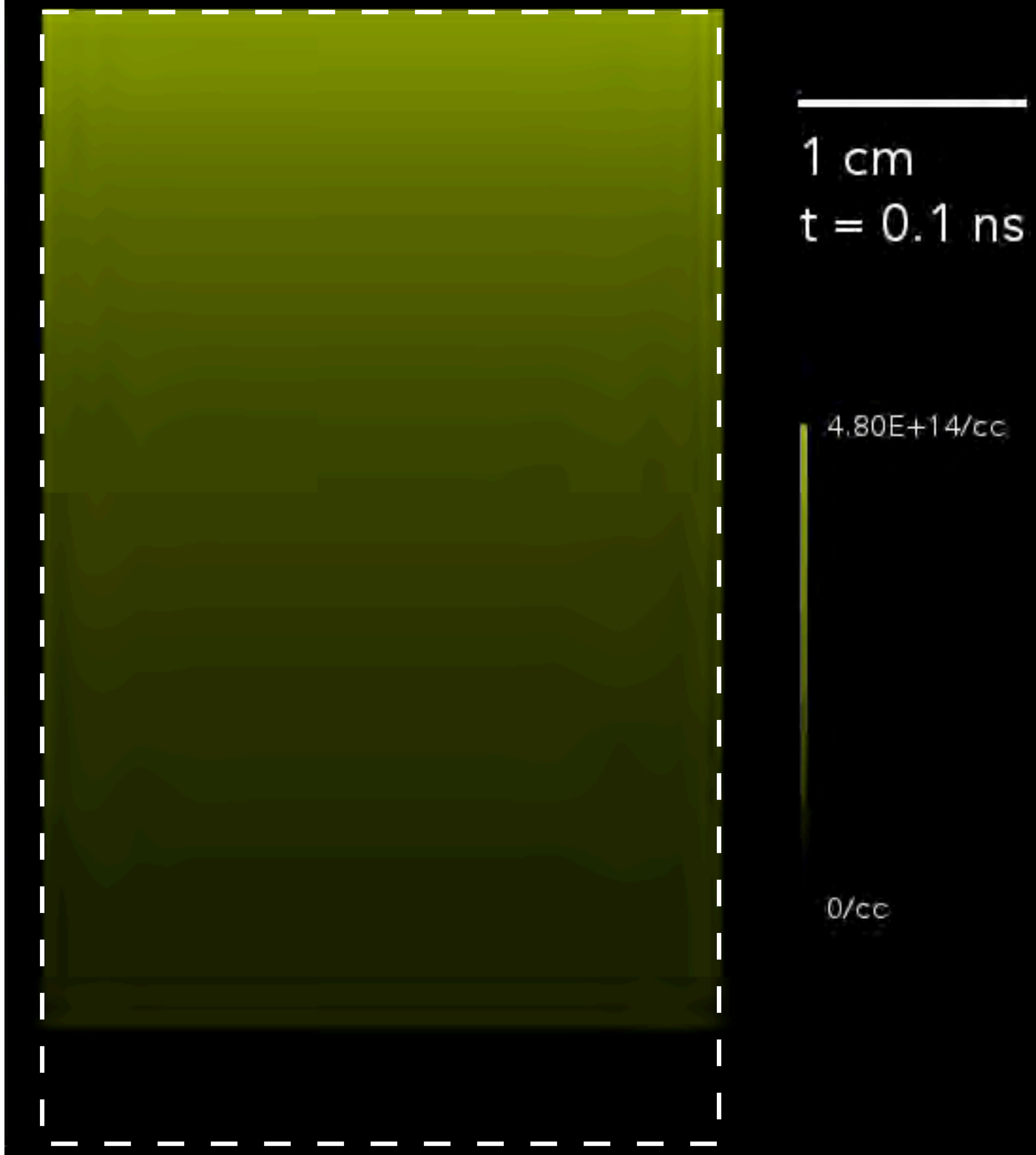


20% B IONS SIMULATION: MOVIE

B Field



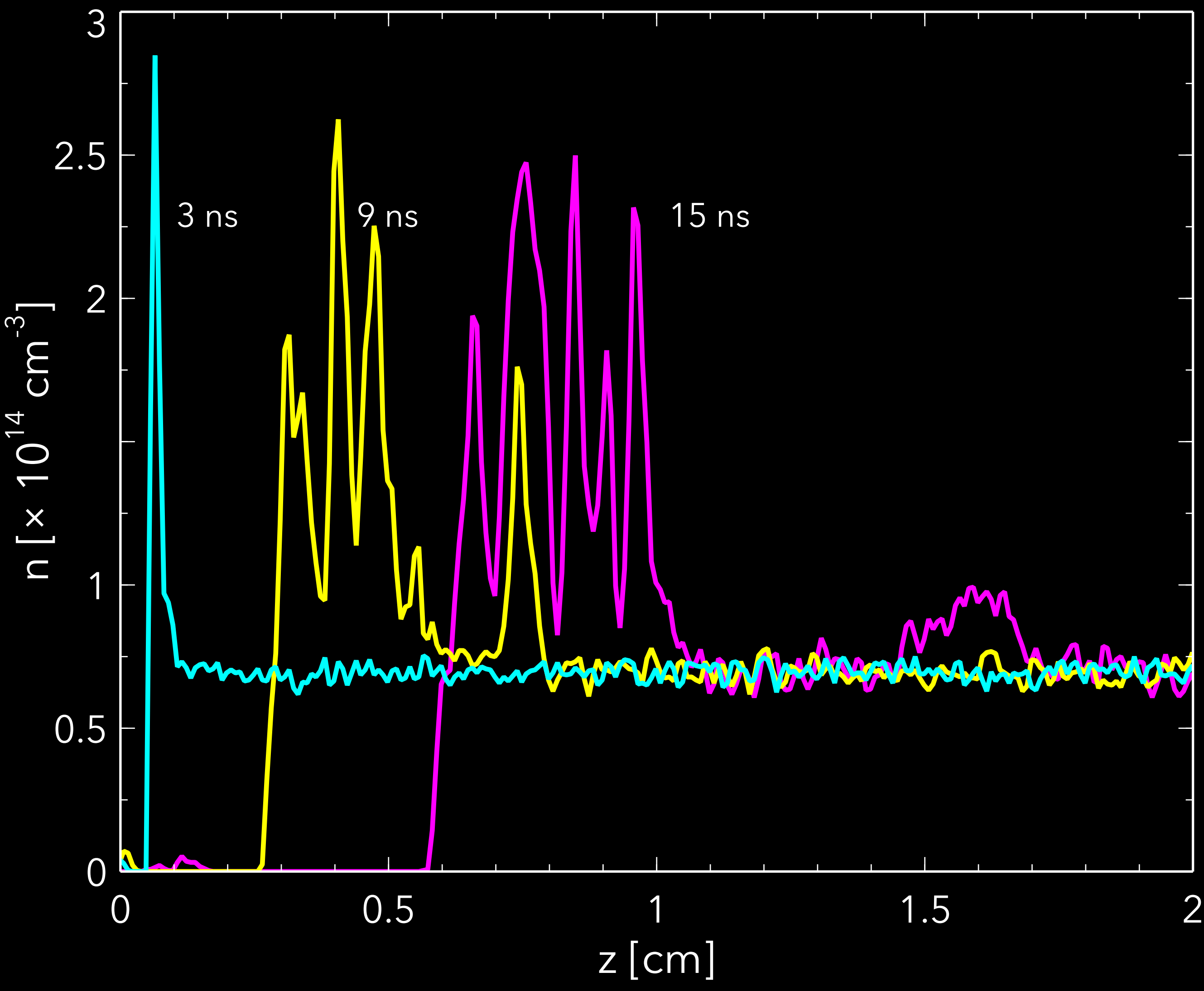
Light Ion Density



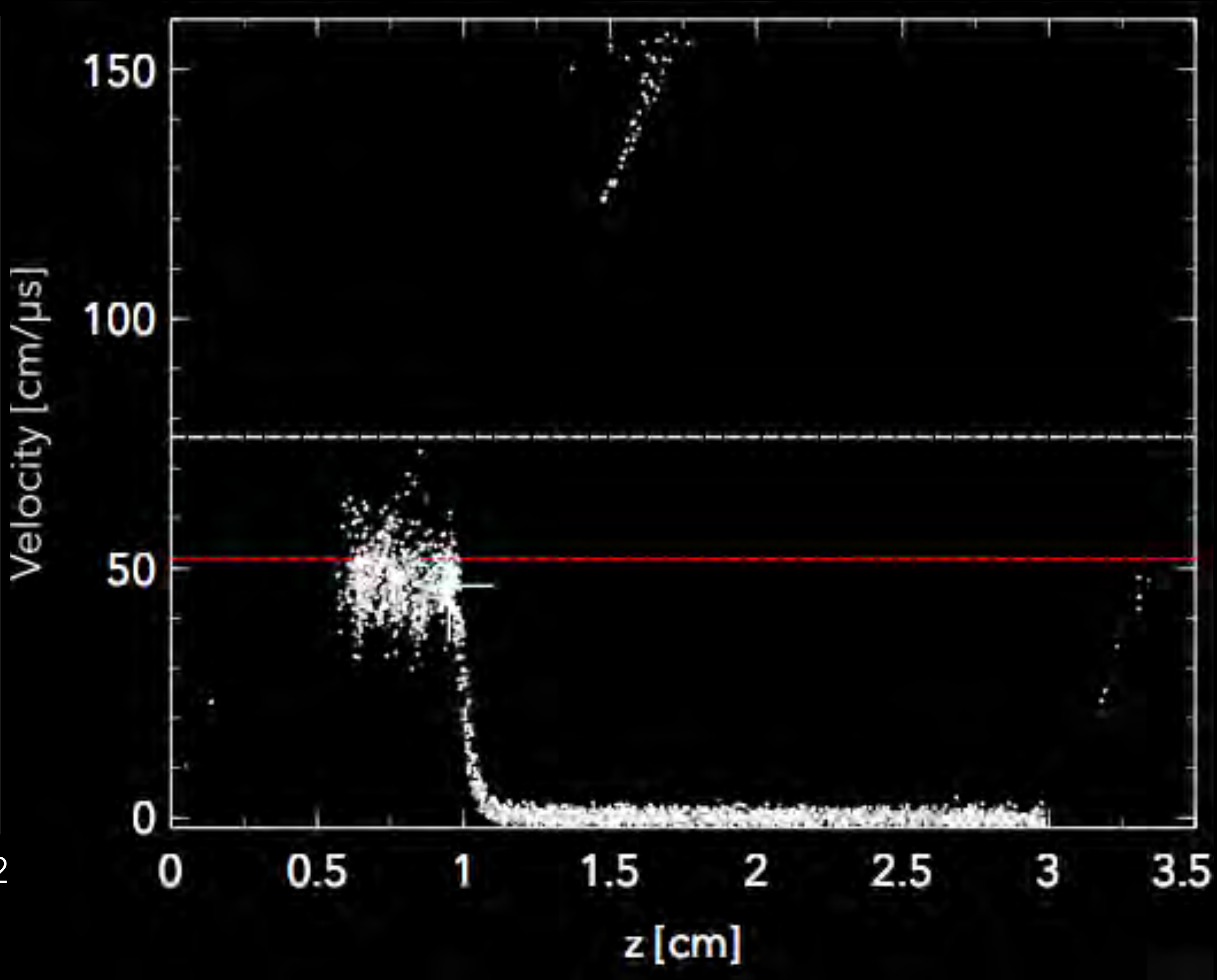
ION VELOCITY, X=2.4 TO 2.6 CM, 15 NS

20% B IONS SIMULATION

Ion Density at x = 2.5 cm



Phase Space Particle Plot

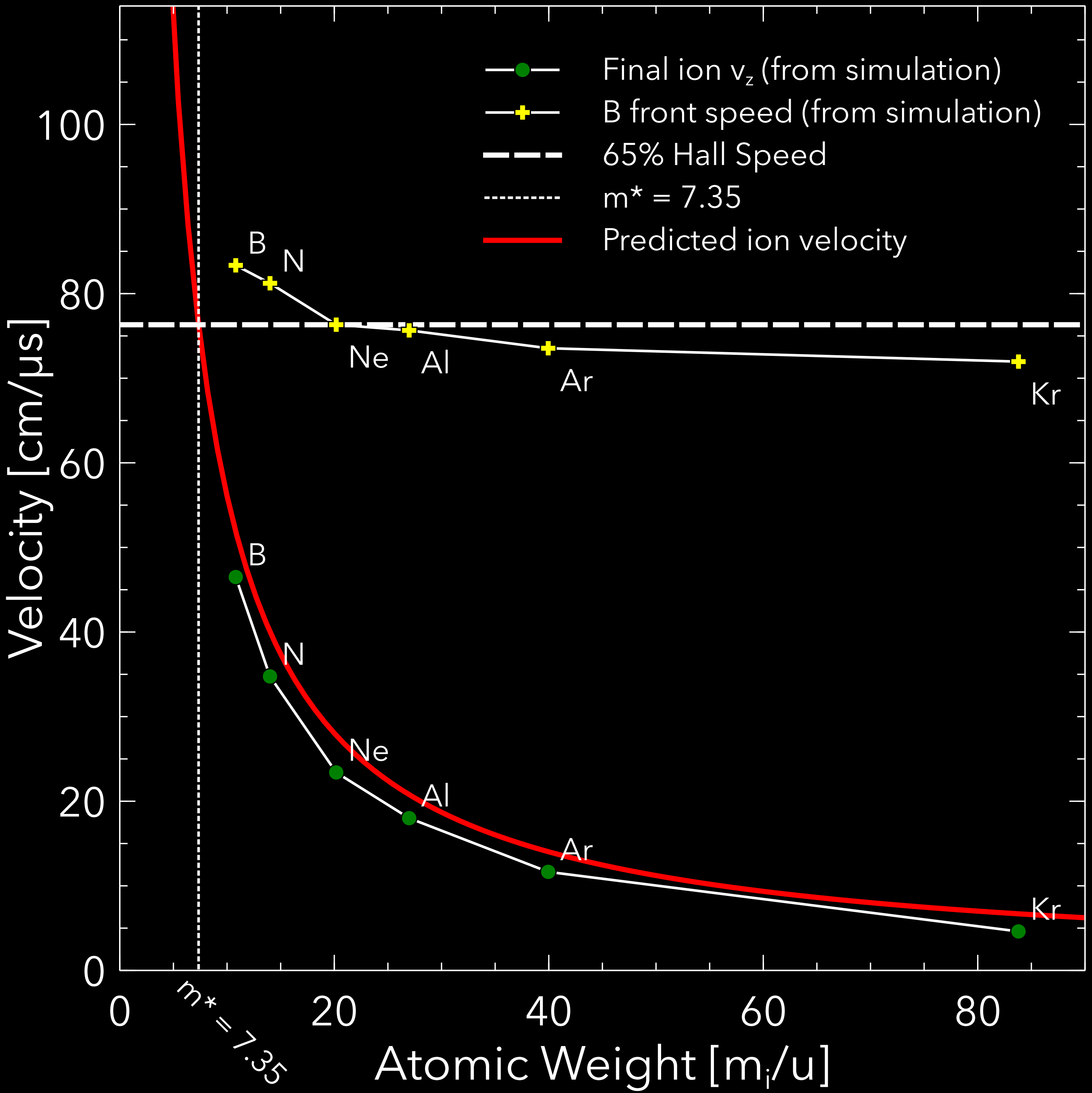


FINAL ION VELOCITY: SIMULATIONS

$$v_H = \frac{cB}{4\pi n e L_n}$$

$$v_b = \alpha v_H$$

$$v_{final} = \frac{B^2}{8\pi n m_i v_b}$$

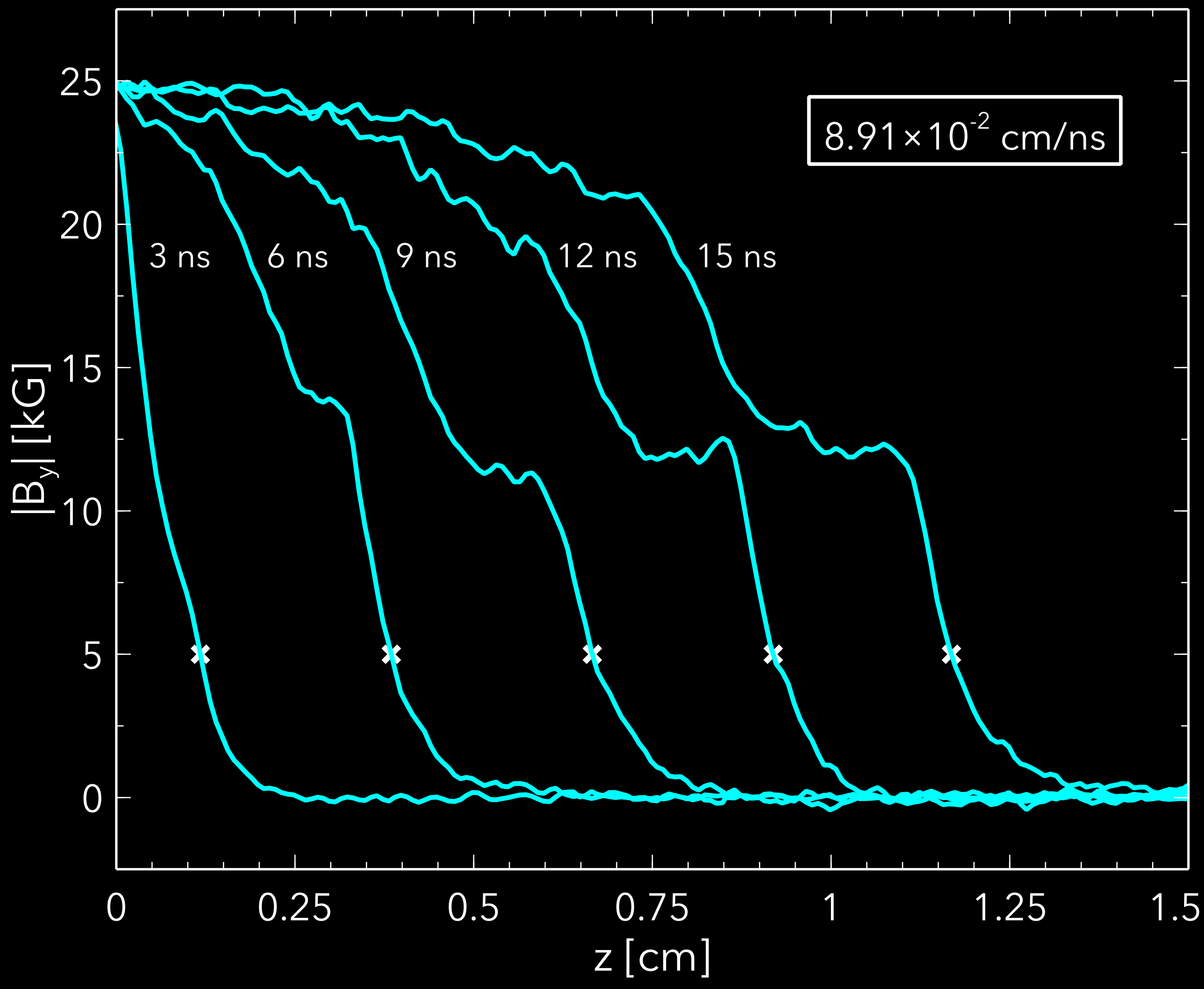


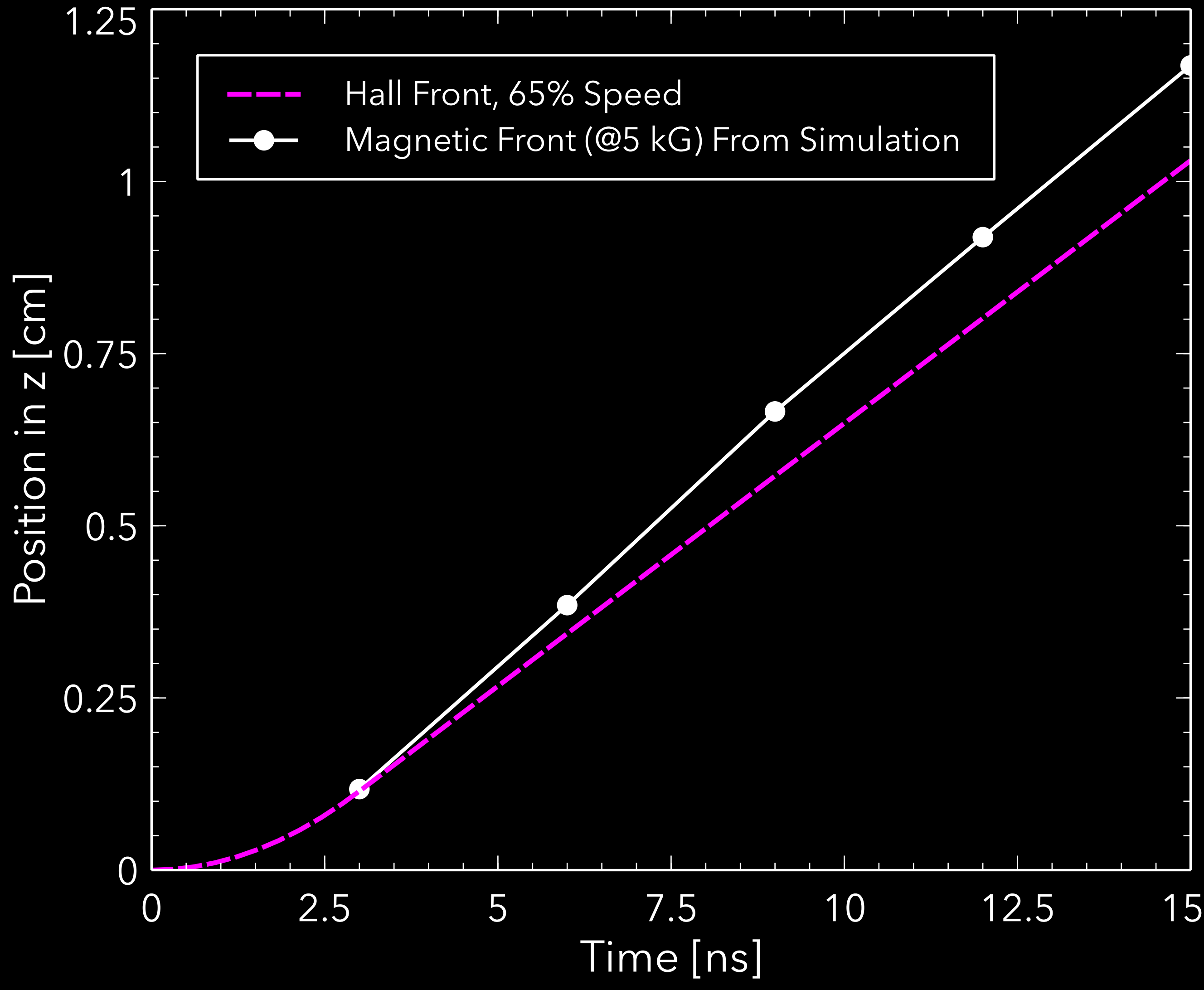
SIMULATION 4

80% FIXED IONS
20% HE IONS

FIELD PENETRATION AT X=2.5 CM

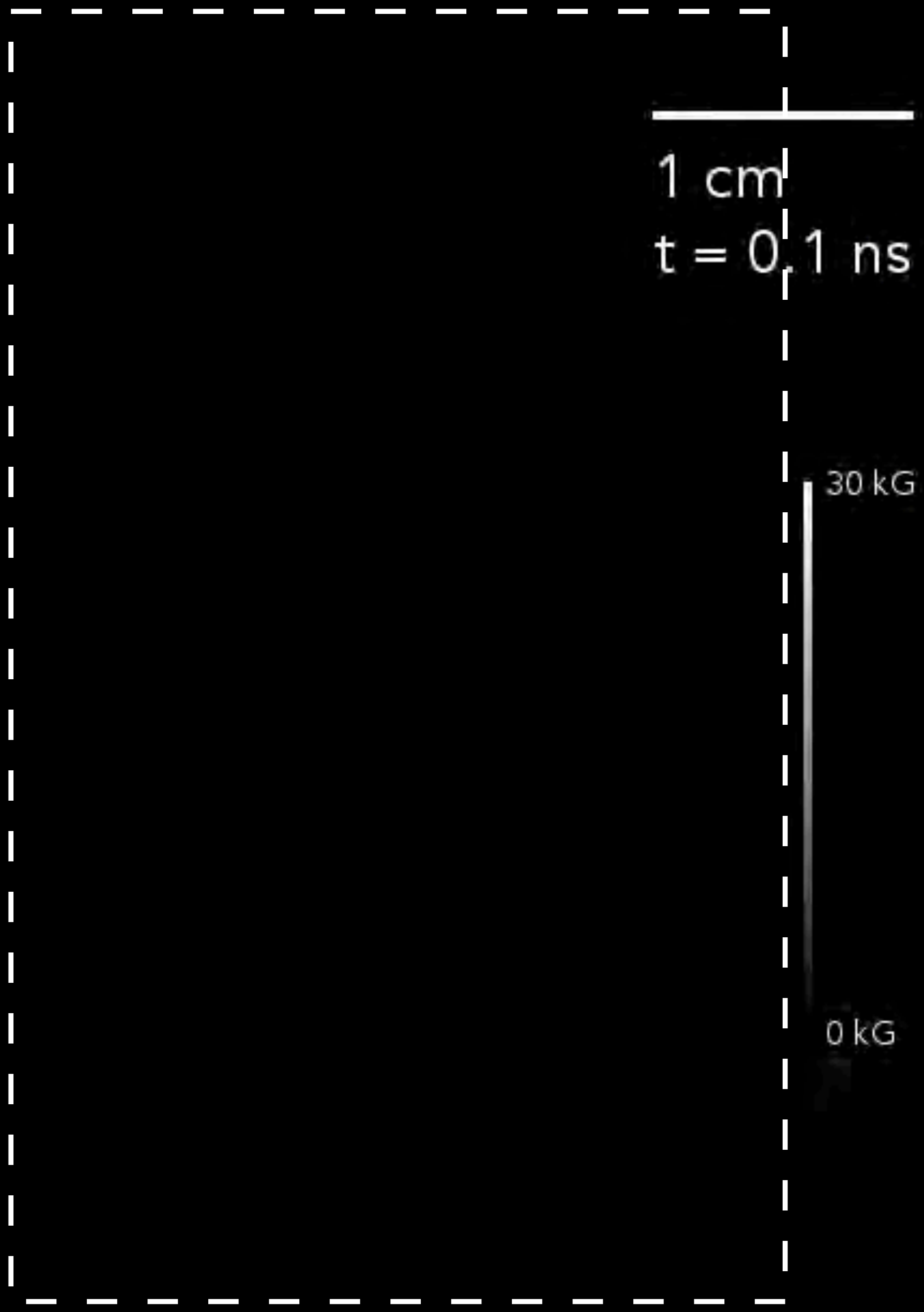
20% HE IONS SIMULATION



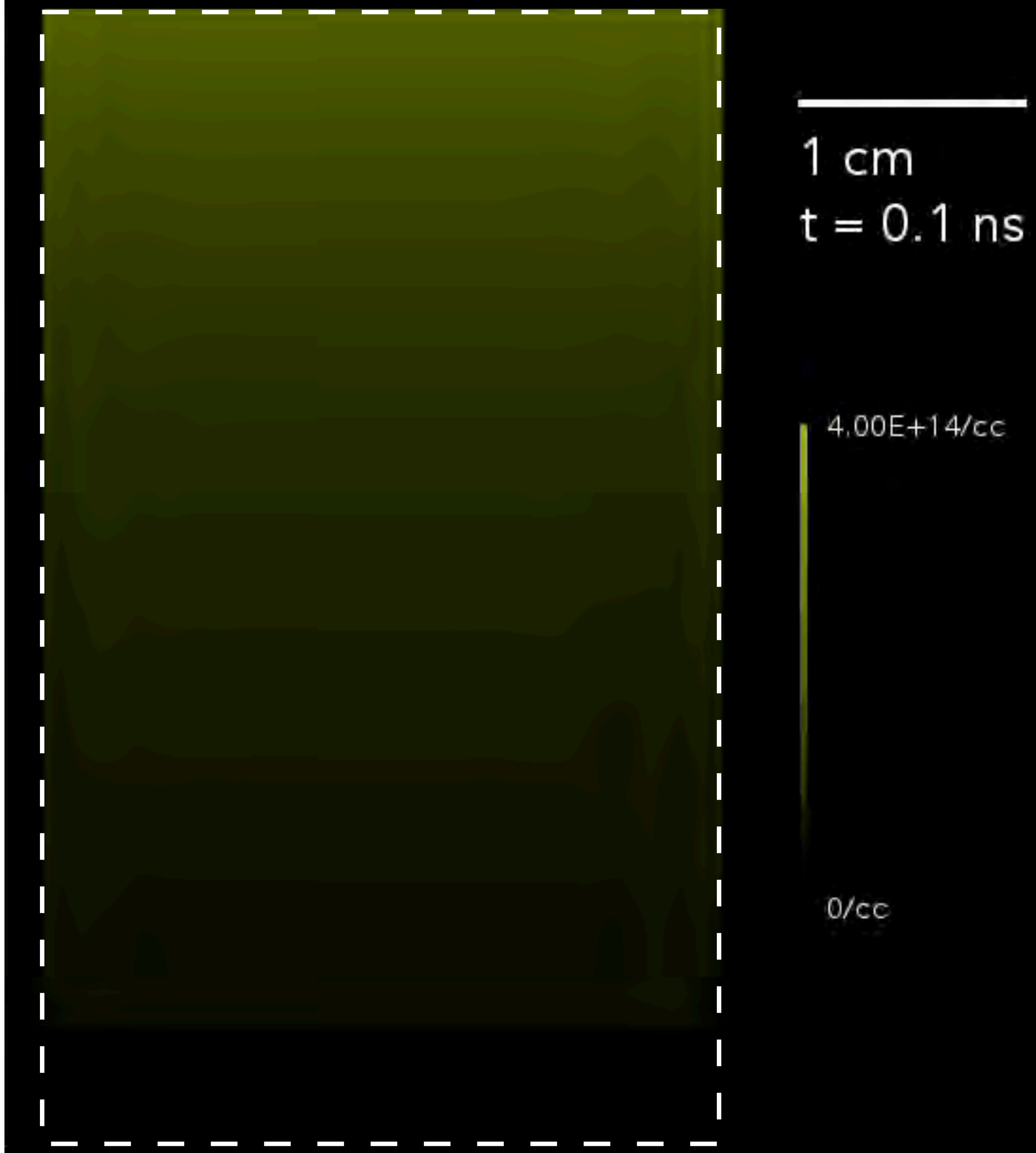


20% HE IONS SIMULATION: MOVIE

B Field

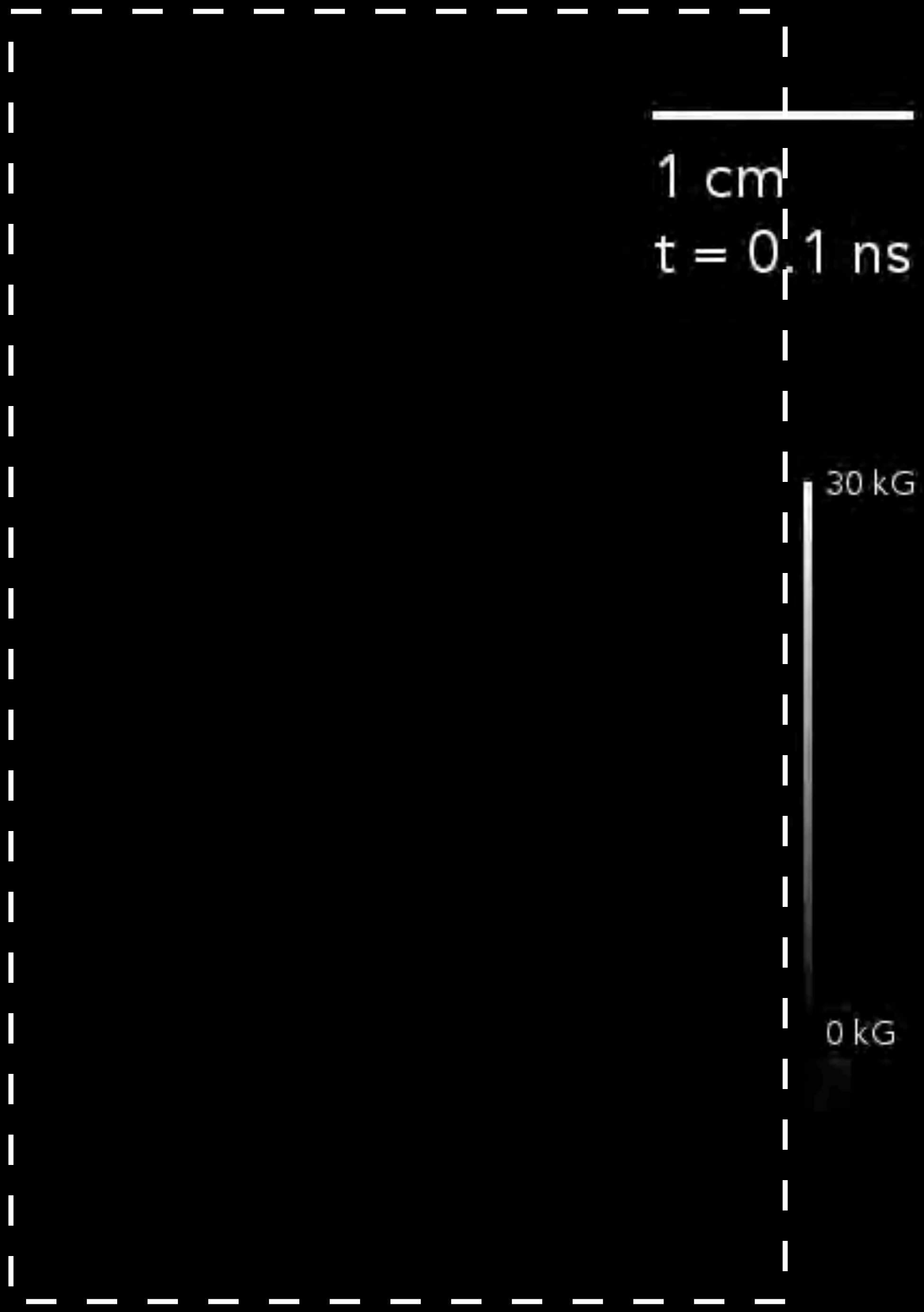


Light Ion Density

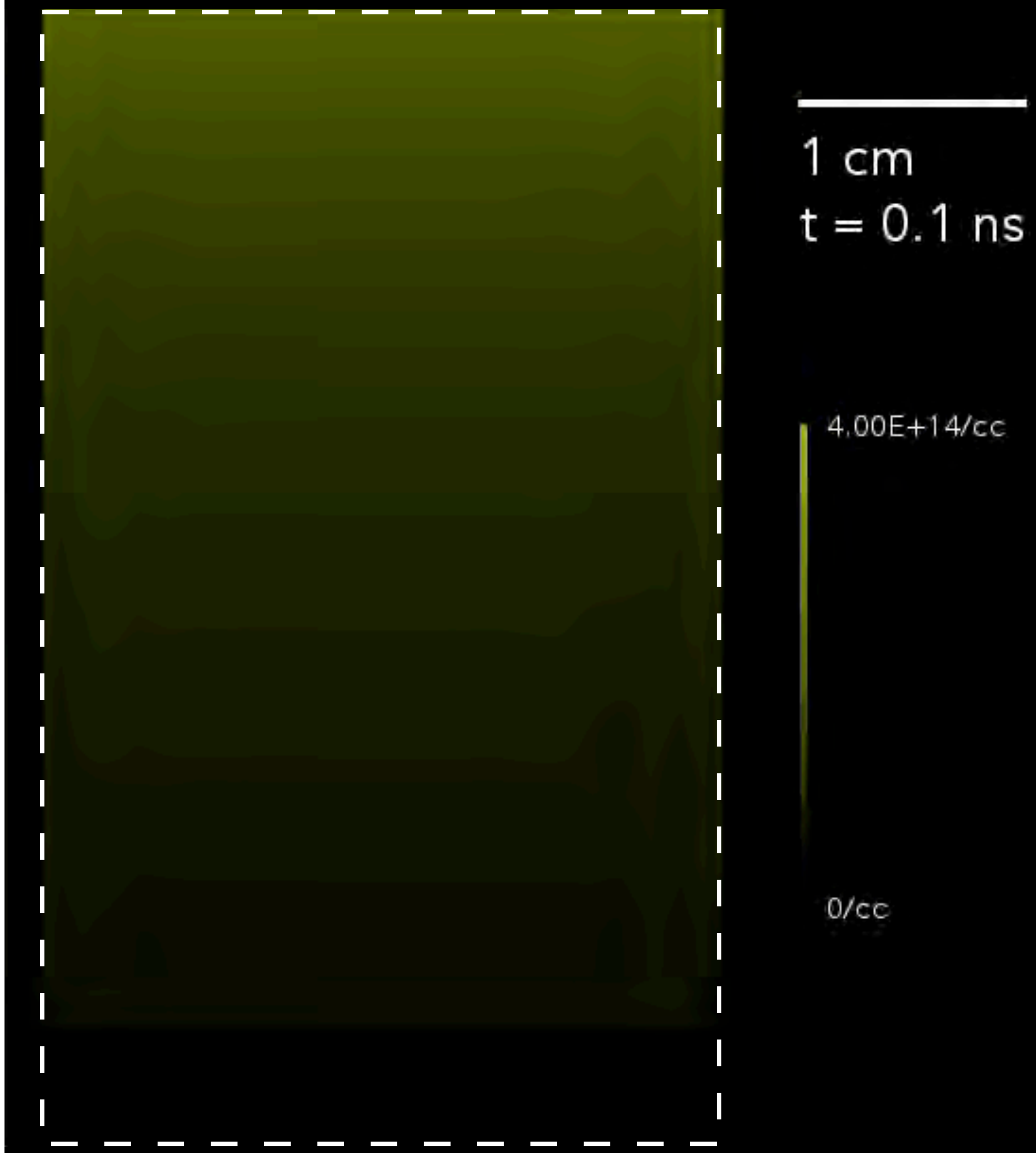


20% HE IONS SIMULATION: MOVIE

B Field



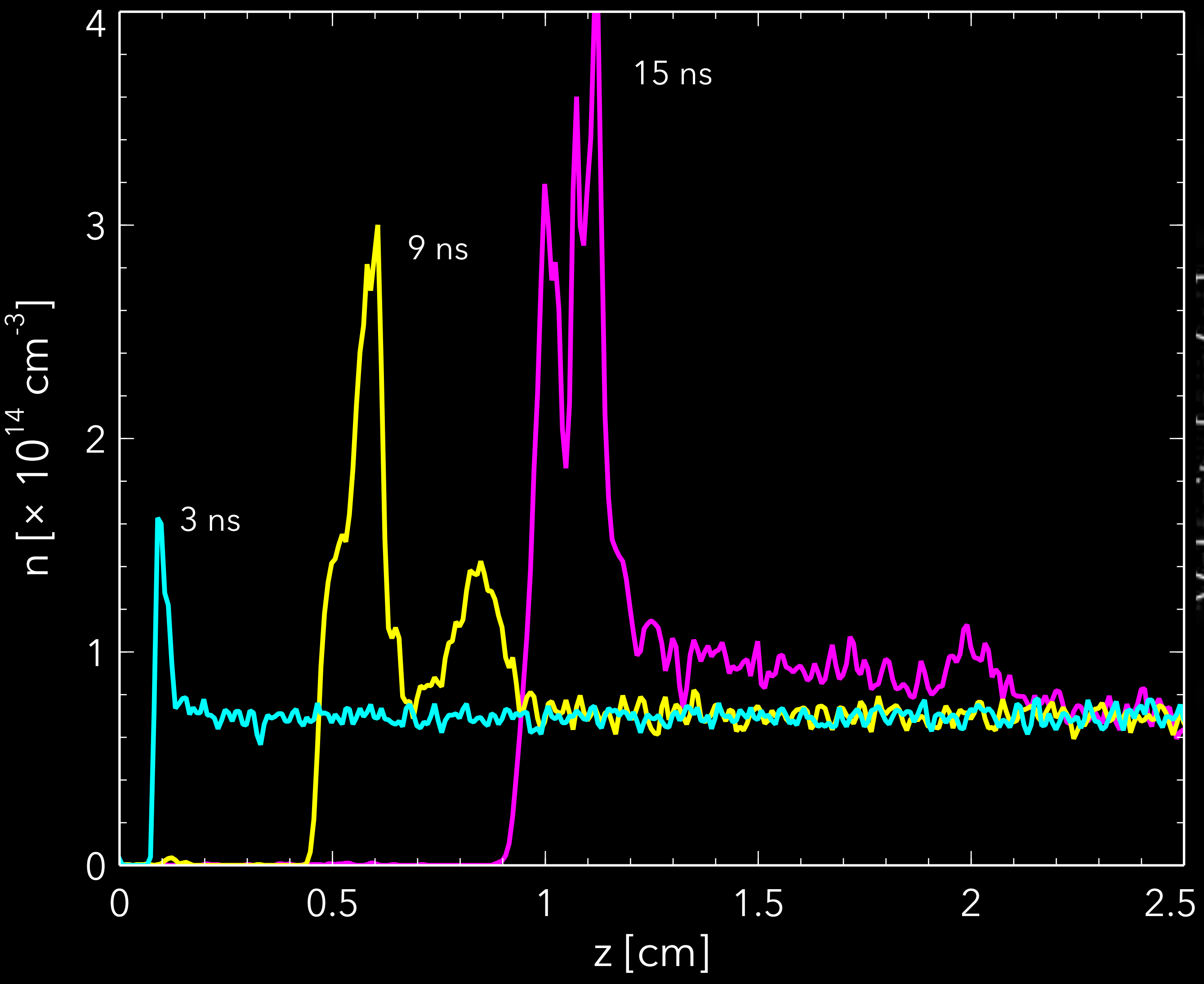
Light Ion Density



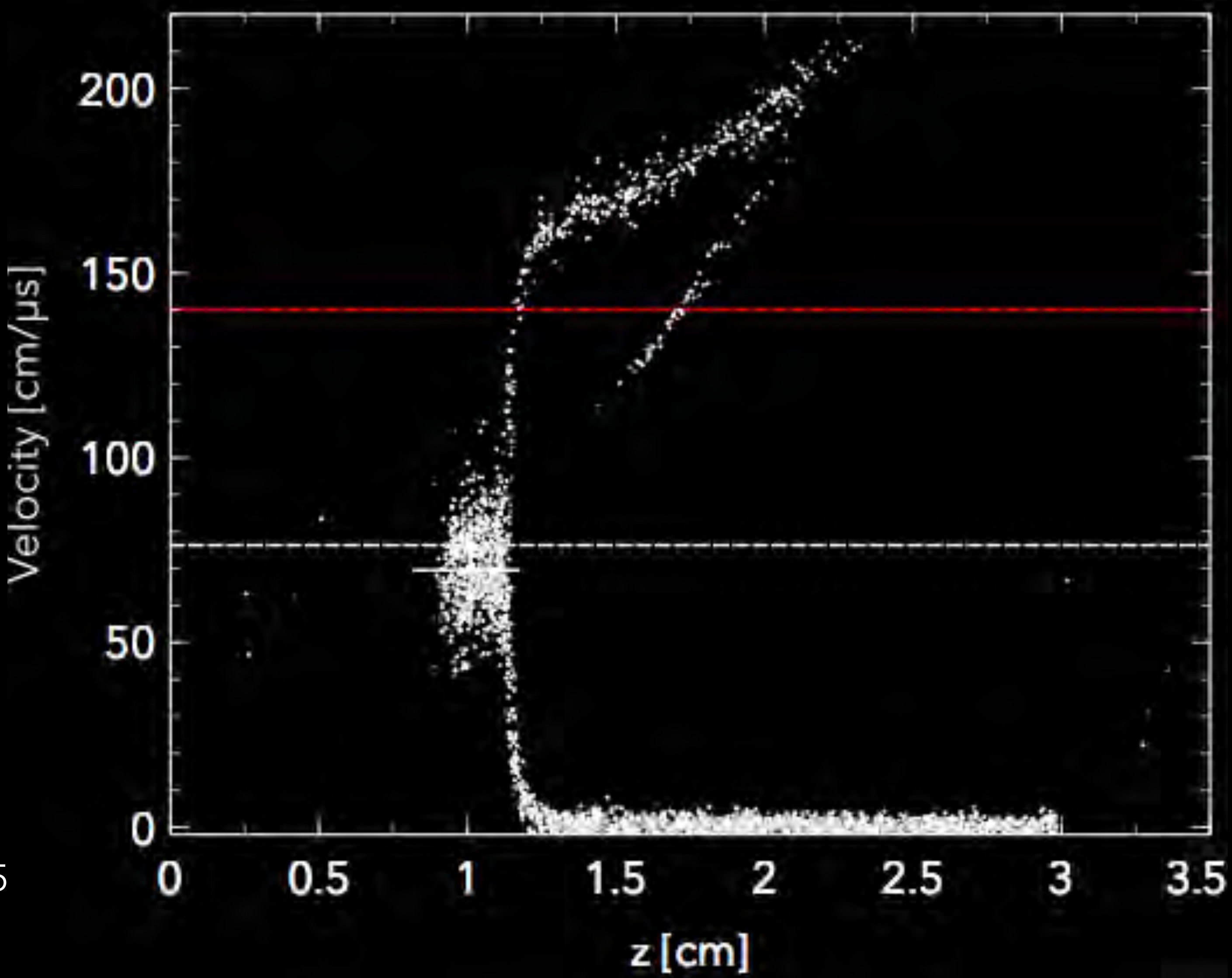
ION VELOCITY, X=2.4 TO 2.6 CM, 15 NS

20% HE IONS SIMULATION

Ion Density at x = 2.5 cm



Phase Space Particle Plot

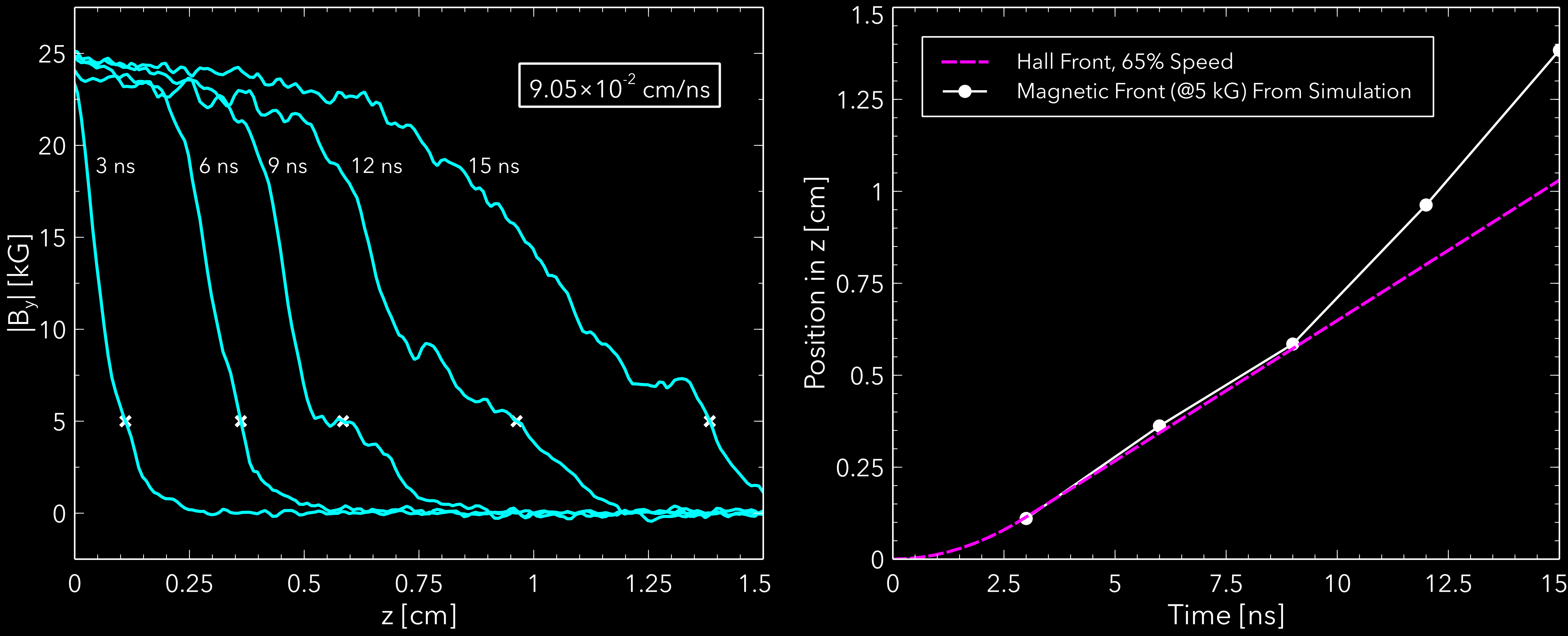


SIMULATION 5

80% FIXED IONS
20% H IONS

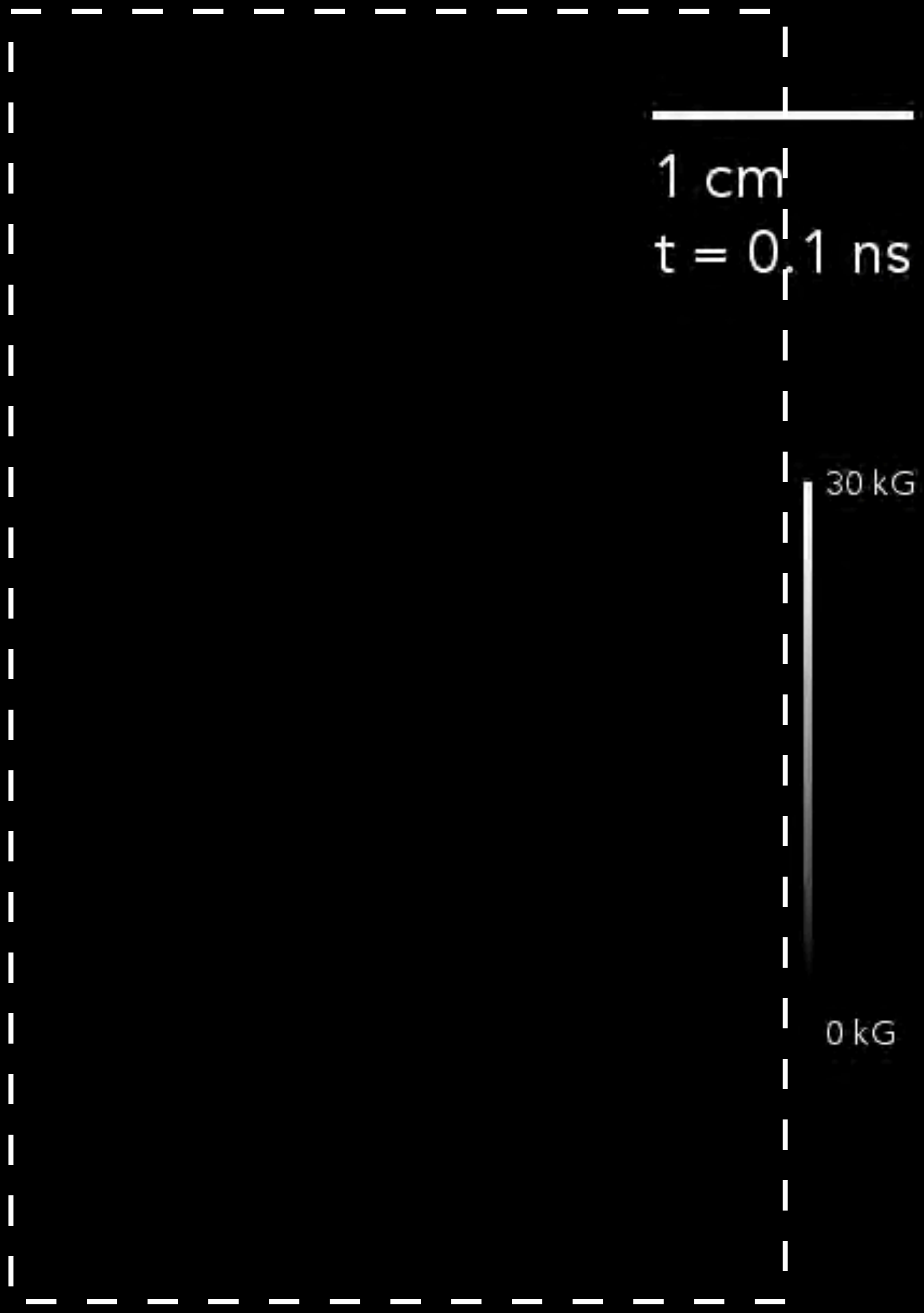
FIELD PENETRATION AT X=2.5 CM

20% H IONS SIMULATION

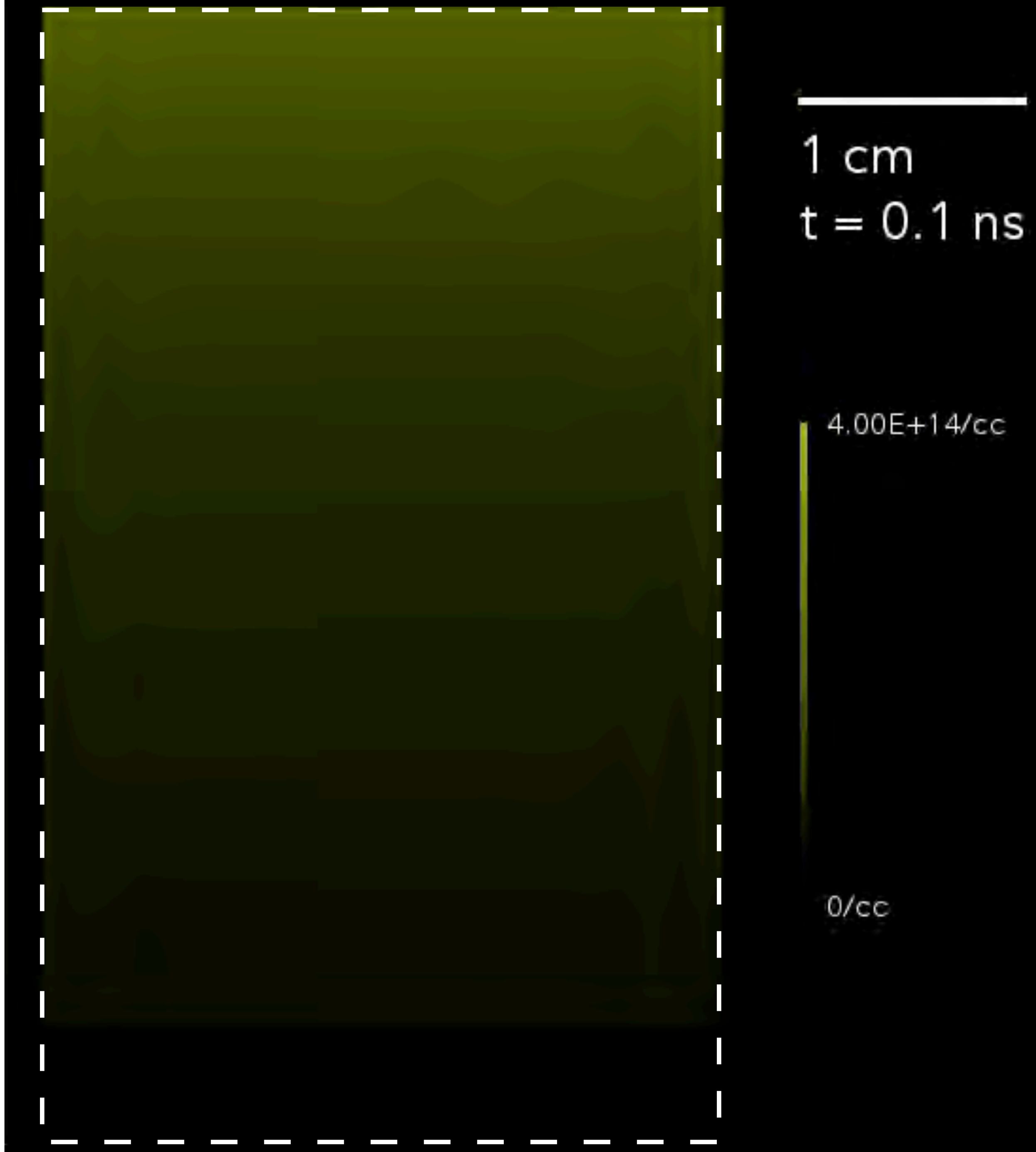


20% H IONS SIMULATION: MOVIE

B Field

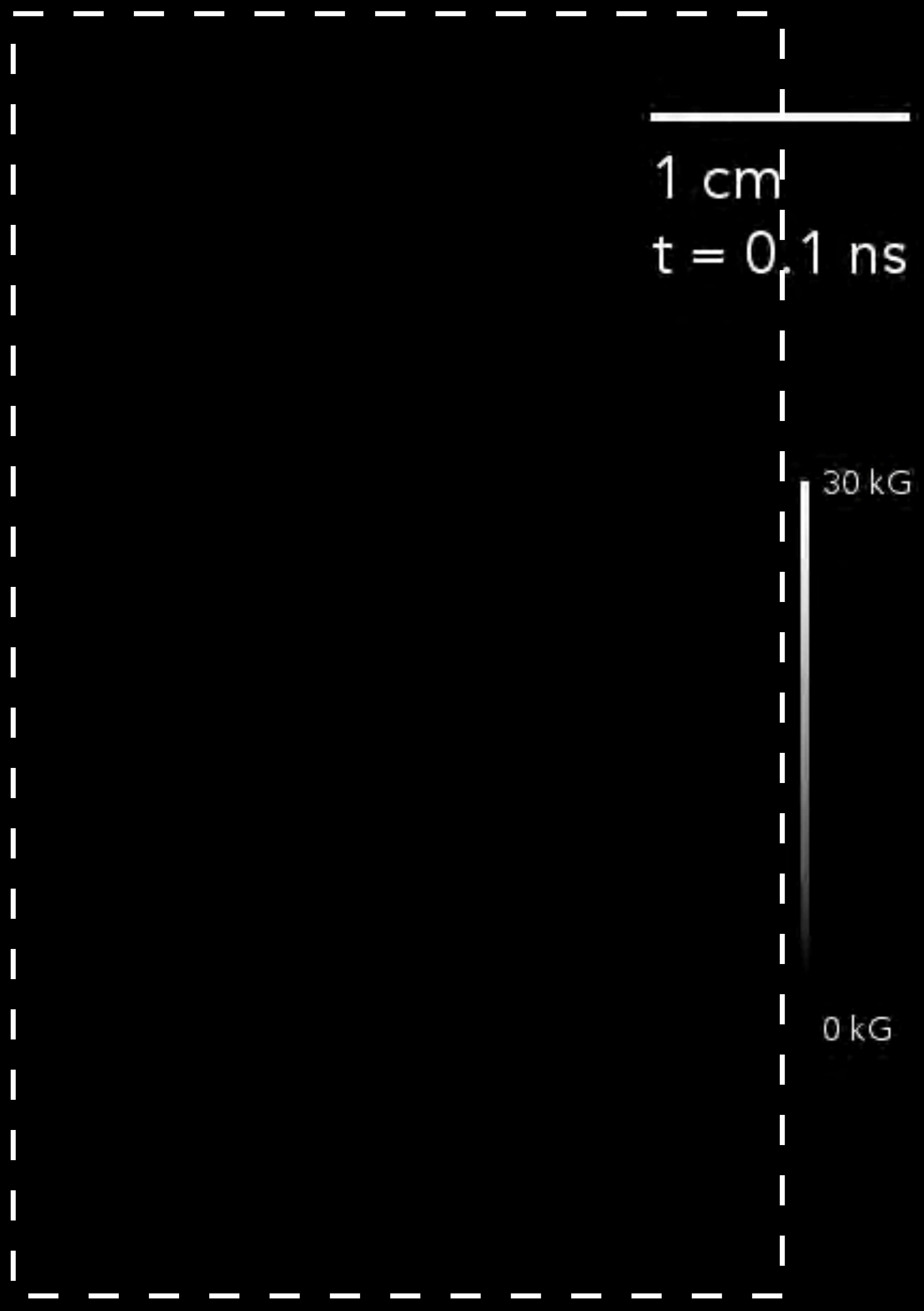


Light Ion Density

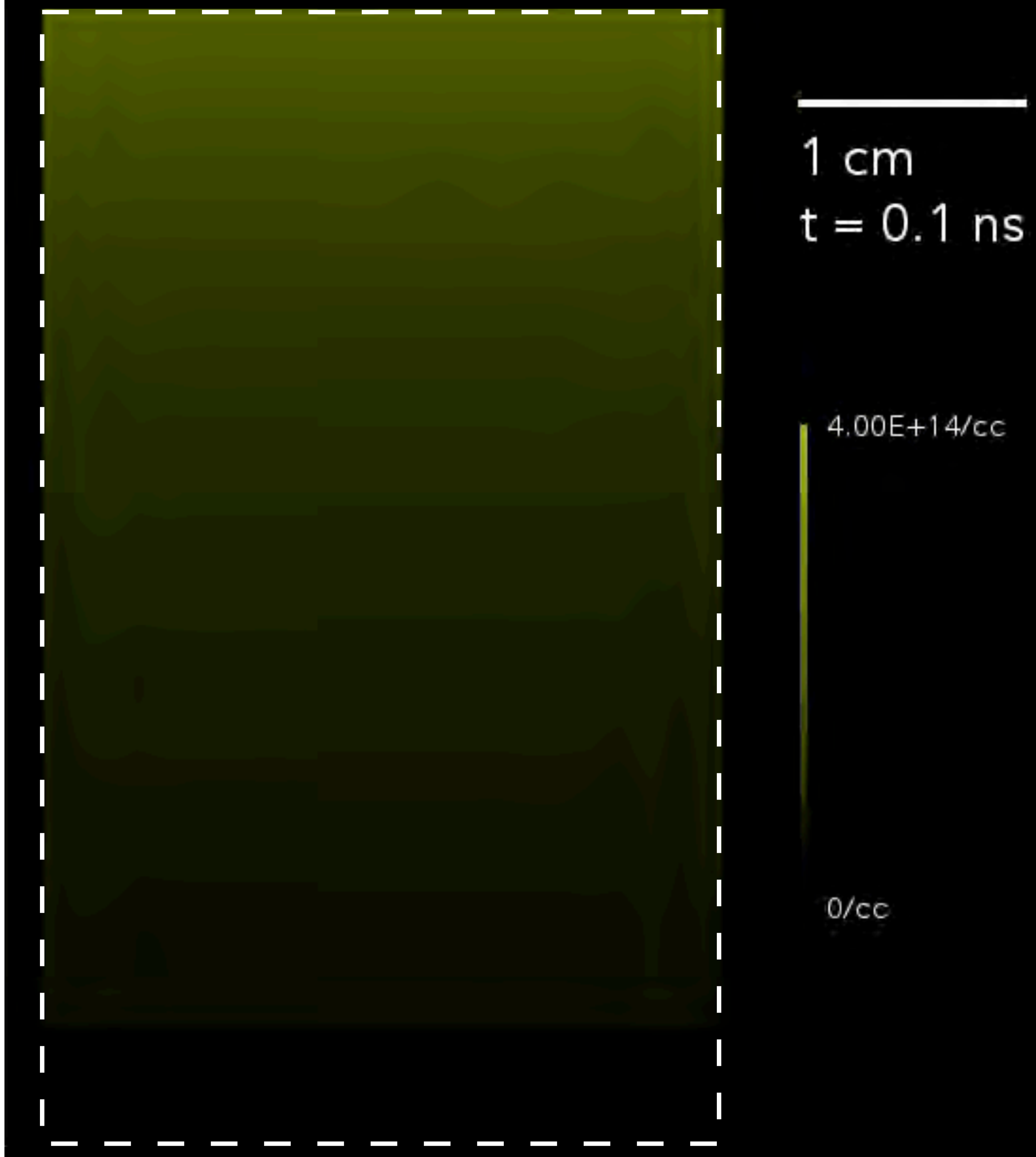


20% H IONS SIMULATION: MOVIE

B Field



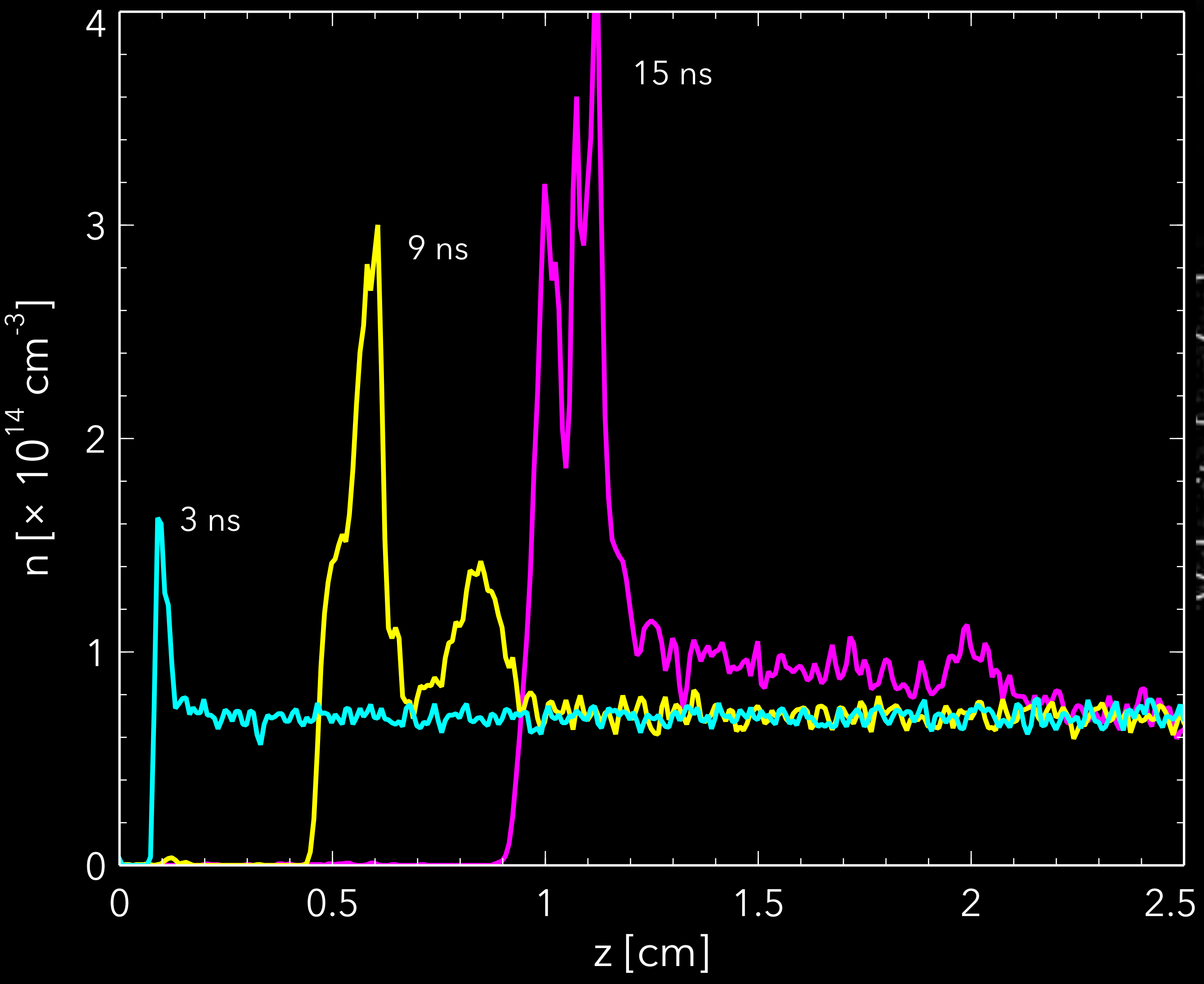
Light Ion Density



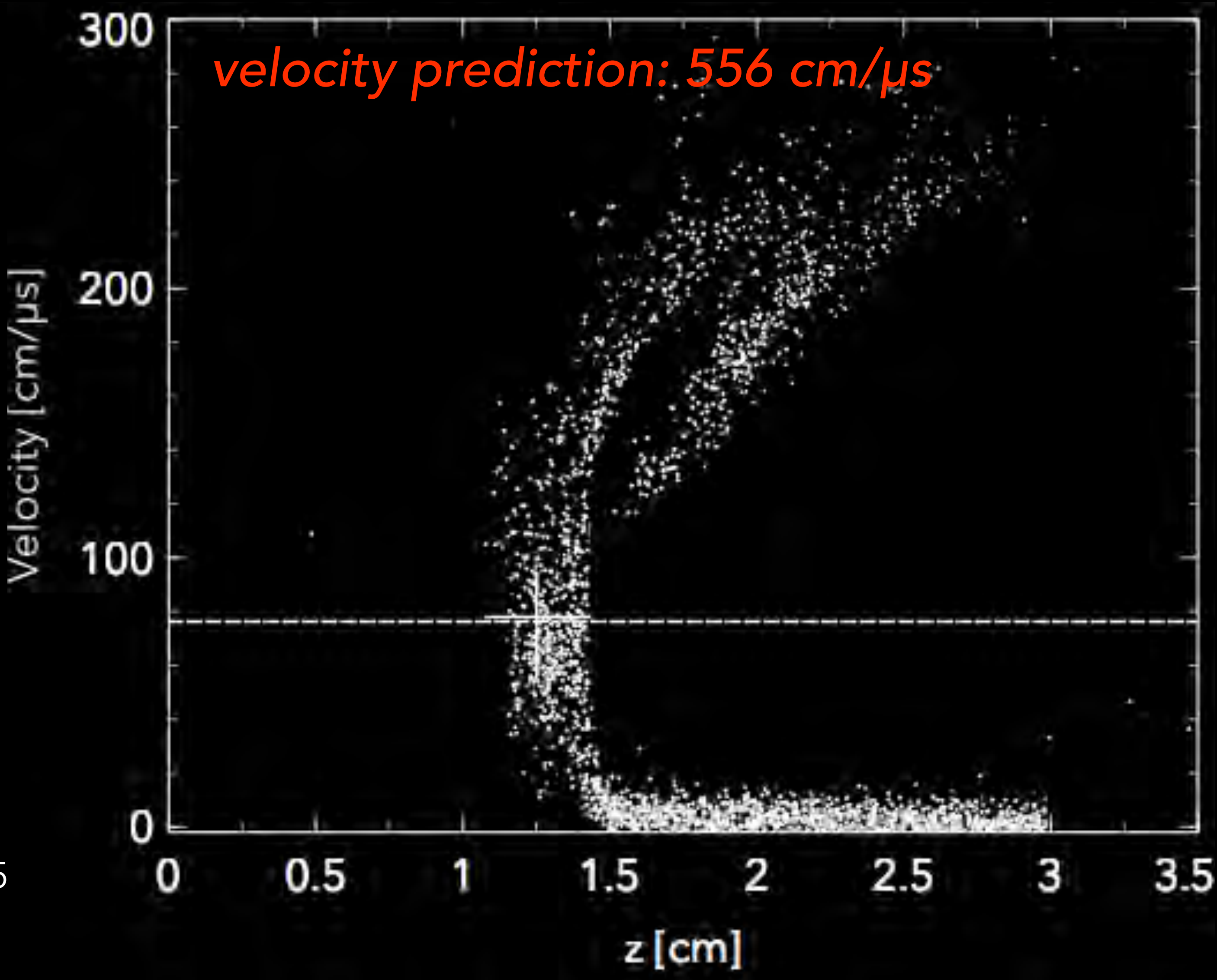
ION VELOCITY, X=2.4 TO 2.6 CM, 15 NS

20% HE IONS SIMULATION

Ion Density at x = 2.5 cm



Phase Space Particle Plot

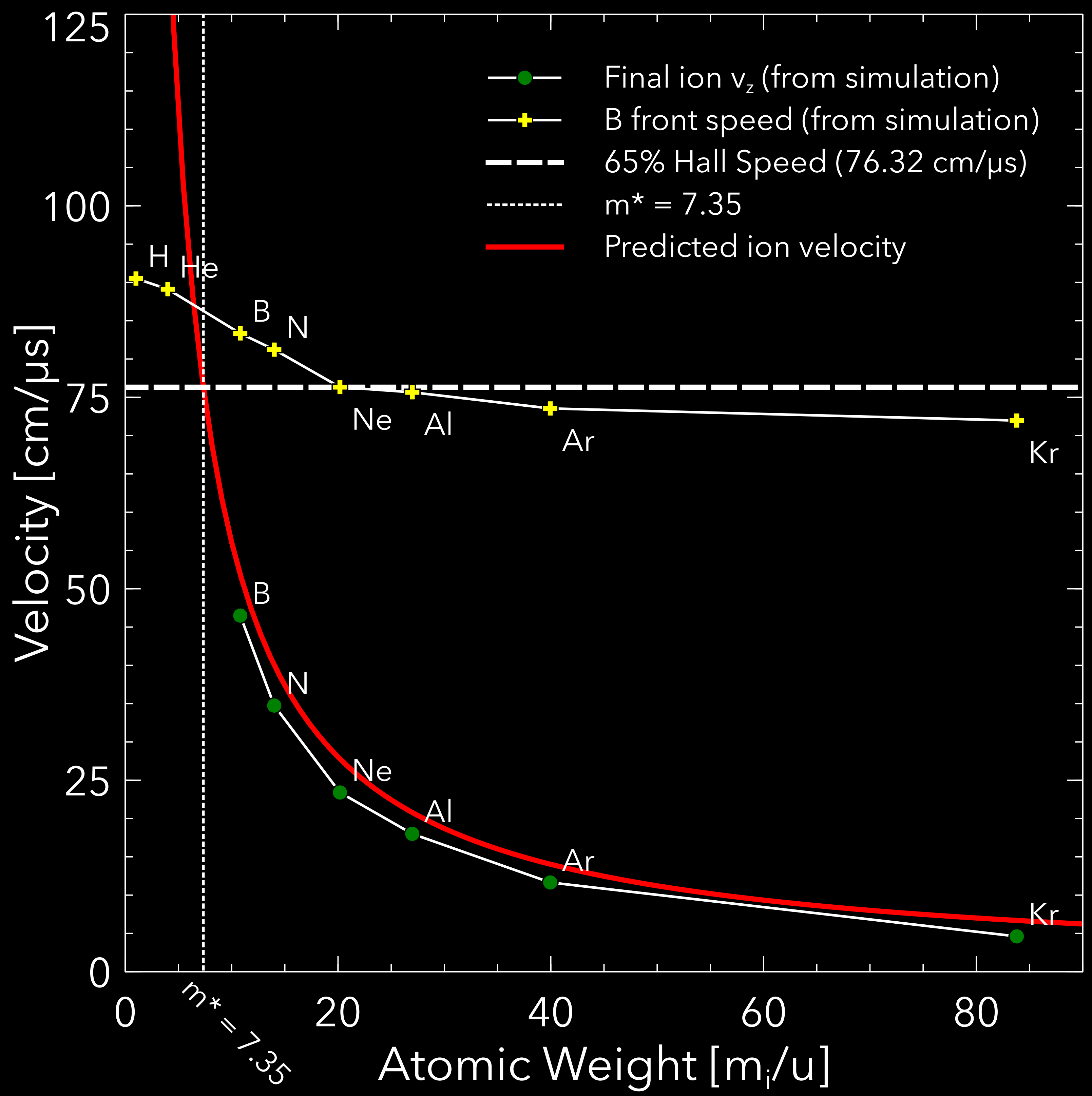


FINAL ION VELOCITY: SIMULATIONS

$$v_H = \frac{cB}{4\pi n e L_n}$$

$$v_b = \alpha v_H$$

$$v_{final} = \frac{B^2}{8\pi n m_i v_b}$$



SUMMARY

- Hall-driven magnetic penetration observed in single species plasma simulations
- Leaky piston behavior observed in multi-component plasma simulations, specifically:
  - Final ion velocity $v_{final,i}$ $=B^2/(8\pi n m_i v_b)$ scaling agrees with simulation results, with simulated velocity slightly less than predicted
  - Species separation observed in multi-component plasma simulations where $m_{light} < m^*$
  - Mass threshold $m^*=B^2/8\pi n v_b{}^2$ agrees with simulation results

QUESTIONS?